PEARSON

Math Makes Sense 7

Marc Garneau
John Pusic
Kanwal Neel
Sharon Jeroski
Susan Ludwig
Robert Sidley
Ralph Mason
Trevor Brown

With Contributions from

Cynthia Pratt Nicolson
Margaret Sinclair
Antonietta Lenjosek
Michael Davis
Elizabeth A. Wood
Daryl M.J. Chichak
Jason Johnston
Steve Thomas
Don Jones
Ken Harper
Mary Doucette
Bryn Keyes
Ralph Connelly
Nora Alexander

PEARSON Education Canada

Publisher
Claire Burnett

Publishing Team
Enid Haley
Lesley Haynes
Alison Rieger
Ioana Gagea
Lynne Gulliver
Stephanie Cox
Cheri Westra
Judy Wilson

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Math Team Leader
Diane Wyman

Product Manager
Kathleen Crosbie

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PEARSON
Education Canada

Consultants and Advisers

Consultants

Craig Featherstone
Mignonne Wood
Trevor Brown

Assessment Consultant
Sharon Jeroski

Elementary Mathematics Adviser
John A. Van de Walle

Advisers

Pearson Education thanks its Advisers, who helped shape the vision for *Pearson Math Makes Sense* through discussions and reviews of manuscript.

Joanne Adomeit
Bob Belcher
Bob Berglind
Auriana Burns
Steve Cairns
Edward Doolittle
Brenda Foster
Marc Garneau
Angie Harding
Florence Glanfield
Jodi Mackie
Ralph Mason
Christine Ottawa
Gretha Pallen
Shannon Sharp
Cheryl Shields
Gay Sul
Chris Van Bergeyk
Denise Vuignier

Reviewers

Field Test Teachers

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Aboriginal Content Reviewers

Glenda Bristow, First Nations, Métis and Inuit Coordinator,
St. Paul Education Regional Div. No. 1
Edward Doolittle, Assistant Professor of Mathematics, University of Regina
Patrick Loyer, Consultant, Aboriginal Education, Calgary Catholic

Grade 7 Reviewers

Angie Balkwill
Regina Public School Board, SK

Betty Barabash
Edmonton Catholic School District, AB

Lorraine M. Baron
School District 23 (Central Okanagan), BC

Warren Brownell
Moose Jaw School Division 1, SK

Laura Corsi
Edmonton Catholic School District, AB

Tricia L. Erlendson
Regina Separate School Division, SK

Kira Fladager
Regina Public School Board, SK

Daniel Gallays
Greater Saskatoon Catholic Schools, SK

Lise Hantelmann
School District 91 (Nechako Lakes), BC

Tammy L. Hartmann
Simon Fraser University, BC

Jacinthe Hodgson
Regina Public School Board, SK

Mary-Elizabeth Kaiser
Calgary Board of Education, AB

Geri Lorway
Consultant, AB

Rob Marshall
School District 22 (Vernon), BC

Sandra Maurer
Livingstone Range School Division No. 68, AB

Stephanie Miller
School District 41 (Burnaby), BC

Kanwal Neel
School District 38 (Richmond), BC

Jackie Ratkovic
Consultant, AB

Suzanne Vance
Moose Jaw School Division 1, SK

Randi-Lee Weninger
Greater Saskatoon Catholic School Division, SK

Michele Wiebe
School District 60 (Peace River North), BC

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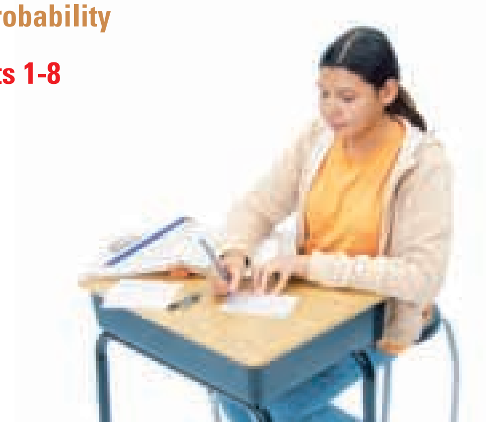
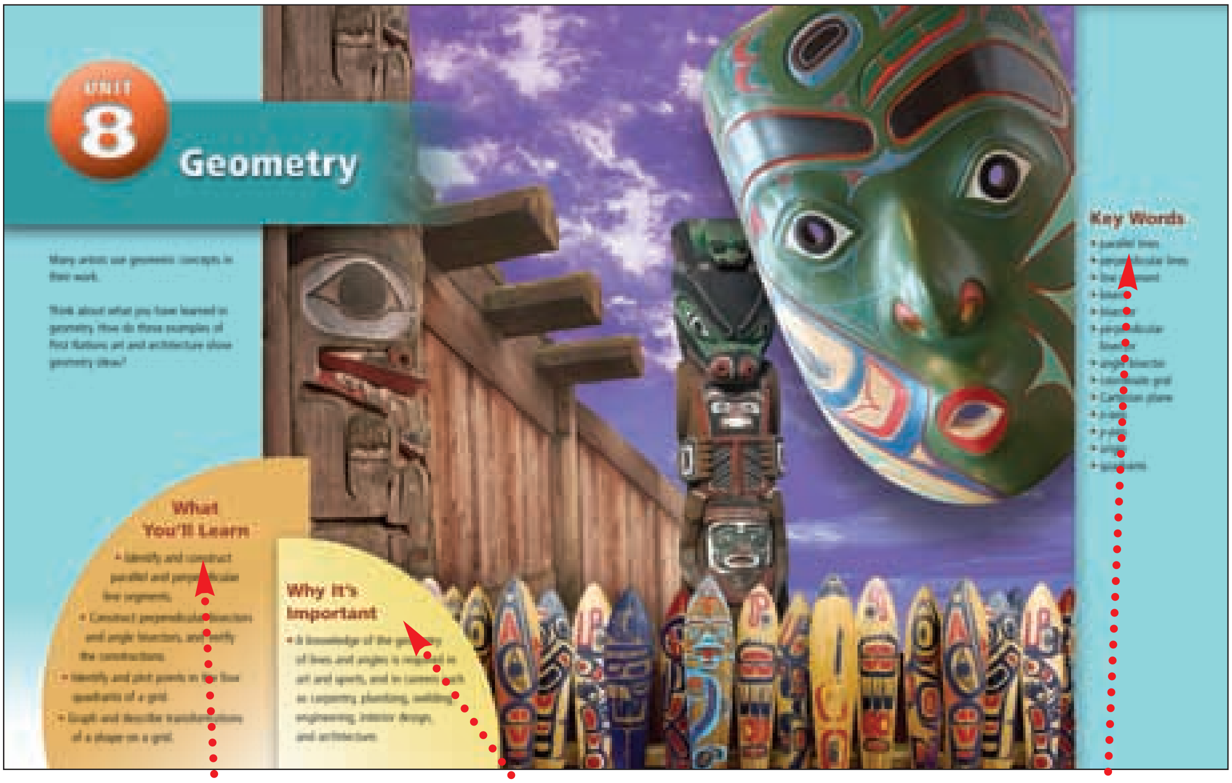


Welcome to
Pearson Math Makes Sense 7

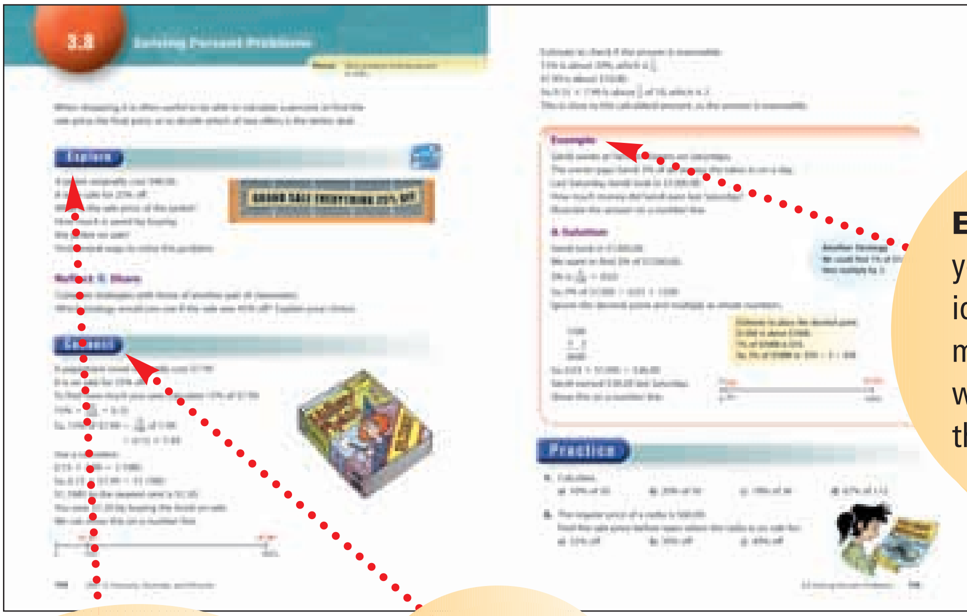
Math helps you understand your world.

This book will help you improve your problem-solving skills and show you how you can use your math now, and in your future career.

The opening pages of **each unit** are designed to help you prepare for success.



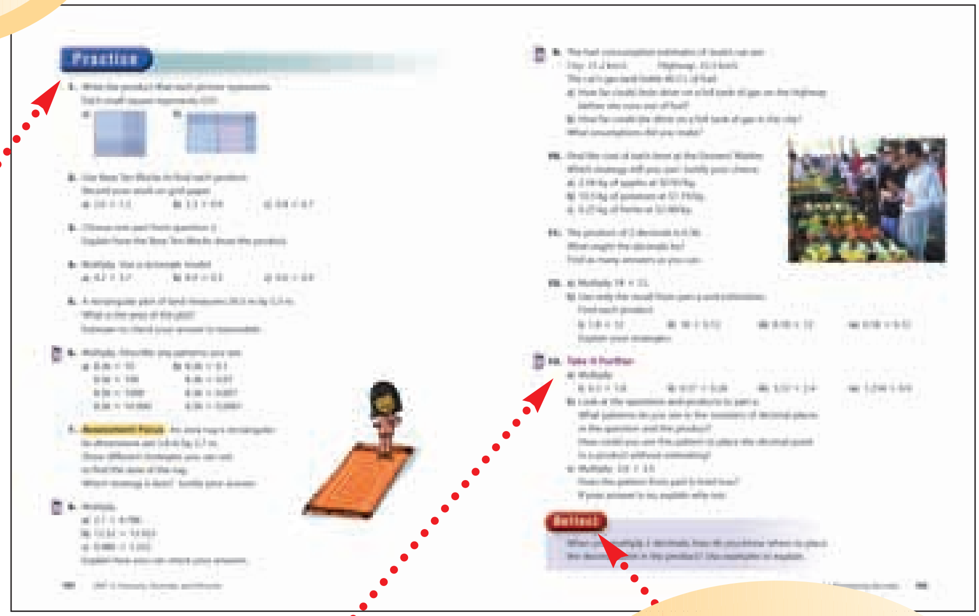
Find out **What You'll Learn** and **Why It's Important**. Check the list of **Key Words**.



Examples show you how to use the ideas and that there may be different ways to approach the question.

Explore an idea or problem, usually with a partner, and often using materials.

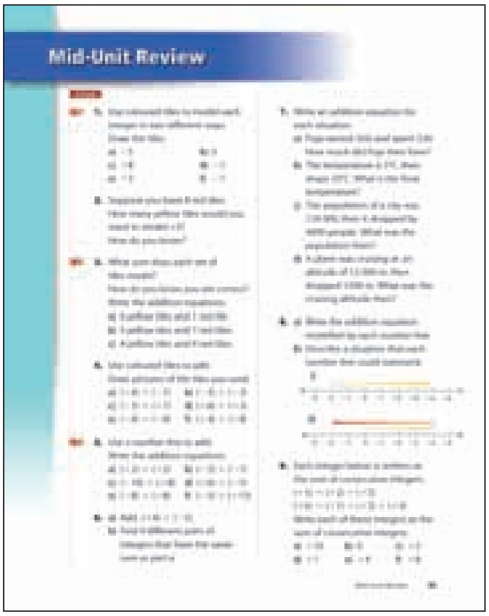
Connect summarizes the math.



Practice questions reinforce the math.

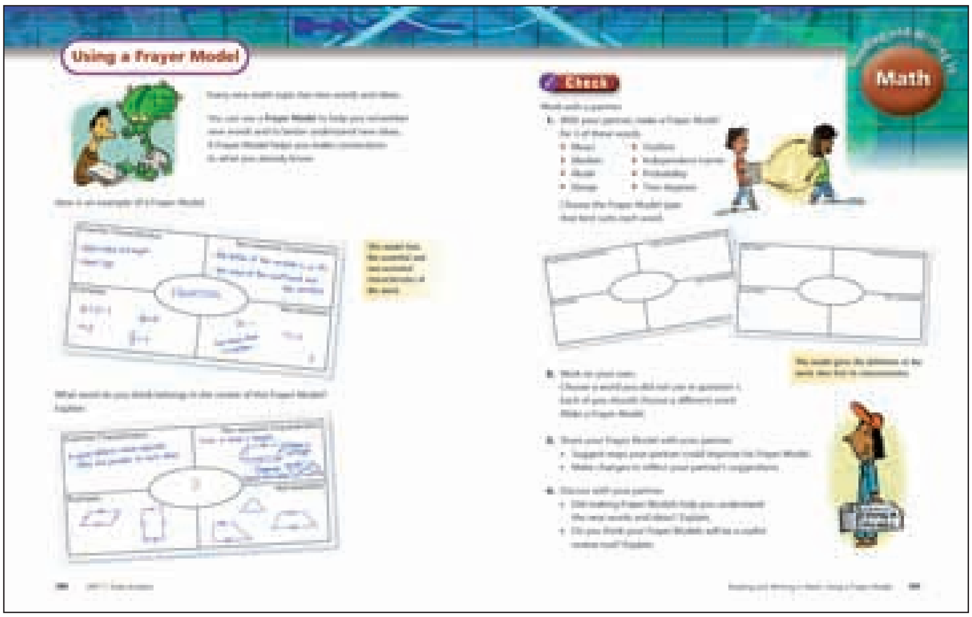
Take It Further questions offer enrichment and extension.

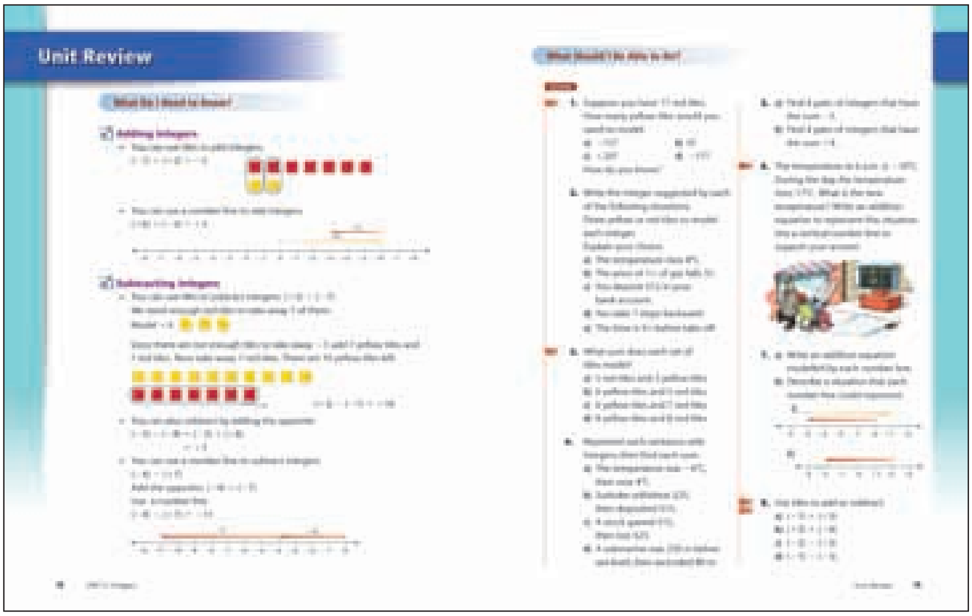
Reflect on the big ideas of the lesson. Think about your learning style and strategies.



Use the **Mid-Unit Review** to refresh your memory of key concepts.

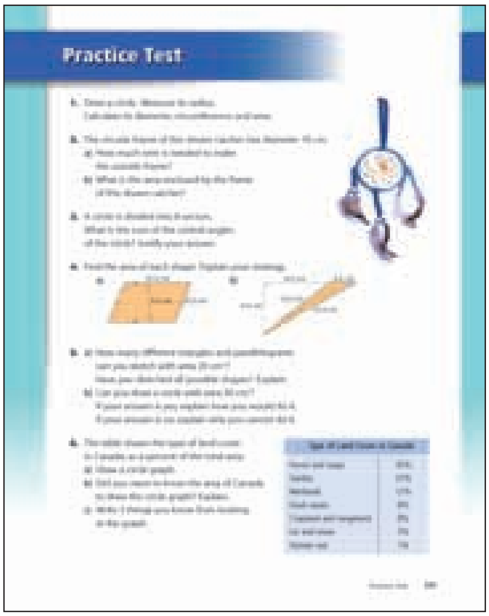
Reading and Writing in Math helps you understand how reading and writing about math differs from other language skills you use. It may suggest ways to help you study.





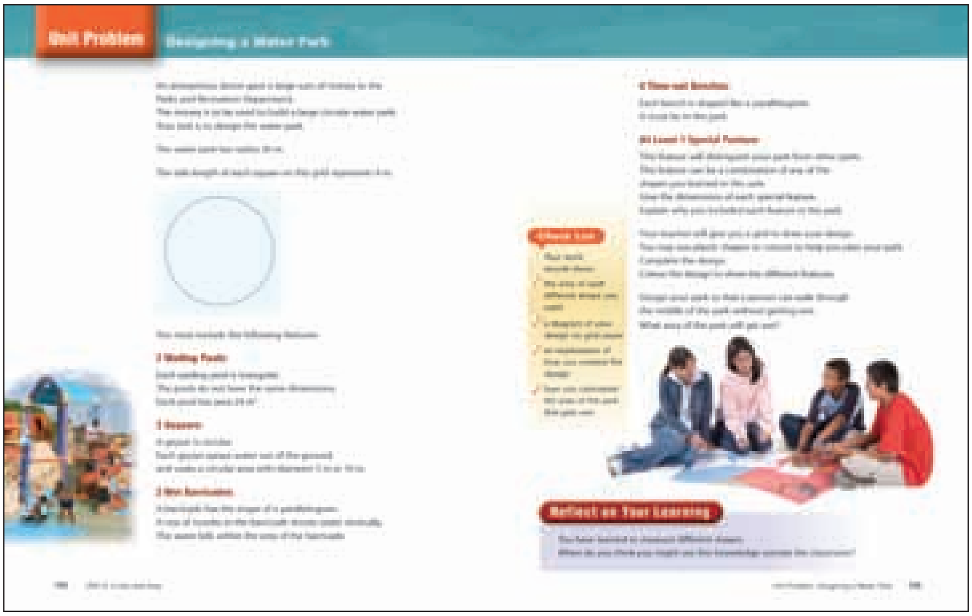
What Do I Need to Know? summarizes key ideas from the unit.

What Should I Be Able to Do? allows you to find out if you are ready to move on. The Practice and Homework book provides additional support.



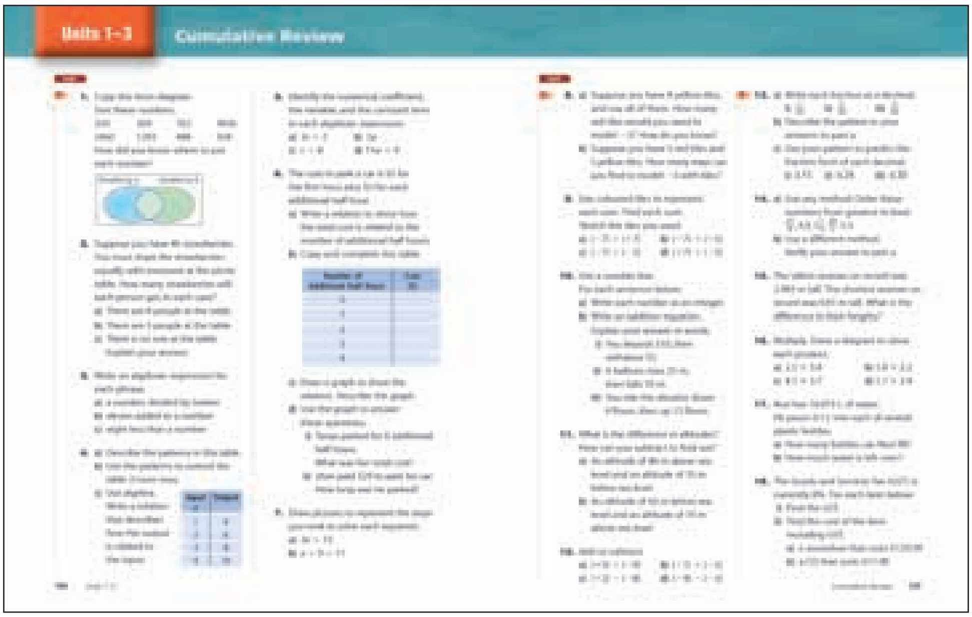
The **Practice Test** models the kind of test your teacher might give.

The **Unit Problem** presents problems to solve, or a project to do, using the math of the unit.

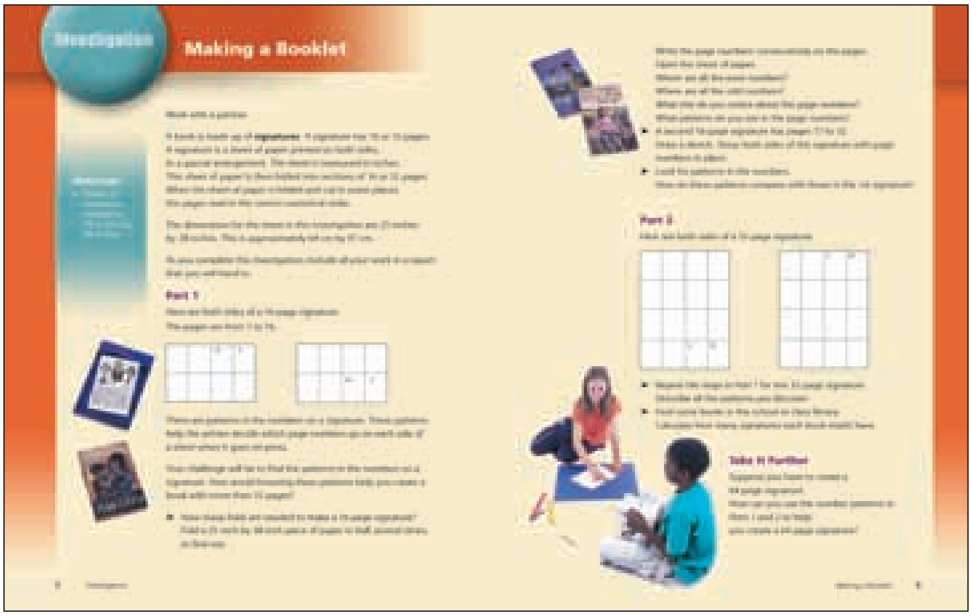

Unit Problem Designing a Water Park

Reflect on Your Learning

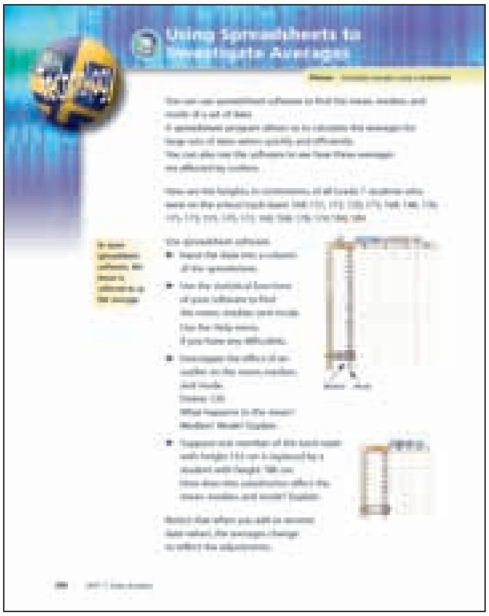
Keep your skills sharp with **Cumulative Review.**

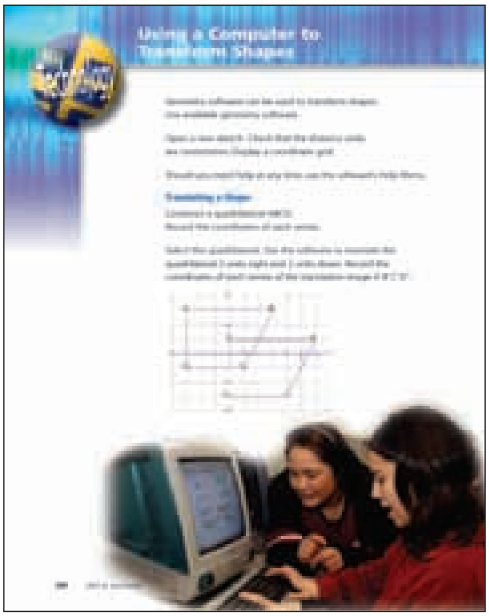

Units 1–3 Cumulative Review

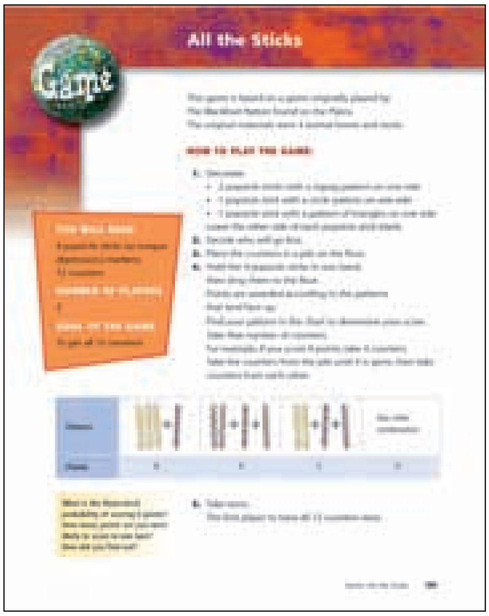
Explore some interesting math when you do the **Investigations**.



Icons remind you to use **technology**.
Follow the instructions for using a computer or calculator to do math.

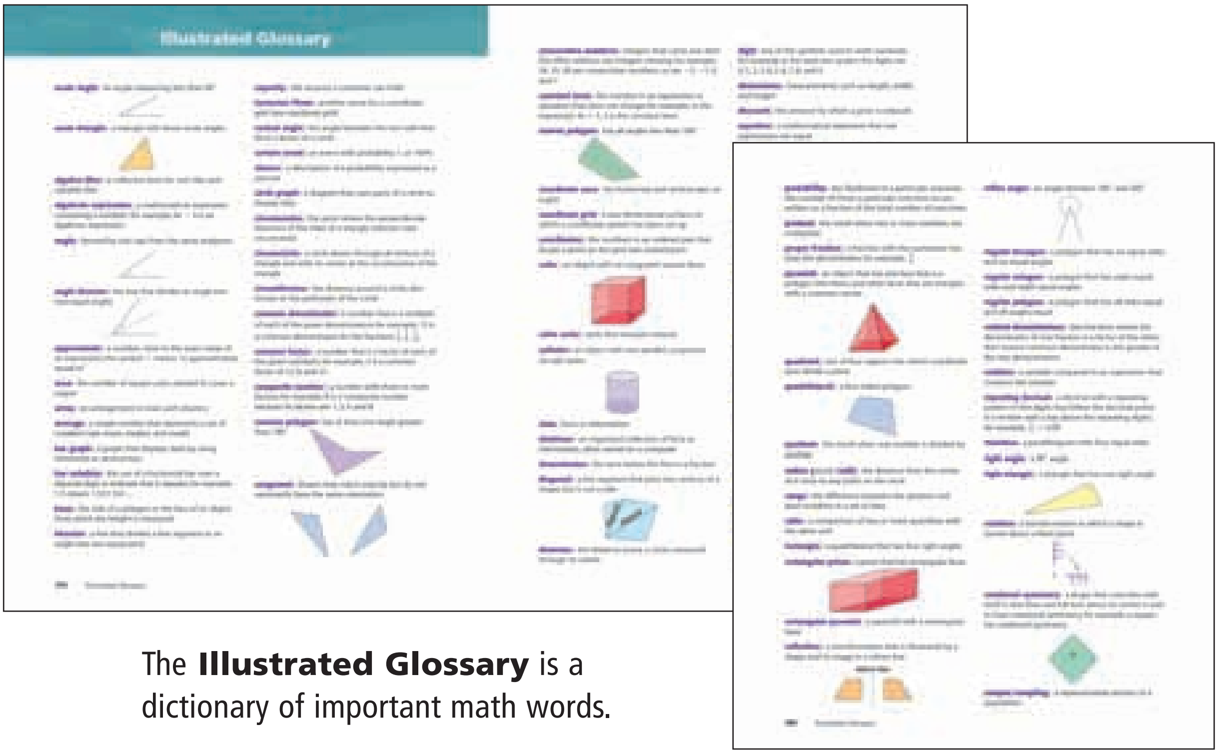






Play a **Game** with your classmates or at home to reinforce your skills.

The World of Work describes how people use mathematics in their careers.



The **Illustrated Glossary** is a dictionary of important math words.

Investigation

Making a Booklet

Materials:

- sheets of newsprint measuring 25 inches by 38 inches

Work with a partner.

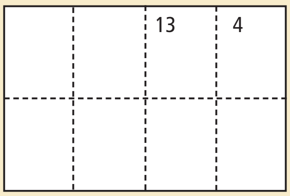
A book is made up of **signatures**. A signature has 16 or 32 pages. A signature is a sheet of paper printed on both sides, in a special arrangement. The sheet is measured in inches. This sheet of paper is then folded into sections of 16 or 32 pages. When the sheet of paper is folded and cut in some places, the pages read in the correct numerical order.

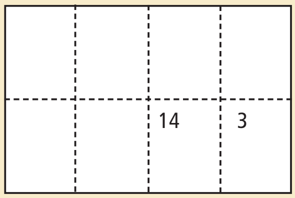
The dimensions for the sheet in this *Investigation* are 25 inches by 38 inches. This is approximately 64 cm by 97 cm.

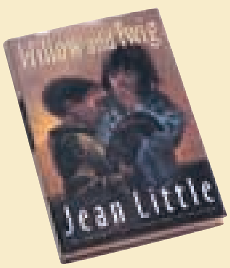
As you complete this *Investigation*, include all your work in a report that you will hand in.

Part 1

Here are both sides of a 16-page signature.
The pages are from 1 to 16.







There are patterns in the numbers on a signature. These patterns help the printer decide which page numbers go on each side of a sheet when it goes on press.

Your challenge will be to find the patterns in the numbers on a signature. How would knowing these patterns help you create a book with more than 32 pages?

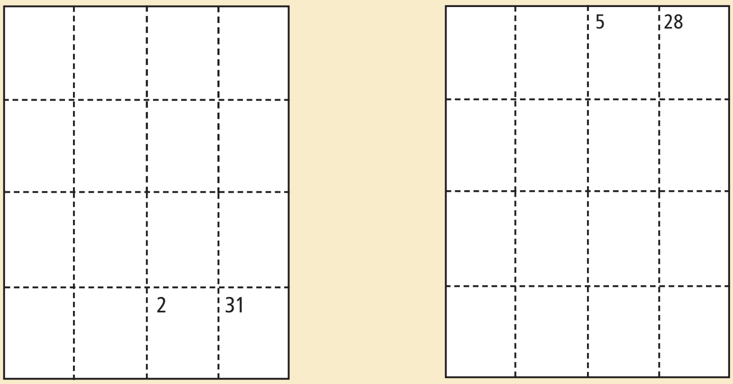
➤ How many folds are needed to make a 16-page signature? Fold a 25-inch by 38-inch piece of paper in half, several times, to find out.

Write the page numbers consecutively on the pages.
Open the sheet of paper.
Where are all the even numbers?
Where are all the odd numbers?
What else do you notice about the page numbers?
What patterns do you see in the page numbers?

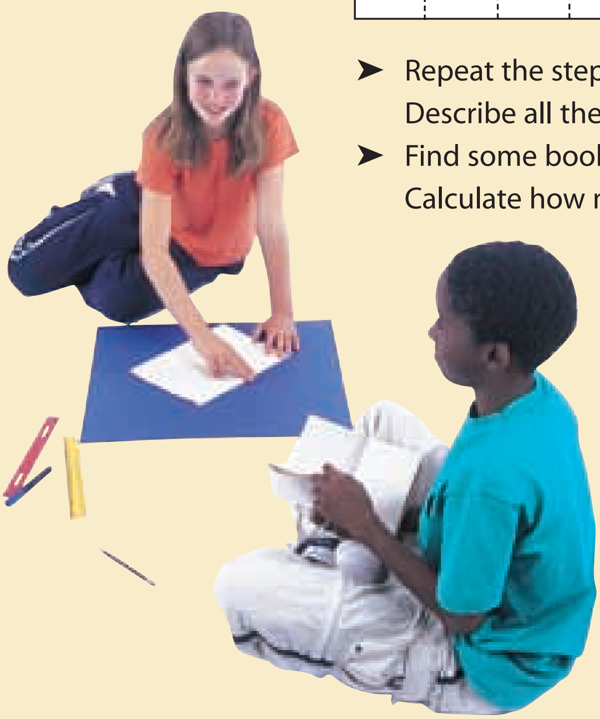
- ➤ A second 16-page signature has pages 17 to 32.
Draw a sketch. Show both sides of this signature with page numbers in place.
- ➤ Look for patterns in the numbers.
How do these patterns compare with those in the 1st signature?

Part 2

Here are both sides of a 32-page signature.



- ➤ Repeat the steps in *Part 1* for this 32-page signature.
Describe all the patterns you discover.
- ➤ Find some books in the school or class library.
Calculate how many signatures each book might have.



Take It Further

Suppose you have to create a 64-page signature.
How can you use the number patterns in *Parts 1* and *2* to help you create a 64-page signature?

UNIT 1 Patterns and Relations

Students in a Grade 7 class were raising money for charity.
Some students had a "bowl-a-thon."

This table shows the money that one student raised for different bowling times.

Time (h)	Money Raised ($)
1	8
2	16
3	24
4	32
5	40
6	48

- What patterns do you see in the table?
- Extend the table.
  For how long would the student have to bowl to raise $72?

What You'll Learn

- Use patterns to explore divisibility rules.
- Translate between patterns and equivalent linear relations.
- Evaluate algebraic expressions by substitution.
- Represent linear relations in tables and graphs.
- Solve simple equations, then verify the solutions.

Why It's Important

- Divisibility rules help us find the factors of a number.
- Graphs provide information and are a useful problem-solving tool.
- Efficient ways to represent a pattern can help us describe and solve problems.

Key Words

- divisibility rules
- algebraic expression
- numerical coefficient
- constant term
- relation
- linear relation
- unit tile
- variable tile
- algebra tiles

1.1 Patterns in Division

Focus Explore divisibility by 2, 4, 5, 8, and 10.

Which of these numbers are divisible by 2? By 5? By 10?
How do you know?

- 78
- 27
- 35
- 410
- 123
- 2100
- 4126
- 795

Explore

You will need a hundred chart numbered 301–400, and three different coloured markers.

➤ Use a marker. Circle all numbers on the hundred chart that are divisible by 2.
Use a different marker.
Circle all numbers that are divisible by 4.
Use a different marker.
Circle all numbers that are divisible by 8.
Describe the patterns you see in the numbers you circled.

➤ Choose 3 numbers greater than 400.
Which of your numbers do you think are divisible by 2? By 4? By 8?
Why do you think so?

Reflect & Share

Share your work with another pair of classmates.
Suppose a number is divisible by 8.
What else can you say about the number?
Suppose a number is divisible by 4.
What else can you say about the number?

Connect

We know that 100 is divisible by 4: $100 \div 4 = 25$
So, any multiple of 100 is divisible by 4.
To find out if any whole number with 3 or more digits is divisible by 4, we only need to check the last 2 digits.

To find out if 352 is divisible by 4, check if 52 is divisible by 4.
$52 \div 4 = 13$
52 is divisible by 4, so 352 is divisible by 4.

To check if a number, such as 1192, is divisible by 8,
think: $1192 = 1000 + 192$
We know 1000 is divisible by 8: $1000 \div 8 = 125$
So, we only need to check if 192 is divisible by 8.
Use mental math. $192 \div 8 = 24$
192 is divisible by 8, so 1192 is divisible by 8.

Another way to check if a number is divisible by 8 is to divide by 4. If the quotient is even, then the number is divisible by 8.

All multiples of 1000 are divisible by 8.
So, for any whole number with 4 or more digits, we only need to check the last 3 digits to find out if the number is divisible by 8.

A number that is divisible by 8 is also divisible by 2 and by 4 because $8 = 2 \times 4$.
So, a number divisible by 8 is even.

2 and 4 are factors of 8.

You can use patterns to find **divisibility rules** for other numbers.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

➤ All multiples of 10, such as 30, 70, and 260, end in 0.

Any number whose ones digit is 0, is divisible by 10.

➤ Here are some multiples of 5.
5, 10, 15, 20, 25, 30, 35, 40, …, 150, 155, 160, …
The ones digits form a repeating pattern.
The core of the pattern is: 5, 0

Any number whose ones digit is 0 or 5, is divisible by 5.

Every multiple of 5 has a ones digit of 0 or 5.

➤ Multiples of 2 are even numbers: 2, 4, 6, 8, 10, …
All even numbers are divisible by 2.

Any number whose ones digit is even, is divisible by 2.

Example

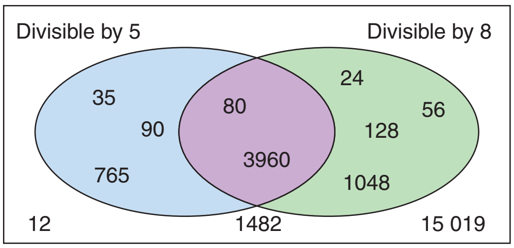
Which numbers are divisible by 5? By 8? Both by 5 and by 8?
How do you know?
12, 24, 35, 56, 80, 90, 128, 765, 1048, 1482, 3960, 15 019

A Solution

Any number with 0 or 5 in the ones place is divisible by 5.
So, the numbers divisible by 5 are: 35, 80, 90, 765, 3960

The divisibility rule for 8 only applies when a number is 1000 or greater.
For numbers less than 1000, use mental math or a calculator.
All multiples of 8 are even, so reject 35, 765, and 15 019.
Use mental math to identify that 12 and 90 are not divisible by 8.
Use mental math to identify that 24, 56, 80, and 128 are divisible by 8.
1048 and 3960 are divisible by 8 because 48 and 960 are divisible by 8.
1482 is not divisible by 8 because 482 is not divisible by 8.

We can display the results in a Venn diagram.



The numbers in the overlapping region are divisible both by 5 and by 8.
So, 80 and 3960 are also divisible by 40, since $5 \times 8 = 40$.

Practice

1. Which numbers are divisible by 2? By 5?
How do you know?

a) 106 **b)** 465 **c)** 2198
d) 215 **e)** 1399 **f)** 4530

2. Explain why a number with 0 in the ones place is divisible by 10.

3. Which numbers are divisible by 4? By 8? By 10?
How do you know?

a) 212 b) 512 c) 5450
d) 380 e) 2132 f) 12 256

4. Maxine and Tony discuss divisibility.
Maxine says, "260 is divisible by 4 and by 5.
$4 \times 5 = 20$, so 260 is also divisible by 20."
Tony says, "148 is divisible by 2 and by 4.
$2 \times 4 = 8$, so 148 is also divisible by 8."
Are both Maxine and Tony correct? Explain your thinking.

5. Write 3 numbers that are divisible by 8.
How did you choose the numbers?

6. **Assessment Focus**

a) Use the divisibility rules for 2, 4, and 8 to sort these numbers.

1046	322	460	1784	28
54	1088	224	382	3662

b) Draw a Venn diagram with 3 loops.
Label the loops: "Divisible by 2," "Divisible by 4," and "Divisible by 8"
Explain why you drew the loops the way you did.
Place the numbers in part a in the Venn diagram.
How did you decide where to place each number?

c) Find and insert 3 more 4-digit numbers in the Venn diagram.

7. Use the digits 0 to 9. Replace the □ in each number to make a number divisible by 4. Find as many answers as you can.

a) 822□ b) 211 4□8 c) 15 □32

8. **Take It Further** A leap year occurs every 4 years.
The years 1992 and 2004 were leap years.
What do you notice about these numbers?
Was 1964 a leap year? 1852? 1788? Explain.

Reflect

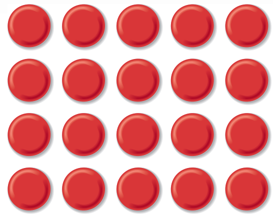
Compare the divisibility rules for 4 and 8.
How can you use one rule to help you remember the other?

1.2 More Patterns in Division

Focus Explore divisibility by 0, 3, 6, and 9.

Division can be thought of as making equal groups.

For 20 ÷ 4, we make 4 equal groups of 5.



Explore

Use a calculator.

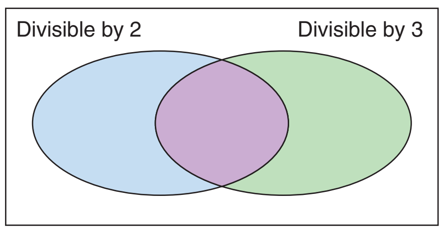
➤ Choose 10 different numbers.
Divide each number by 0.
What do you notice?
What do you think this means?

➤ Choose 15 consecutive 2-digit numbers.
Divide each number by 3 and by 9.
Repeat for 15 consecutive 3-digit numbers.

List the numbers that were divisible by 3 and by 9.
Find the sum of the digits of each number.
What do you notice?

Choose 4 different 4-digit numbers you think are divisible by 3 and by 9.
Divide each number by 3 and by 9 to check.
Add the digits in each number. What do you notice?

➤ Draw this Venn diagram.
Sort these numbers.
12 21 42 56 88 135 246 453 728
What can you say about the numbers in the overlapping region?



Reflect & Share

Share your work with another pair of classmates.
Explain how to choose a 4-digit number that is divisible by 3.
Without dividing, how can you tell if a number is divisible by 6? By 9?
Why do you think a number cannot be divided by 0?

Connect

We can use divisibility rules to find the factors of a number, such as 100.
Any number is divisible by 1 and itself,
so 1 and 100 are factors of 100.
100 is even, so 100 is divisible by 2.
We know 100 is divisible by 4.
The ones digit is 0, so 100 is divisible
by 5 and by 10.
100 is not divisible by 3, 6, 8, or 9.
The factors of 100, from least to greatest, are:
1, 2, 4, 5, 10, 20, 25, 50, 100

$100 \div 1 = 100$
$100 \div 2 = 50$
$100 \div 4 = 25$
$100 \div 5 = 20$
$100 \div 10 = 10$

Factors occur in pairs.
When we find one factor of a number, we also find a second factor.

A whole number cannot be divided by 0.
We cannot take a given number and share it into zero equal groups.
We cannot make sets of zero from a given number of items.

Example

Edward has 16 souvenir miniature hockey sticks.
He wants to share them equally among his cousins.
How many sticks will each cousin get if Edward has:

a) 8 cousins? **b)** 0 cousins?

Explain your answer to part b.

A Solution

a) There are 16 sticks. Edward has 8 cousins.
$16 \div 8 = 2$
Each cousin will get 2 sticks.

b) There are 16 sticks. Edward has no cousins.
16 sticks cannot be shared equally among no cousins.
This answer means that we cannot divide a number by zero.
We cannot divide 16 by 0 because 16 cannot be shared
into zero equal groups.

You have sorted numbers in a Venn diagram. You can also use a *Carroll diagram* to sort numbers.

Here is an example:

	Divisible by 3	Not Divisible by 3
Divisible by 8	24, 120, 1104, 12 096	32, 224, 2360
Not Divisible by 8	12, 252, 819, 11 337	10, 139, 9212

Divisibility Rules

A whole number is divisible by:

2 if the number is even

3 if the sum of the digits is divisible by 3

4 if the number represented by the last 2 digits is divisible by 4

5 if the ones digit is 0 or 5

6 if the number is divisible by 2 and by 3

8 if the number represented by the last 3 digits is divisible by 8

9 if the sum of the digits is divisible by 9

10 if the ones digit is 0

Practice

1. Which numbers are divisible by 3? By 9? How do you know?

a) 117 b) 216 c) 4125 d) 726 e) 8217 f) 12 024

2. Write 3 numbers that are divisible by 6. How did you choose the numbers?

3. Which of these numbers is 229 344 divisible by? How do you know?

a) 2 b) 3 c) 4 d) 5 e) 6 f) 8 g) 9 h) 10

4. Use the divisibility rules to find the factors of each number.
How do you know you have found all the factors?

a) 150 b) 95 c) 117 d) 80

5. Use a Carroll diagram.
Which numbers are divisible by 4? By 9? By 4 and by 9? By neither 4 or 9?

144 128 252 153 235 68 120 361 424 468

6. I am a 3-digit number that has a 2 in the hundreds place.
I am divisible by 3, 4, and 5. Which number am I?

7. **Assessment Focus**
a) Write a 3-digit number that is divisible by 5 and by 9.
How did you choose the number?
b) Find the factors of the number in part a. Use the divisibility rules to help you.
c) How would you find the greatest 3-digit number that is divisible by 5 and by 9? The least 3-digit number? Explain your methods.

8. Use the digits 0 to 9.
Replace the □ in each number to make a number divisible by 3.
Find as many answers as you can.
a) 4□6 b) 1□32 c) 24 71□

9. Suppose you have 24 cereal bars.
You must share the bars equally with everyone in the classroom.
How many cereal bars will each person get, in each case?
a) There are 12 people in the classroom.
b) There are 6 people in the classroom.
c) There is no one in the classroom.
d) Use your answer to part c.
Explain why a number cannot be divided by 0.

10. **Take It Further** Universal Product Codes (UPCs) are used to identify retail products.
The codes have 12 digits, and sometimes start with 0.
To check that a UPC is valid, follow these steps:
- Add the digits in the odd-numbered positions (1st, 3rd, 5th, …).
- Multiply this sum by 3.
- To this product, add the digits in the even-numbered positions.
- The result should be a number divisible by 10.

Look at this UPC. Is it a valid code? Explain.
Find 2 UPC labels on products at home.
Check to see if the codes are valid. Record your results.

Reflect

Which divisibility rules do you find easiest to use?
Which rules do you find most difficult? Justify your choices.

Writing to Explain Your Thinking

Have you ever tried to explain how you solved a problem to a classmate?

Communicating your thinking can be difficult.

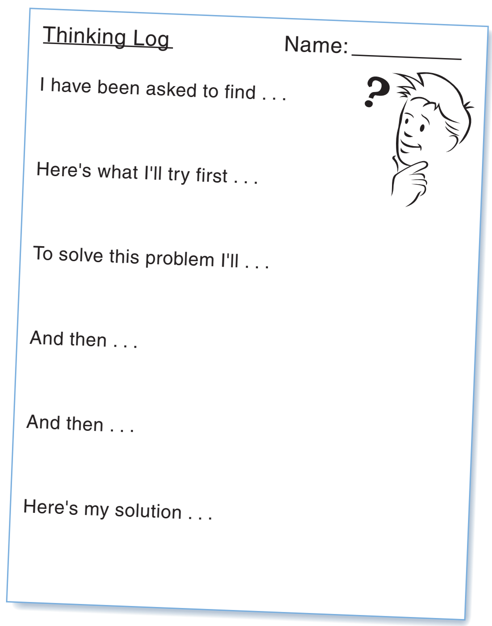
A *Thinking Log* can be used to record what and how you are thinking as you solve a problem.
It is a good way to organize your thoughts.

A classmate, teacher, or parent should be able to follow your thinking to understand how you solved the problem.

Using a Thinking Log

Complete a Thinking Log as you work through this problem.

Nine players enter the Saskatchewan Thumb Wrestling Championship.
In the first round, each player wrestles every other player once.
How many matches are there in the first round?



Thinking Log Name: ___________

I have been asked to find . . .

Here's what I'll try first . . .

To solve this problem I'll . . .

And then . . .

And then . . .

Here's my solution . . .

Reflect & Share

- Read over what you have written.
  Will someone else be able to follow your thinking?
- Share your Thinking Log with a classmate.
  Was your classmate able to follow your thinking and understand your solution? Explain.
- Describe any changes you would make to improve your Thinking Log.

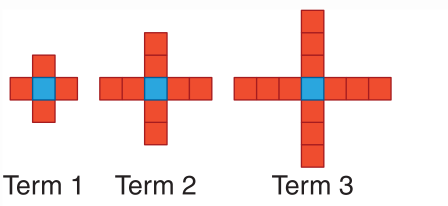
✓ Check

Complete a Thinking Log for each of these problems.

1. I am a 2-digit number. I have three factors.
I am divisible by five. Which number am I?

2. A book contains 124 pages numbered from 1 to 124.
How many times does the digit 7 appear?

3. Here is a pattern of tiles.



a) How many tiles will there be in the 10th term?
b) Which term has 37 tiles? How do you know?

1.3 Algebraic Expressions

Focus Use a variable to represent a set of numbers.

We can use symbols to represent a pattern.

Explore

Tehya won some money in a competition.
She has two choices as to how she gets paid.
Choice 1: \$20 per week for one year
Choice 2: \$400 cash now plus \$12 per week for one year

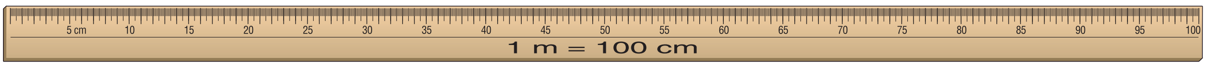
Which method would pay Tehya more money?
For what reasons might Tehya choose each method of payment?

Reflect & Share

Work with another pair of classmates.
For each choice, describe a rule you can use to calculate the total money Tehya has received at any time during the year.

Connect

We can use a variable to represent a number in an expression.
For example, we know there are 100 cm in 1 m.



We can write 1×100 cm in 1 m.
There are 2×100 cm in 2 m.
There are 3×100 cm in 3 m.

Recall that a variable is a letter, such as *n*, that represents a quantity that can vary.

To write an expression for the number of centimetres in any number of metres, we say there are $n \times 100$ cm in n metres.
n is a variable.
n represents any number we choose.

We can use any letter, such as n or x, as a variable.
The expression $n \times 100$ is written as $100n$.
$100n$ is an **algebraic expression**.

Variables are written in italics so they are not confused with units of measurement.

Here are some other algebraic expressions, and their meanings.
In each case, n represents the number.

- Three more than a number: $3 + n$ or $n + 3$
- Seven times a number: $7n$
- Eight less than a number: $n - 8$
- A number divided by 20: $\frac{n}{20}$

$7n$ means $7 \times n$.

When we replace a variable with a number in an algebraic expression, we *evaluate* the expression. That is, we find the value of the expression for a particular value of the variable.

Example

Write each algebraic expression in words.
Then evaluate for the value of the variable given.

a) $5k + 2$ for $k = 3$ **b)** $32 - \frac{x}{4}$ for $x = 20$

A Solution

a) $5k + 2$ means 5 times a number, then add 2.
Replace k with 3 in the expression $5k + 2$.
Then use the order of operations.

$$\begin{aligned} 5k + 2 &= 5 \times 3 + 2 && \text{Multiply first.} \\ &= 15 + 2 && \text{Add.} \\ &= 17 \end{aligned}$$

b) $32 - \frac{x}{4}$ means 32 minus a number divided by 4.
Replace x with 20 in the expression $32 - \frac{x}{4}$.
Then use the order of operations.

$\frac{x}{4}$ means $x \div 4$.

$$\begin{aligned} 32 - \frac{x}{4} &= 32 - \frac{20}{4} && \text{Divide first.} \\ &= 32 - 5 && \text{Subtract.} \\ &= 27 \end{aligned}$$

In the expression $5k + 2$,

- 5 is the **numerical coefficient** of the variable.
- 2 is the **constant term**.
- k is the *variable*.
  The variable represents any number in a set of numbers.

Practice

1. Identify the numerical coefficient, the variable, and the constant term in each algebraic expression.
   a) $3x + 2$ b) $5n$ c) $w + 3$ d) $2p + 4$

2. An algebraic expression has variable p, numerical coefficient 7, and constant term 9.
   Write as many different algebraic expressions as you can that fit this description.

3. Write an algebraic expression for each phrase.
   a) six more than a number
   b) a number multiplied by eight
   c) a number decreased by six
   d) a number divided by four

4. A person earns \$4 for each hour he spends baby-sitting.
   a) Find the money earned for each time.
      i) 5 h ii) 8 h
   b) Write an algebraic expression you could use to find the money earned in t hours.

5. Write an algebraic expression for each sentence.
   a) Double a number and add three.
   b) Subtract five from a number, then multiply by two.
   c) Divide a number by seven, then add six.
   d) A number is subtracted from twenty-eight.
   e) Twenty-eight is subtracted from a number.

6. a) Write an algebraic expression for each phrase.
      i) four more than a number
      ii) a number added to four
      iii) four less than a number
      iv) a number subtracted from four
   b) How are the expressions in part a alike?
      How are they different?

7. Evaluate each expression by replacing x with 4.

a) $x + 5$ b) $3x$ c) $2x - 1$

d) $\frac{x}{2}$ e) $3x + 1$ f) $20 - 2x$

8. Evaluate each expression by replacing z with 7.

a) $z + 12$ b) $10 - z$ c) $5z$

d) $3z - 3$ e) $35 - 2z$ f) $3 + \frac{z}{7}$

9. **Assessment Focus** Jason works at a local fish and chips restaurant.
He earns \$7/h during the week, and \$9/h on the weekend.

a) Jason works 8 h during the week and 12 h on the weekend.
Write an expression for his earnings.

b) Jason works x hours during the week and 5 h on the weekend.
Write an expression for his earnings.

c) Jason needs \$115 to buy sports equipment.
He worked 5 h on the weekend.
How many hours does Jason have to work during the week to have the money he needs?

10. **Take It Further** A value of n is substituted in each expression to get the number in the box.
Find each value of n.

a) $5n$ $\boxed{30}$ b) $3n - 1$ $\boxed{11}$

c) $4n + 7$ $\boxed{15}$ d) $5n - 4$ $\boxed{11}$

e) $4 + 6n$ $\boxed{40}$ f) $\frac{n}{8}$ $\boxed{5}$

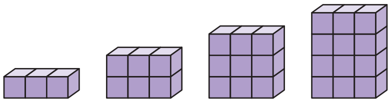
Reflect

Explain why it is important to use the order of operations when evaluating an algebraic expression.
Use an example in your explanation.

1.4 Relationships in Patterns

Focus Determine a relation to represent a pattern.

Here is a pattern made from linking cubes.



A pattern rule is: Start at 3. Add 3 each time.
This rule relates each term to the term that comes before it.

We can also describe this pattern using the term number.

Term Number	1	2	3	4
Term	3	6	9	12

How does each term relate to the term number?

Explore

On Enviro-Challenge Day, Grade 7 classes compete to see which class can collect the most garbage.

Each student in Ms. Thomson's class pledges to pick up 6 pieces of garbage.

- ➤ How many pieces of garbage will be picked up when the number of students is 5? 10? 15? 20? 25? 30?
- ➤ What pattern do you see in the numbers of pieces of garbage?
- ➤ Write a rule to find how many pieces of garbage will be picked up, when you know the number of students.
- ➤ Write an algebraic expression for the number of pieces of garbage picked up by n students.

Reflect & Share

Share your work with another pair of classmates.
Find the number of pieces of garbage picked up by 35 students.
How can you do this using the pattern?
Using the rule? Using the algebraic expression?

Connect

Miss Jackson's class pledges to pick up a total of 10 more pieces of garbage than Ms. Thomson's class.
Here are the numbers of pieces of garbage picked up by different numbers of students.

Number of students	2	4	6	8	10	12
Number of pieces of garbage picked up by Ms. Thomson's class	12	24	36	48	60	72
Number of pieces of garbage picked up by Miss Jackson's class	22	34	46	58	70	82

Pieces of garbage picked up by Miss Jackson's class = 10 + Pieces of garbage picked up by Ms. Thomson's class

Let n represent the number of students who pick up garbage in Ms. Thomson's class.
Then the number of pieces of garbage picked up by Ms. Thomson's class is $6n$.
And, the number of pieces of garbage picked up by Miss Jackson's class is $10 + 6n$.

Recall that 10 is the constant term.

The number of pieces of garbage is *related* to the number of students.
When we compare or *relate* a variable to an expression that contains the variable, we have a **relation**.
That is, $10 + 6n$ is related to n. **This is a relation.**

Example

Mr. Prasad plans to hold a party for a group of his friends.
The cost of renting a room is $35.
The cost of food is $4 per person.

a) Write a relation for the cost of the party, in dollars, for n people.
b) How much will a party cost for 10 people? For 15 people?
c) How does the relation change if the cost of food doubles? How much more would a party for 10 people cost? How do you know the answer makes sense?

A Solution

a) The cost of renting a room is \$35.
This does not depend on how many people come.
The cost of food is \$4 per person.
If 5 people come, the cost of food in dollars is: $4 \times 5 = 20$
If n people come, the cost of food in dollars is: $4 \times n$, or $4n$
So, n is related to $35 + 4n$.

b) To find the cost for 10 people, substitute $n = 10$ into $35 + 4n$.

$$\begin{aligned} 35 + 4n &= 35 + 4(10) \\ &= 35 + 40 \\ &= 75 \end{aligned}$$

4(10) means 4 × 10.

The party will cost \$75.

To find the cost for 15 people, substitute $n = 15$ into $35 + 4n$.

$$\begin{aligned} 35 + 4n &= 35 + 4(15) \\ &= 35 + 60 \\ &= 95 \end{aligned}$$

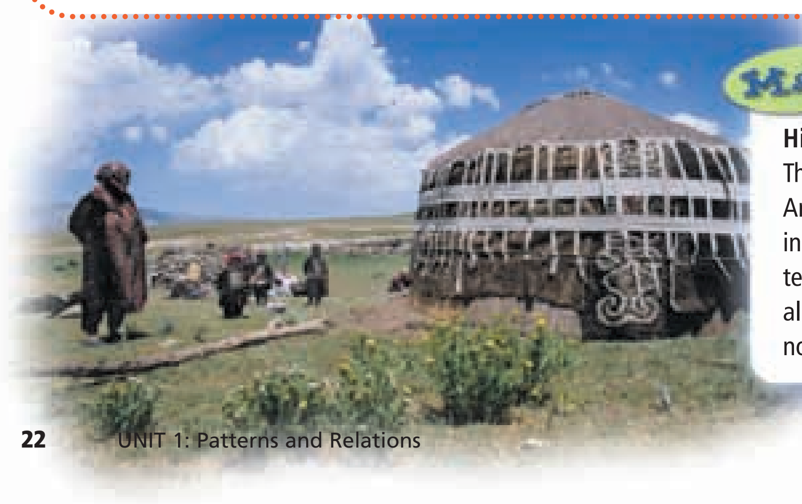
The party will cost \$95.

c) If the cost of food doubles, Mr. Prasad will pay \$8 per person.
If n people come, the cost for food, in dollars, is $8n$.
For n people, the cost of the party, in dollars, is now $35 + 8n$.
If 10 people come, the cost is now:

$$\begin{aligned} 35 + 8n &= 35 + 8(10) \\ &= 35 + 80 \\ &= 115 \end{aligned}$$

The party will cost \$115.

This is an increase of \$115 − \$75 = \$40.
The answer makes sense because the cost is now \$4 more per person.
So, the extra cost for 10 people would be \$4 × 10, or \$40 more.



Math Link

History
The word "algebra" comes from the Arabic word "al-jabr." This word appeared in the title of one of the earliest algebra texts, written around the year 825 by al-Khwarizmi. He lived in what is now Uzbekistan.

Practice

1. **i)** For each number pattern, how is each term related to the term number?

ii) Let n represent any term number. Write a relation for the term.

a)

Term Number	1	2	3	4	5	6
Term	2	4	6	8	10	12

b)

Term Number	1	2	3	4	5	6
Term	3	4	5	6	7	8

c)

Term Number	1	2	3	4	5	6
Term	8	16	24	32	40	48

d)

Term Number	1	2	3	4	5	6
Term	6	7	8	9	10	11

2. There are n students in a class. Write a relation for each statement.

a) the total number of pencils, if each student has three pencils

b) the total number of desks, if there are two more desks than students

c) the total number of geoboards, if each pair of students shares one geoboard

d) the total number of stickers, if each student gets four stickers and there are ten stickers left over

3. A person earns $10 for each hour worked.

a) Write a relation for her earnings for n hours of work.

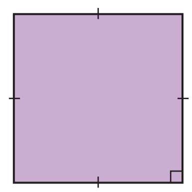
b) How much does she earn for 30 h of work?

4. **a)** Write a relation for the perimeter of a square with side length n centimetres.

b) What is the perimeter of a square with side length 12 cm?

c) Suggest a situation that could be represented by each relation.

i) $3s$ is related to s **ii)** $8t$ is related to t



5. Suggest a real-life situation that could be represented by each relation.

a) $n + 5$ is related to n **b)** $15 + 2p$ is related to p

c) $3t + 1$ is related to t

How do you know each situation fits the relation?

6. Koko is organizing an overnight camping trip. The cost to rent a campsite is $20. The cost of food is $9 per person.

a) How much will the trip cost if 5 people go? 10 people go?
b) Write a relation for the cost of the trip when *p* people go.
c) Suppose the cost of food doubles.
Write a relation for the total cost of the trip for *p* people.
d) Suppose the cost of the campsite doubles.
Write a relation for the total cost of the trip for *p* people.
e) Explain why using the variable *p* is helpful.

7. Assessment Focus A pizza with cheese and tomato toppings costs $8.00. It costs $1 for each extra topping.

a) Write a relation for the cost of a pizza with *e* extra toppings.
b) What is the cost of a pizza with 5 extra toppings?
c) On Tuesdays, the cost of the same pizza with cheese and tomato toppings is $5.00. Write a relation for the cost of a pizza with *e* extra toppings on Tuesdays.
d) What is the cost of a pizza with 5 extra toppings on Tuesdays?
e) How much is saved by buying the pizza on Tuesday?

8. Write a relation for the pattern rule for each number pattern.
Let *n* represent any term number.

a) 4, 8, 12, 16, … b) 7, 8, 9, 10, … c) 0, 1, 2, 3, …

9. Take It Further

i) For each number pattern, how is each term related to the term number?
ii) Let *n* represent any term number. Write a relation for the term.

a)

Term Number	1	2	3	4	5	6
Term	3	5	7	9	11	13

b)

Term Number	3	4	5	6	7	8
Term	7	10	13	16	19	22

c)

Term Number	2	3	4	5	6	7
Term	5	9	13	17	21	25

Reflect

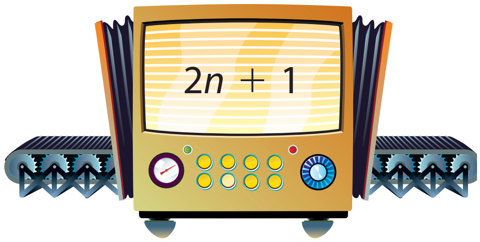
How did your knowledge of patterning help you in this lesson?

1.5 Patterns and Relationships in Tables

Focus Create a table of values for a relation.

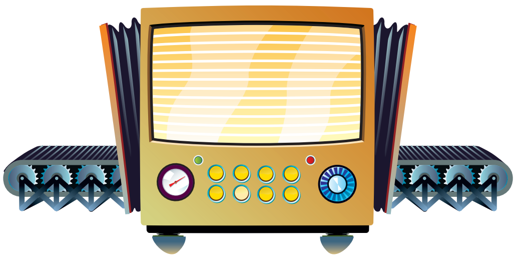
An Input/Output machine represents a relation.
Any Input number can be represented by n.

Suppose you input $n = 8$.
What will the output be?
How is the output related to the input?



Explore

Sketch an Input/Output machine like this one.



Write an algebraic expression to go in the machine.

➤ Use the numbers 1 to 6 as input.
Find the output for each Input number.
Record the input and output in a table like this.

Input	Output
1	
2	
3	
4	
5	
6	

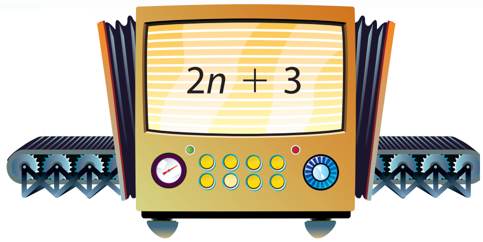
➤ How is the output related to the input?

➤ Describe the pattern in the Output numbers.

Reflect & Share

Share your work with another pair of classmates.
Describe how you would find the next 3 Output numbers for your classmates' Input/Output machine.
How is the output related to the input?

Connect



This Input/Output machine relates n and $2n + 3$.
To create a table of values,
select a set of Input numbers.
To get each Output number, multiply the
Input number by 2, then add 3.

When $n = 1$, $2n + 3 = 2(1) + 3$
$= 2 + 3$
$= 5$
When $n = 2$, $2n + 3 = 2(2) + 3$
$= 4 + 3$
$= 7$
When $n = 3$, $2n + 3 = 2(3) + 3$
$= 6 + 3$
$= 9$,
and so on.

Remember the *order of operations*. Multiply before adding.

Input n	Output $2n + 3$
1	5
2	7
3	9
4	11
5	13

We used consecutive Input numbers.
The Output numbers form a pattern. They increase by 2 each time.
This is because the expression contains $2n$,
which means that the Input number is doubled.
When the Input number increases by 1, the Output number increases by 2.

The expression $2n + 3$ can also be written as $3 + 2n$.

When a relation is represented as a table of values,
we can write the relation using algebra.

Example

Write the relation represented by this table.

Input	Output
1	2
2	5
3	8
4	11
5	14

A Solution

Let any Input number be represented by n.
The input increases by 1 each time.
The output increases by 3 each time.
This means that the expression for the output contains $3n$.

Input	Output
1	2
2	5
3	8
4	11
5	14

Substitute several values of n in $3n$, then look for a pattern.

When $n = 1, 3n = 3(1) = 3$
When $n = 2, 3n = 3(2) = 6$
When $n = 3, 3n = 3(3) = 9$
When $n = 4, 3n = 3(4) = 12$
When $n = 5, 3n = 3(5) = 15$

Each value is 1 more than the output above. That is, the output is 1 less than each value.

So, the output is $3n - 1$.
The table shows how $3n - 1$ relates to n.

Another Solution

Another way to solve this problem is to notice that each output is 1 less than a multiple of 3.
So, the output is $3 \times n - 1$, or $3n - 1$.
The table shows how $3n - 1$ relates to n.

Input	Output
1	$2 = 3 \times 1 - 1$
2	$5 = 3 \times 2 - 1$
3	$8 = 3 \times 3 - 1$
4	$11 = 3 \times 4 - 1$
5	$14 = 3 \times 5 - 1$
n	$3 \times n - 1$

Practice

1. Copy and complete each table.
Explain how the Output number is related to the Input number.

a)

Input x	Output $2x$
1	
2	
3	
4	
5	

b)

Input m	Output $10 - m$
1	
2	
3	
4	
5	

c)

Input p	Output $3p + 5$
1	
2	
3	
4	
5	

2. Use algebra. Write a relation for each Input/Output table.

a)

Input n	Output
1	7
2	14
3	21
4	28

b)

Input n	Output
1	4
2	7
3	10
4	13

c)

Input n	Output
1	1
2	3
3	5
4	7

3. **Assessment Focus** For each table, find the output.
Explain how the numbers 3 and 4 in each relation affect the output.

a)

Input n	Output $3n + 4$
1	
2	
3	
4	

b)

Input n	Output $4n + 3$
1	
2	
3	
4	

4. Use algebra. Write a relation for each Input/Output table.

a)

Input x	Output
1	5
2	8
3	11
4	14

b)

Input x	Output
1	1
2	7
3	13
4	19

c)

Input x	Output
1	8
2	13
3	18
4	23

5. **Take It Further**

a) Describe the patterns in this table.

b) Use the patterns to extend the table 3 more rows.

c) Use algebra.
Write a relation that describes how the output is related to the input.

Input x	Output
5	1
15	3
25	5
35	7
45	9
55	11

Reflect

Your friend missed today's lesson. Explain how to write the relation represented by an Input/Output table.

Mid-Unit Review

LESSON

1.1

1. Which numbers are divisible by 4? By 8? How do you know?

a) 932 **b)** 1418 **c)** 5056
d) 12 160 **e)** 14 436

1.2

2. Draw a Venn diagram with 2 loops. Label the loops: "Divisible by 3" and "Divisible by 5." Sort these numbers:
54 85 123 735 1740 3756 6195
What is true about the numbers in the overlapping region?

3. Use the divisibility rules. Find the factors of each number.

a) 85 **b)** 136 **c)** 270

1.3

4. Write an algebraic expression for each statement.
Let *n* represent the number.

a) seven more than a number
b) a number multiplied by eleven
c) a number divided by six
d) three less than four times a number
e) the sum of two and five times a number

1.4

5. Predict which expression in each pair will have the greater value when *y* is replaced with 8.
Evaluate to check your predictions.

a) i) $y + 7$ **ii)** $2y$
b) i) $6y$ **ii)** $9 - y$
c) i) $\frac{y + 4}{2}$ **ii)** $\frac{y}{2} + 4$
d) i) $2y + 6$ **ii)** $3y - 6$

6. i) For each number pattern, how is each term related to the term number?

ii) Let *n* represent the term number. Write a relation for the term.

a)

Term Number	1	2	3	4	5	6
Term	6	12	18	24	30	36

b)

Term Number	1	2	3	4	5	6
Term	5	6	7	8	9	10

7. Dave pays to practise in a music studio. He pays \$12 each month, plus \$2 for each hour he practises.

a) Write a relation for the total cost for one month, in dollars, when Dave practises *t* hours.
b) How much will Dave pay to practise 10 h in one month? 20 h?
c) How does the relation change when the cost per hour doubles?

1.5

8. Use algebra. Write a relation for each Input/Output table.

a)

Input x	Output
1	7
2	11
3	15
4	19

b)

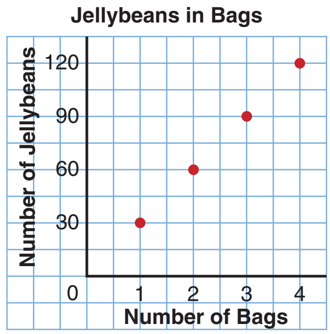
Input x	Output
1	5
2	13
3	21
4	29

1.6 Graphing Relations

Focus Create a table of values, then graph.

We can use a graph to show the relationship between two quantities.
What does this graph show?

How many jellybeans are in each bag?
Write a relation for the total number of jellybeans in *n* bags.



Explore

You will need grid paper.

The cost of *n* CDs, in dollars, is 12*n*.
➤ What is the cost of one CD?
➤ Copy and complete this table.
➤ Graph the data.

Number of CDs *n*	Cost ($) 12*n*
0	
2	
4	
6	
8	
10	

Use the graph to answer these questions:
➤ What is the cost of 5 CDs?
➤ How many CDs could you buy with $72?

Reflect & Share

Describe the patterns in the table. How are these patterns shown in the graph?
If you had $50, how many CDs could you buy?

Connect

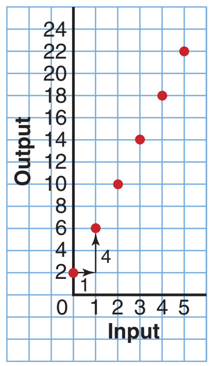
This table shows how $4n + 2$ relates to n, where n is a whole number.
We could have chosen any Input numbers, but to see patterns it helps to use consecutive numbers.

Input n	Output $4n + 2$
0	$4(0) + 2 = 2$
1	$4(1) + 2 = 6$
2	$4(2) + 2 = 10$
3	$4(3) + 2 = 14$
4	$4(4) + 2 = 18$
5	$4(5) + 2 = 22$

These data are plotted on a graph.
The input is plotted on the horizontal axis.
The output is plotted on the vertical axis.
On the vertical axis, the scale is 1 square for every 2 units.
The graph also shows how $4n + 2$ relates to n.

When we place a ruler along the points, we see the graph is a set of points that lie on a straight line.
When points lie on a straight line, we say the relation is a **linear relation**.
Since no numbers lie between the Input values in the table, it is not meaningful to join the points with a solid line.

Graph of $4n + 2$ against n



The graph shows that each time the input increases by 1, the output increases by 4.

Example

Mr. Beach has 25 granola bars.
He gives 3 granola bars to each student who stays after school to help prepare for the school concert.

a) Write a relation to show how the number of granola bars that remain is related to the number of helpers.
b) Make a table to show this relation.
c) Graph the data. Describe the graph.
d) Use the graph to answer these questions:
  i) How many granola bars remain when 7 students help?
  ii) When will Mr. Beach not have enough granola bars?

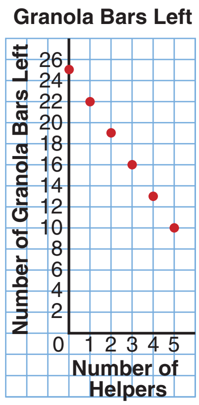
A Solution

a) Let n represent the number of helpers.
Each helper is given 3 granola bars.
So, the number of granola bars given to n helpers is $3n$.
There are 25 granola bars.
The number of granola bars that remain is $25 - 3n$.
So, n is related to $25 - 3n$.

b) Substitute each value of n into $25 - 3n$.

Number of Helpers n	Number of Granola Bars Left $25 - 3n$
0	$25 - 3(0) = 25$
1	$25 - 3(1) = 22$
2	$25 - 3(2) = 19$
3	$25 - 3(3) = 16$
4	$25 - 3(4) = 13$
5	$25 - 3(5) = 10$

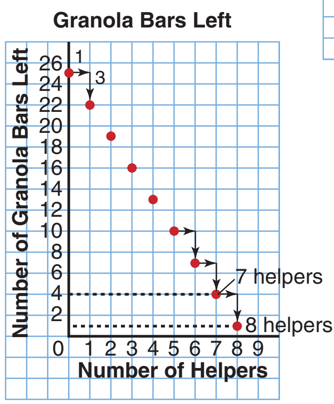
c) On the vertical axis, use a scale of 1 square for every 2 units.
The points lie on a line so the graph represents a linear relation.
When the input increases by 1, the output decreases by 3.



The graph goes down to the right.
This is because the number of granola bars that remain decreases as the number of helpers increases.

d) i) To find the number of granola bars that remain, extend the graph.
The points lie on a straight line.
Extend the graph to 7 helpers.
There are 4 granola bars left.

ii) Continue to extend the graph.
25 granola bars are enough for 8 helpers, but not for 9 helpers.
Mr. Beach will not have enough granola bars for 9 or more helpers.



To graph a relation, follow these steps:

- Select appropriate Input numbers. Make a table of values.
- Choose scales for the horizontal and vertical axes.
- Use a ruler to draw the axes on grid paper.
  Use numbers to indicate the scale.
- Label the axes. Give the graph a title.
- Plot the data in the table.

Another Strategy
We could have solved part d of the *Example* by extending the table.

Practice

1. Copy and complete this Input/Output table for each relation.

a) $4n$ is related to n.

b) $x + 3$ is related to x.

c) $4c + 6$ is related to c.

Input n	Output
1	
2	
3	
4	
5	

2. Graph each relation in question 1.
Suggest a real-life situation it could represent.

3. a) Copy and complete this Input/Output table to show how $6a - 4$ is related to a.

b) Graph the relation.
What scale did you use on the vertical axis?
How did you make your choice?

c) Explain how the graph illustrates the relation.

Input a	Output
2	
4	
6	
8	
10	

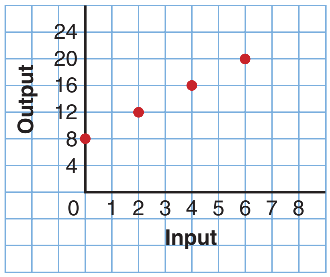
4. Look at the graph on the right.

a) What is the output when the input is 1?

b) Which input gives the output 18?

c) Extend the graph. What is the output when the input is 8?

d) Suggest a real-life situation this graph could represent.



5. Admission to Fun Place is $5.
Each go-cart ride costs an additional $3.

a) Write a relation to show how the total cost is related to the number of go-cart rides.

b) Copy and complete this table.

c) Draw a graph to show the relation.
Describe the graph.

d) Use the graph to answer these questions:

i) Erik goes on 6 go-cart rides.
What is his total cost?

ii) Before entering the park, Lydia has $30.
How many go-cart rides can she afford?

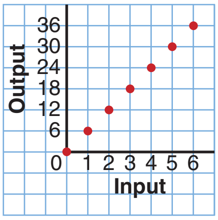
Number of Go-Cart Rides	Total Cost ($)
0	
1	
2	
3	
4	
5	

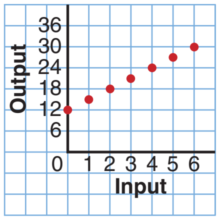
6. Match each graph to its relation.

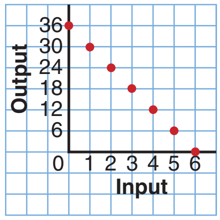
a) The number of seashells collected is related to the number of students who collected.
There are 12 seashells to start.
Each student collects 3 seashells.

b) The number of counters on the teacher's desk is related to the number of students who remove counters.
There are 36 counters to start.
Each student removes 6 counters.

c) The money earned baby-sitting is related to the number of hours worked. The baby-sitter earns \$6/h.

i)


ii)


iii)


7. Akuti borrows \$75 from her mother to buy a new lacrosse stick. She promises to pay her mother \$5 each week until her debt is paid off.

a) Write a relation to show how the amount Akuti owes is related to the number of weeks.

b) Make a table for the amount owing after 2, 4, 6, 8, and 10 weeks.

c) Draw a graph to show the relation. Describe the graph.

d) Use the graph to answer these questions:

i) How much does Akuti owe her mother after 13 weeks?

ii) When will Akuti finish paying off her debt?

8. **Assessment Focus** Use the relation: $5n + 6$ is related to n

a) Describe a real-life situation that could be represented by this relation.

b) Make a table of values using appropriate Input numbers.

c) Graph the relation. Describe the graph.

d) Write 2 questions you could answer using the graph.
Answer the questions.

Reflect

How can the graph of a relation help you answer questions about the relation? Use an example to show your thinking.

1.7 Reading and Writing Equations

Focus Translate statements into equations.

Explore

Part 1

➤ Write an algebraic expression for these statements:
Think of a number.
Multiply it by 3.
Add 4.

➤ The answer is 13. What is the original number?

Part 2

➤ Each of you writes your own number riddle.
Trade riddles with your partner.

➤ Write an algebraic expression for your partner's statements.
Find your partner's original number.

Reflect & Share

Compare your answer to *Part 1* with that of another pair of classmates.
If you found different values for the original number, who is correct?
Can both of you be correct? How can you check?

Connect

Zena bought 3 CDs.
Each CD costs the same amount.
The total cost was $36.
What was the cost of 1 CD?

$? + $? + $? = $36

We can write an equation for this situation.
Let p dollars represent the cost of 1 CD.
Then, the cost of 3 CDs is $3p$. This is equal to $36.
We can write an equation to represent this situation:
$3p = 36$

When we write one quantity equal to another quantity, we have an *equation*.
Each quantity may be a number or an algebraic expression.
For example, $3x + 2$ is an algebraic expression; 11 is a number.
When we write $3x + 2 = 11$, we have an equation.
An equation is a statement that two quantities are equal.
Each side of the equation has the same value.

In an equation, the variable represents a specific unknown number.
When we find the value of the unknown number, we *solve* the equation.

Example

Write an equation for each sentence.

a) Three more than a number is 15.
b) Five less than a number is 7.
c) A number subtracted from 5 is 1.
d) A number divided by 3 is 10.
e) Eight added to 3 times a number is 26.

A Solution

a) Three more than a number is 15.
Let x represent the number.
Three more than x: $x + 3$
The equation is: $x + 3 = 15$

b) Five less than a number is 7.
Let z represent the number.
Five less than z: $z - 5$
The equation is: $z - 5 = 7$

c) A number subtracted from 5 is 1.
Let g represent the number.
g subtracted from 5: $5 - g$
The equation is: $5 - g = 1$

d) A number divided by 3 is 10.
Let j represent the number.
j divided by 3: $\frac{j}{3}$
The equation is: $\frac{j}{3} = 10$

e) Eight added to 3 times a number is 26.
Let h represent the number.
3 times h: $3h$
8 added to $3h$: $3h + 8$
The equation is: $3h + 8 = 26$

Practice

1. Write an equation for each sentence.
a) Eight more than a number is 12.
b) Eight less than a number is 12.

2. Write a sentence for each equation.
a) $12 + n = 19$
b) $3n = 18$
c) $12 - n = 5$
d) $\frac{n}{2} = 6$

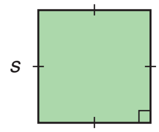
3. Write an equation for each sentence.

   a) Six times the number of people in the room is 258.

   b) One-half the number of students in the band is 21.

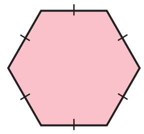
   c) The area of a rectangle with base 6 cm and height h centimetres is 36 cm^2.

4. The perimeter of a square is 156 cm.
   Write an equation you could use to find the side length of the square.



Recall that perimeter is the distance around a shape.

5. The side length of a regular hexagon is 9 cm.
   Write an equation you could use to find the perimeter of the hexagon.



6. Match each equation with the correct sentence.

   a) $n + 4 = 8$ — **A.** Four less than a number is 8.

   b) $4n = 8$ — **B.** Four more than four times a number is 8.

   c) $n - 4 = 8$ — **C.** The sum of four and a number is 8.

   d) $4 + 4n = 8$ — **D.** The product of four and a number is 8.

7. Alonso thinks of a number.
   He divides the number by 4, then adds 10.
   The answer is 14.
   Write an equation for the problem.

8. **Assessment Focus**

   a) Write an equation for each sentence.

      i) Five times the number of students is 295.

      ii) The area of a rectangle with base 7 cm and height h centimetres is 28 cm^2.

      iii) The cost of 2 tickets at x dollars each and 5 tickets at \$4 each is \$44.

      iv) Bhavin's age 7 years from now will be 20 years old.

   b) Which equation was the most difficult to write? Why?

   c) Write your own sentence, then write it as an equation.

Give an example of an algebraic expression and of an equation.
How are they similar? How are they different?

1.8 Solving Equations Using Algebra Tiles

Focus Use algebra tiles and symbols to solve simple equations.

We can use tiles to represent an expression.
One yellow tile  can represent $+1$.

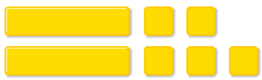
We call it a **unit tile**.

We also use tiles to represent variables.
This tile represents *x*. 

We call it an *x*-tile, or a **variable tile**.

A unit tile and a variable tile are collectively algebra tiles.

What algebraic expression do these tiles represent?



In this lesson, you will learn how to use tiles to solve equations.
In Unit 6, you will learn other ways to solve equations.

Explore

Alison had $13.
She bought 5 gift bags.
Each bag costs the same amount.
Alison then had $3 left.
How much was each gift bag?

➤ Let *d* dollars represent the cost of 1 gift bag.
Write an equation to represent the problem.

➤ Use tiles. Solve the equation to find the value of *d*.
How much was each gift bag?

Reflect & Share

Compare your equation with that of another pair of classmates.
If the equations are different, try to find out why.
Discuss your strategies for using tiles to solve the equation.

Connect

Owen collects model cars.
His friend gives him 2 cars.
Owen then has 7 cars.
How many cars did he have at the start?

We can write an equation that we can solve to find out. Let x represent the number of cars Owen had at the start.
2 more than x is: $x + 2$
The equation is: $x + 2 = 7$

We can use tiles to solve this equation.
We draw a vertical line in the centre of the page.
It represents the equals sign in the equation.

We arrange tiles on each side of the line to represent the expression or number on each side of the equation.

We want to get the x-tile on its own.
This is called *isolating the variable*.
When we solve an equation, we must *preserve* the equality.
That is, whatever we do to one side of the equation, we must also do to the other side.

To solve the equation $x + 2 = 7$:

On the left side, put tiles to represent $x + 2$.

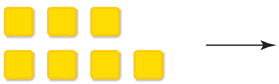
On the right side, put tiles to represent 7.

To isolate the x-tile, remove the 2 unit tiles from the left side.
To preserve the equality, remove 2 unit tiles from the right side, too.

The tiles show the solution is $x = 5$.

To *verify* the solution, replace x with 5 yellow tiles.

Left side:  7 yellow tiles

Right side:  7 yellow tiles

Since the left side and right side have equal numbers of tiles, the solution $x = 5$ is correct.
Owen had 5 cars at the start.

Example

Two more than three times a number is 14.

a) Write an equation you can solve to find the number.
b) Use tiles to solve the equation.
c) Verify the solution.

A Solution

a) Two more than three times a number is 14.
Let x represent the number.
Three times x: $3x$
Two more than $3x$: $3x + 2$
The equation is: $3x + 2 = 14$

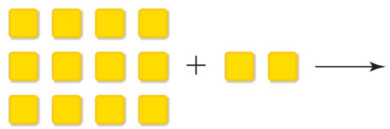
b) $3x + 2 = 14$

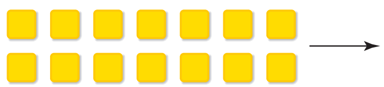
Remove 2 unit tiles from each side to isolate the x-tiles.

There are 3 x-tiles.
Arrange the tiles remaining on each side into 3 equal groups.

One x-tile equals 4 unit tiles.
So, $x = 4$

c) To verify the solution, replace x with 4 yellow tiles.

Left side:  14 yellow tiles

Right side:  14 yellow tiles

Since the left side and right side have equal numbers of tiles,
the solution $x = 4$ is correct.

Practice

Use tiles to solve each equation.

1. Draw pictures to represent the steps you took to solve each equation.

a) $x + 6 = 13$ **b)** $4 + x = 12$ **c)** $11 = x + 7$

d) $2x = 16$ **e)** $18 = 3x$ **f)** $4x = 12$

2. Seven more than a number is 12.

a) Write an equation for this sentence.

b) Solve the equation. Verify the solution.

3. For each equation in question 1, identify a constant term, the numerical coefficient, and the variable.

4. At the used bookstore, one paperback book costs \$3.
How many books can be bought for \$12?
   a) Write an equation you can solve to find how many books can be bought.
   b) Solve the equation. Verify the solution.

5. Kiera shared 20 hockey cards equally among her friends.
Each friend had 4 cards.
   a) Write an equation that describes this situation.
   b) Solve the equation to find how many friends shared the cards.

6. In Nirmala's Grade 7 class, 13 students walk to school. There are 20 students in the class.
   a) Write an equation you can solve to find how many students do not walk to school.
   b) Solve the equation. Verify the solution.

7. Jacob is thinking of a number. He multiplies it by 3 and then adds 4. The result is 16.
   a) Write an equation to represent this situation.
   b) Solve the equation to find Jacob's number.

8. **Assessment Focus** Tarana had 2 paper plates. She bought 4 packages of paper plates.
Each package had the same number of plates. Tarana now has a total of 18 plates.
How many paper plates were in each package?
   a) Write an equation you can solve to find how many plates were in each package.
   b) Solve the equation. Verify the solution.

9. **Take It Further** Dominique has 20 comic books. She gives 5 to her sister,
then gives 3 to each of her friends. Dominique has no comic books left.
   a) Write an equation you can solve to find how many friends were given comic books.
   b) Solve the equation. Verify the solution.

10. **Take It Further**
   a) Write an equation whose solution is $x = 4$.
   b) Write a sentence for your equation.
   c) Solve the equation.
   d) Describe a situation that can be represented by your equation.

Reflect

When you solve an equation, how can you be sure that your solution is correct?

Unit Review

What Do I Need to Know?

- A whole number is divisible by:
  2 if the number is even
  3 if the sum of the digits is divisible by 3
  4 if the number represented by the last 2 digits is divisible by 4
  5 if the ones digit is 0 or 5
  6 if the number is divisible by 2 and by 3
  8 if the number represented by the last 3 digits is divisible by 8
  9 if the sum of the digits is divisible by 9
  10 if the ones digit is 0
  A whole number cannot be divided by 0.

- A *variable* is a letter or symbol.
  It represents a set of numbers in an *algebraic expression*.
  A variable can be used to write an algebraic expression:
  "3 less than a number" can be written as $n - 3$.

  A variable represents a number in an *equation*.
  A variable can be used to write an equation.
  "4 more than a number is 11" can be written as $x + 4 = 11$.

- An algebraic expression can be *evaluated* by substituting a number for the variable.
  $6r + 3$ when $r = 2$ is: $6 \times 2 + 3 = 12 + 3$
  $= 15$

- A *relation* describes how the output is related to the input.
  A relation can be displayed using a table of values or a graph.
  When points of a relation lie on a straight line, it is a *linear relation*.

- An equation can be solved using tiles.

What Should I Be Able to Do?

LESSON

1.1 1.2 **1.** Use the divisibility rules to find the factors of 90.

2. Which of these numbers is 23 640 divisible by? How do you know?

a) 2	**b)** 3	**c)** 4
d) 5	**e)** 6	**f)** 8
g) 9	**h)** 10	**i)** 0

3. I am a 3-digit number.
I am divisible by 4 and by 9.
My ones digit is 2.
I am less than 500.
Which number am I?
Find as many numbers as you can.

4. Draw a Venn diagram with 2 loops. Label the loops "Divisible by 6," and "Divisible by 9."

a) Should the loops overlap? Explain.

b) Write these numbers in the Venn diagram.

330	639	5598	10 217
2295	858	187	12 006

How did you know where to put each number?

1.3 **5.** **i)** Write an algebraic expression for each statement.

ii) Evaluate each expression by replacing the variable with 8.

a) five less than a number

b) a number increased by ten

c) triple a number

d) six more than three times a number

1.4 **6.** There are n women on a hockey team.
Write a relation for each statement.

a) the total number of hockey sticks, if each player has 4 sticks

b) the total number of lockers in the dressing room, if there are 3 more lockers than players

c) the total number of water jugs on the bench, if each group of 4 players shares 1 jug

1.5 **7.** Copy and complete each table.
Explain how the Output number is related to the Input number.

a)

Input n	Output $n + 13$
1	
2	
3	
4	
5	

b)

Input n	Output $5n + 1$
1	
2	
3	
4	
5	

c)

Input n	Output $6n - 3$
1	
2	
3	
4	
5	

LESSON

8. Use algebra. Write a relation for each Input/Output table.

a)

Input n	Output
1	12
2	13
3	14
4	15

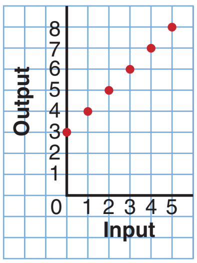
b)

Input n	Output
1	2
2	7
3	12
4	17

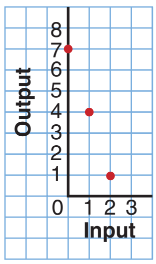
1.6

9. Match each graph with one of the relations below.

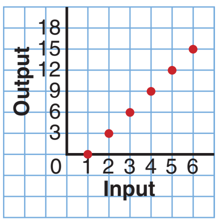
a)



b)



c)



i) $7 - 3n$ is related to n.

ii) $4n + 3$ is related to n.

iii) $n - 1$ is related to n.

iv) $n + 3$ is related to n.

v) $3n - 3$ is related to n.

vi) $7 - n$ is related to n.

10. For each relation below:

i) Describe a real-life situation that could be represented by the relation.

ii) Make a table of values.

iii) Graph the relation.

iv) Describe the graph.

v) Write 2 questions you could answer using the graph. Answer the questions.

a) $4 + 2m$ is related to m.

b) $15 - 2d$ is related to d.

11. Gerad is paid \$6 to supervise a group of children at a day camp. He is paid an additional \$2 per child.

a) Write a relation to show how the total amount Gerad is paid is related to the number of children supervised, c.

b) Copy and complete this table of values for the relation.

c	Amount Paid (\$)
0	
5	
10	
15	

c) Draw a graph to show the relation. Describe the graph.

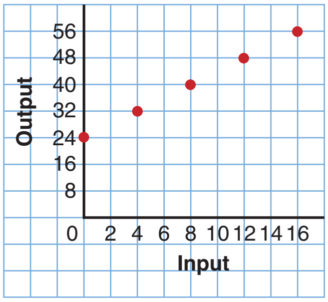
d) Use the graph to answer these questions:

i) How much money is Gerad paid when he supervises 25 children?

ii) Gerad was paid \$46. How many children did he supervise? Show your work.

LESSON

12. Suggest a real-life situation that could be represented by this graph.



1.7

13. Write an equation for each sentence.
   a) A pizza with 15 slices is shared equally among *n* students. Each student gets 3 slices.
   b) Four less than three times the number of red counters is 20.

14. The drum ring of this hand drum is a regular octagon. It has perimeter 48 cm. Write an equation you could use to find the side length of the drum ring.

15. i) Write an equation you can use to solve each problem.
   ii) Use tiles to solve each equation.
   iii) Draw pictures to represent the steps you took to solve each equation.
   iv) Use tiles to verify each solution.
   a) Thirty-six people volunteered to canvas door-to-door for the Heart and Stroke Foundation. They were divided into groups of 3. How many groups were there?
   b) A garden has 7 more daffodils than tulips. There are 18 daffodils. How many tulips are there?
   c) A sleeve of juice contains 3 juice boxes. Marty buys 24 juice boxes. How many sleeves does he buy?
   d) Jan collects foreign stamps. Her friend gives her 8 stamps. Jan then has 21 stamps. How many stamps did Jan have to start with?

16. A number is multiplied by 4, then 5 is added. The result is 21. What is the number?
   a) Write an equation to represent this situation.
   b) Solve the equation to find the number.
   c) Verify the solution.

Practice Test

1. Use the digits 0 to 9.
Replace the □ in 16 21□ so that the number is divisible by:

a) 2 **b)** 3 **c)** 4 **d)** 5
e) 6 **f)** 8 **g)** 9 **h)** 10

Find as many answers as you can.

2. Here are 3 algebraic expressions:
$2 + 3n$; $2n + 3$; $3n - 2$
Are there any values of n that will produce the same number when substituted into two or more of the expressions?
Investigate to find out. Show your work.

3. Jamal joined a video club. The annual membership fee is \$25.
The cost of each video rental is an additional \$2.

a) Write a relation for the total cost when Jamal rents v videos in one year.
b) Graph the relation. How much will Jamal pay when he rents 10 videos? 25 videos?
c) How does the relation change when the cost per video rental increases by \$1?
How much more would Jamal pay to rent 10 videos?
How do you know the answer makes sense?

4. **a)** Write an equation for each situation.
b) Solve each equation using tiles. Sketch the tiles you used.
c) Verify each solution.

i) There were 22 students in a Grade 7 class.
Five students went to a track meet.
How many students were left in the class?
ii) Twice the number of dogs in the park is 14.
How many dogs are in the park?
iii) Daphne scored the same number of goals in period one, period two, and period three this season.
She also scored 4 overtime goals, for a total of 19 goals.
How many goals did she score in each period?

Unit Problem

Fund Raising

Two students raised money for charity in a bike-a-thon.
The route was from Lethbridge to Medicine Hat, a distance of 166 km.

Part 1

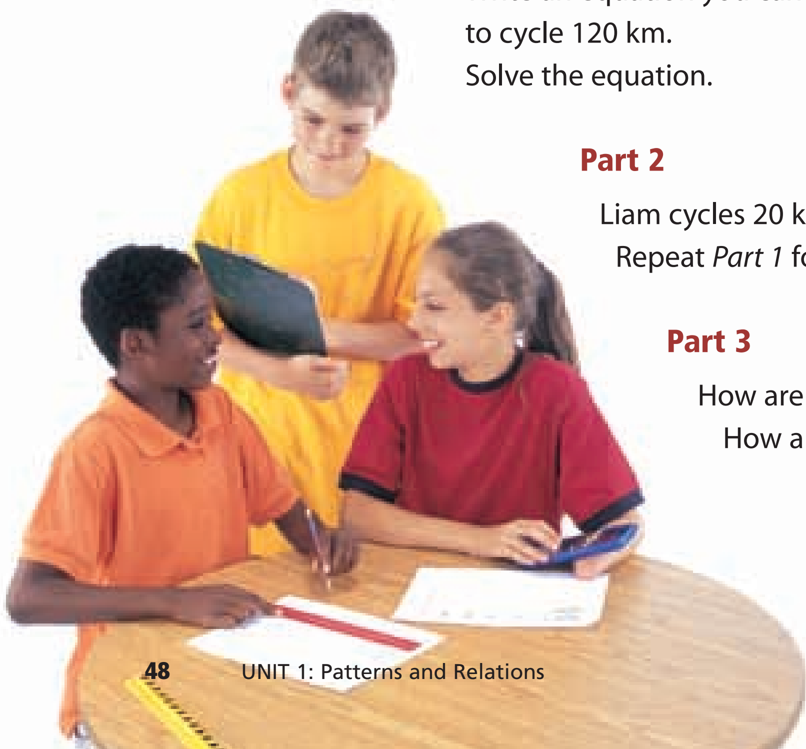
Ingrid cycles 15 km each hour.
How far does Ingrid cycle in 1 h? 2 h? 3 h? 4 h? 5 h?
Record the results in a table.

Time (h)					
Distance (km)					

Graph the data.
Graph *Time* horizontally and *Distance* vertically.

Write a relation for the distance Ingrid travels in t hours.
How far does Ingrid travel in 7 h?
How could you check your answer?

Let t hours represent the time Ingrid cycled.
How far does Ingrid cycle in t hours?
Write an equation you can solve to find the time Ingrid took to cycle 120 km.
Solve the equation.



Part 2

Liam cycles 20 km each hour.
Repeat *Part 1* for Liam.

Part 3

How are the graphs for Ingrid and Liam alike?
How are they different?

Part 4

Ingrid's sponsors paid her $25 per kilometre.
Liam's sponsors paid him $20 per kilometre.
Make a table to show how much money each student raised for every 10 km cycled.

Distance (km)	Money Raised by Ingrid ($)	Money Raised by Liam ($)
10		
20		
30		

How much money did Ingrid raise if she cycled *d* kilometres?
How much money did Liam raise if he cycled *d* kilometres?

Liam and Ingrid raised equal amounts of money.
How far might each person have cycled? Explain.

Check List

Your work should show:

- ✓ all tables and graphs, clearly labelled
- ✓ the equations you wrote and how you solved them
- ✓ how you know your answers are correct
- ✓ explanations of what you found out

Reflect on Your Learning

You have learned different ways to represent a pattern. Which way do you find easiest to use? When might you want to use the other ways?

UNIT 2

Integers

Canada has 6 time zones.
This map shows the summer time zones.

- What time is it where you are now?
- You want to call a friend in Newfoundland. What time is it there?
- In the province or territory farthest from you, what might students be doing now?

What other questions can you ask about this map?

What You'll Learn

- Model integers with coloured tiles.
- Add integers using coloured tiles and number lines.
- Subtract integers using coloured tiles and number lines.
- Solve problems involving the addition and subtraction of integers.

Why It's Important

- We use integers when we talk about weather, finances, sports, geography, and science.
- Integers extend the whole number work from earlier grades.

Key Words

- negative integer
- positive integer
- zero pair
- opposite integers

2.1 Representing Integers

Focus Use coloured tiles to represent integers.

One of the coldest places on Earth is Antarctica, with an average annual temperature of about −58°C.
This is a **negative integer**.

One of the hottest places on Earth is Ethiopia, with an average annual temperature of about +34°C.
This is a **positive integer**.

We can use yellow tiles to represent positive integers and red tiles to represent negative integers.

One yellow tile can represent +1.

One red tile can represent −1.

A red tile and a yellow tile combine to model 0:

−1
+1

We call this a **zero pair.**

Explore

You will need coloured tiles.

➤ One of you uses 9 tiles and one uses 10 tiles.
You can use any combination of red and yellow tiles each time.
How many different integers can you model with 9 tiles?
How many different integers can your partner model with 10 tiles?

➤ Draw a picture to show the tiles you used for each integer you modelled.
Circle the zero pairs. Write the integer each picture represents. How do you know?

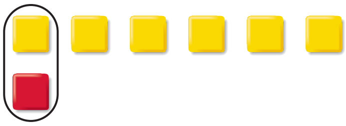
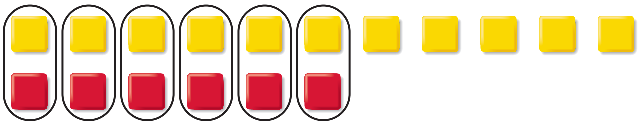
Reflect & Share

Compare your models with those of your partner.
Which integers did you model? Your partner?
Were you able to model any of the same integers?
Why or why not?

Connect

We can model any integer in many ways.

Each set of tiles below models $+5$.

- 
- 
- 

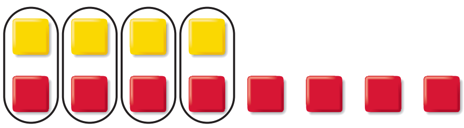
Each pair of 1 yellow tile and 1 red tile makes a zero pair. The pair models 0.

Example

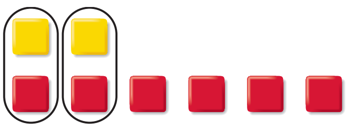
Use coloured tiles to model -4 in three different ways.

A Solution

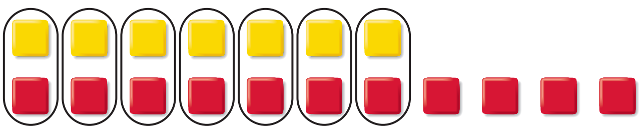
Start with 4 red tiles to model -4.
Add different numbers of zero pairs.
Each set of tiles below models -4.

- 

Adding 4 zero pairs does not change the value.

- 

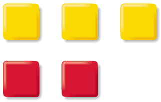
Adding 2 zero pairs does not change the value.

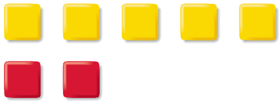
- 

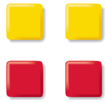
Adding 7 zero pairs does not change the value.

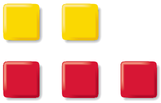
Practice

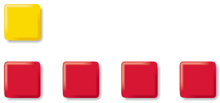
1. Write the integer modelled by each set of tiles.

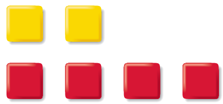
a) 

b) 

c) 

d) 

e) 

f) 

2. Draw yellow and red tiles to model each integer in two different ways.

a) -6 **b)** $+7$ **c)** $+6$ **d)** -2
e) $+9$ **f)** -4 **g)** 0 **h)** $+10$

3. Work with a partner.
Place 10 yellow and 10 red tiles in a bag.

a) Suppose you draw 6 tiles from the bag.
What integers might the tiles model?
List all seven possible integers.

b) Without looking, draw 6 tiles from the bag.
Record the integer that these tiles model.
Repeat the experiment 9 more times.
Which integer was modelled most often?

Math Link

Sports

In golf, a hole is given a value called **par**. Par is the number of strokes a good golfer takes to reach the hole.
A score of $+2$ means a golfer took 2 strokes more than par, or 2 strokes over par.
A score of -1 means a golfer took 1 stroke fewer than par, or 1 stroke under par.
Some scores have special names.
A score of $+1$ is a bogey.
A score of -1 is a birdie.
A score of -2 is an eagle.

In a golf tournament, the golfer with the fewest strokes wins the game.

4. Assessment Focus

a) Choose an integer between −9 and +6.
Use coloured tiles to model the integer.

b) How many more ways can you find to model the integer with tiles?
Create a table to order your work.

c) What patterns can you find in your table?

d) Explain how the patterns in your table can help you model an integer between −90 and +60.

5. a) Suppose you have 10 yellow tiles, and use all of them.
How many red tiles would you need to model +2?
How do you know?

b) Suppose you have 100 yellow tiles, and use all of them.
How many red tiles would you need to model +2?
How do you know?

6. Write the integer suggested by each of the following situations.
Draw yellow or red tiles to model each integer.
Explain your choice.

a) You move your game piece forward 9 squares on the game board.

b) You ride down 5 floors on an elevator.

c) You walk up 11 stairs.

d) The temperature drops 9°C.

e) You climb down 7 rungs on a ladder.

7. Write two integers suggested by each of the following situations.

a) You deposit $100 in your bank account, then pay back your friend $20.

b) While shopping in a large department store, you ride the elevator up 6 floors, then down 4 floors.

c) The temperature rises 12°C during the day, then falls 8°C at night.

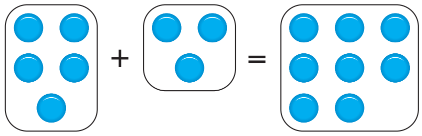
Reflect

How is it possible to use coloured tiles to model any integer in many different ways?

2.2 Adding Integers with Tiles

Focus Use coloured tiles to add integers.

Recall that when you add two numbers, such as 5 + 3, you can show the addition by combining 5 counters with 3 counters to obtain 8 counters.



You can add two integers in a similar way.
You know that +1 and −1 combine to make a zero pair.
We can combine coloured tiles to add integers.

Explore

You will need coloured tiles.

➤ Choose two different positive integers.
Add the integers.
Draw a picture of the tiles you used.
Write the addition equation.

➤ Repeat the activity for a positive integer and a negative integer.

➤ Repeat the activity for two different negative integers.

Reflect & Share

Share your equations with another pair of classmates.
How did you use the tiles to find a sum of integers?
How can you predict the sign of the sum?

Connect

➤ To add two positive integers: (+5) + (+4)
We can model each integer with tiles.

+5: 

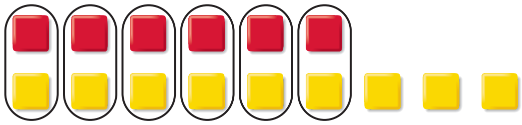
+4: 

Combine the tiles. There are 9 yellow tiles.
They model +9.
So, (+5) + (+4) = +9

This is an addition equation.

➤ To add a negative integer and a positive integer: $(-6) + (+9)$
We can model each integer with tiles. Circle zero pairs.

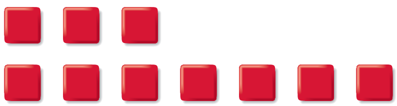
-6:
$+9$:



There are 6 zero pairs.
There are 3 yellow tiles left.
They model $+3$.
So, $(-6) + (+9) = +3$

➤ To add two negative integers: $(-3) + (-7)$
We can model each integer with tiles.

-3:
-7:



Combine the tiles. There are 10 red tiles.
They model -10.
So, $(-3) + (-7) = -10$

Example

The temperature rises 5°C, then falls 8°C.

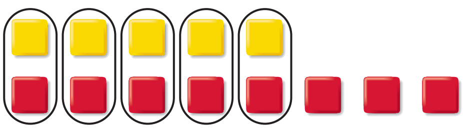
a) Represent the above sentence with integers. **b)** Find the overall change in temperature.

A Solution

a) $+5$ represents a rise of 5°C.
-8 represents a fall of 8°C.
Using integers, the sentence is: $(+5) + (-8)$

b) Model each integer with tiles.
Circle zero pairs.

$+5$
-8

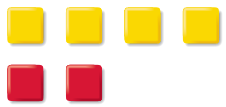


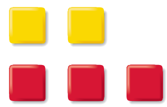
There are 3 red tiles left.
They model -3.
So, $(+5) + (-8) = -3$
The overall change in temperature is -3°C.

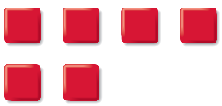
Practice

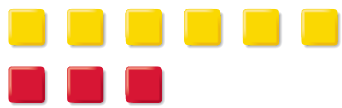
Use coloured tiles.

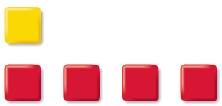
1. What sum does each set of tiles model?
Write the addition equation.

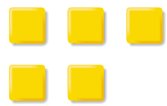
a) 

b) 

c) 

d) 

e) 

f) 

2. What sum does each set of tiles model?
How do you know you are correct?

a) 3 yellow tiles and 2 red tiles
b) 3 yellow tiles and 4 red tiles
c) 2 red tiles and 2 yellow tiles

3. Use coloured tiles to represent each sum. Find each sum.
Sketch the tiles you used. What do you notice?

a) $(+2) + (-2)$ **b)** $(-4) + (+4)$ **c)** $(+5) + (-5)$

4. Add. Sketch coloured tiles to show how you did it.

a) $(+2) + (+3)$ **b)** $(-3) + (+4)$ **c)** $(-4) + (-1)$
d) $(+1) + (-1)$ **e)** $(-3) + (-4)$ **f)** $(+5) + (-2)$

5. Add. Write the addition equations.

a) $(+4) + (+3)$ **b)** $(-7) + (+5)$ **c)** $(-4) + (-5)$
d) $(+8) + (-1)$ **e)** $(-10) + (-6)$ **f)** $(+4) + (-13)$

6. Represent each sentence with integers, then find each sum.

a) The temperature drops 3°C and rises 4°C.
b) Marie earned \$5 and spent \$3.
c) A stock rises 15¢, then falls 7¢.
d) Jerome moves his game piece 3 squares backward, then 8 squares forward.
e) Duma deposits \$12, then withdraws \$5.

7. Use question 6 as a model.
Write 3 integer addition problems.
Trade problems with a classmate.
Solve your classmate's problems with coloured tiles.

8. Copy and complete.

a) $(+5) + \square = +8$
b) $\square + (-3) = -4$
c) $(+3) + \square = +1$
d) $(-5) + \square = -3$
e) $(+2) + \square = +1$
f) $\square + (-6) = 0$

9. **Assessment Focus**

a) Add: $(+3) + (-7)$
b) Suppose you add the integers in the opposite order: $(-7) + (+3)$. Does the sum change?
Use coloured tile drawings and words to explain the result.
c) How is $(-3) + (+7)$ different from $(+3) + (-7)$? Explain.
d) Repeat parts a to c with a sum of integers of your choice.
What do you notice?

10. **Take It Further** Add. Sketch coloured tiles to show how you did it.

a) $(+1) + (+2) + (+3)$
b) $(+2) + (-1) + (+3)$
c) $(-3) + (-1) + (-1)$
d) $(+4) + (-3) + (+1)$

11. **Take It Further** In a magic square, every row, column, and diagonal has the same sum. Copy and complete each magic square. How did you do it?

a)

+3		+1
	0	
−1		

b)

−1		+1
	−2	
		−3

12. **Take It Further** Copy each integer pattern.
What do you add each time to get the next term?
Write the next 4 terms.

a) $+8, +4, 0, -4, \ldots$
b) $-12, -9, -6, -3, \ldots$

Reflect

Talk to a partner. Tell how you used coloured tiles to add two integers when the integers have:

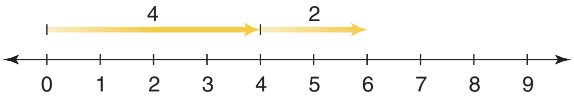
- the same signs
- opposite signs

2.3 Adding Integers on a Number Line

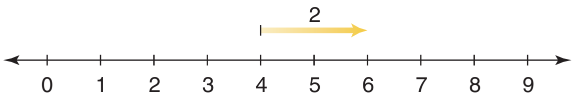
Focus Add integers using number lines.

We can show the addition of whole numbers on a number line: $4 + 2 = 6$

Draw 2 arrows.



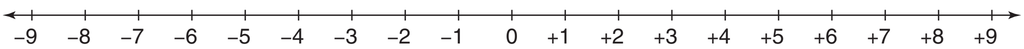
Or, begin at 4, and draw 1 arrow.



We can also show the addition of integers on a number line.

Explore

You will need copies of a number line.



➤ Choose two different positive integers.
Use a number line to add them.
Write the addition equation.

➤ Repeat the activity for a positive integer and a negative integer.

➤ Repeat the activity for two different negative integers.

➤ What happens when you add $+2$ and -2?

Reflect & Share

Compare your strategies for adding on a number line with those of your classmates.
Use coloured tiles to check the sums.
Why do you think integers such as $+2$ and -2 are called **opposite integers**?

Connect

➤ To add a positive integer, move right (in the positive direction).

$(-2) + (+3)$

Start at 0.

Draw an arrow 2 units long, pointing left.

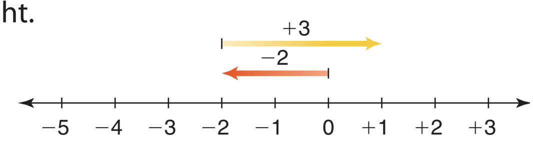
This arrow represents -2.

From -2, draw an arrow 3 units long, pointing right.

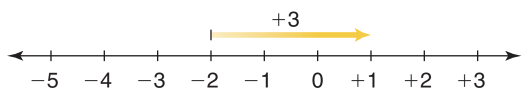
This arrow represents $+3$.

The arrow head is at $+1$.

So, $(-2) + (+3) = +1$



Notice that the first arrow ends at the first integer.
So, we could start at that integer,
and use only 1 arrow to find the sum.



➤ To add a negative integer, move left (in the negative direction).

$(-2) + (-3)$

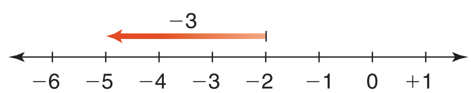
Start at -2.

Draw an arrow 3 units long, pointing left.

This arrow represents -3.

The arrow head is at -5.

So, $(-2) + (-3) = -5$



We can use the same method to add integers
on a vertical number line.

➤ The temperature is 12°C. It falls 5°C.

Find the final temperature.

$(+12) + (-5)$

Start at $+12$.

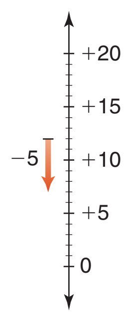
Draw an arrow 5 units long, pointing down.

This arrow represents -5.

The arrow head is at $+7$.

So, $(+12) + (-5) = +7$

The final temperature is 7°C.



Example

Sandra and Joe buy and sell CDs at a flea market.
One day in August, they bought 3 CDs for \$5 each.
They sold 2 CDs for \$9 each.

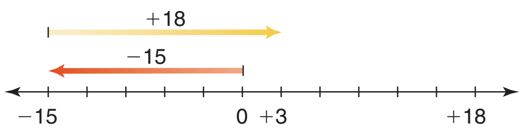
a) Write the expenses and income as integers.

b) Did Sandra and Joe make money or lose money that day in August? Explain.

A Solution

a) Expenses: $(-5) + (-5) + (-5) = -15$; they spent \$15.
Income: $(+9) + (+9) = +18$; they made \$18.

b) Draw a number line.
Add expenses and income.



$(-15) + (+18) = +3$
Since the sum of the expenses and income is positive, Sandra and Joe made money. They made \$3.

Another Strategy
We could use coloured tiles.

Practice

1. Use a number line to represent each sum.

a) $(+1) + (+3)$ **b)** $(-1) + (+3)$ **c)** $(-3) + (+1)$ **d)** $(-1) + (-3)$
e) $(-3) + (-4)$ **f)** $(-3) + (+4)$ **g)** $(+3) + (-4)$ **h)** $(+3) + (+4)$

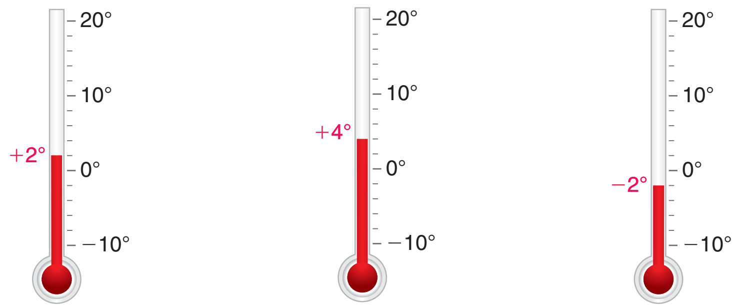
2. Use a number line to add.

a) $(+4) + (+2)$ **b)** $(+5) + (-3)$ **c)** $(-4) + (-2)$ **d)** $(-8) + (+2)$
e) $(-6) + (-7)$ **f)** $(+1) + (-6)$ **g)** $(-5) + (+2)$ **h)** $(+8) + (+4)$

3. **a)** Reverse the order of the integers in question 2, then add.
b) Compare your answers to the answers in question 2.
What do you notice?
c) Make a general statement about your observations.

4. Look at these thermometers. Find each temperature after:

a) it falls 4°C		b) it falls 7°C		c) it rises 6°C

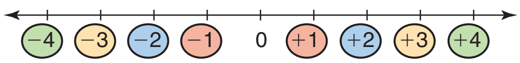


5. a) The temperature rises 7°C, then drops 2°C.
What is the overall change in temperature?

b) Adrian loses $4, then earns $8.
Did Adrian gain or lose overall?

c) The value of a stock went up $3, then down $2.
What was the final change in the value of the stock?

6. Opposite integers are the same distance from 0 but are on opposite sides of 0.



a) Write the opposite of each integer.

i) +2		ii) −5		iii) +6		iv) −8

b) Add each integer to its opposite in part a.

c) What do you notice about the sum of two opposite integers?

7. Use a number line. For each sentence below:

a) Write each number as an integer.

b) Write the addition equation.
Explain your answer in words.

i) You take 5 steps backward, then 10 steps backward.

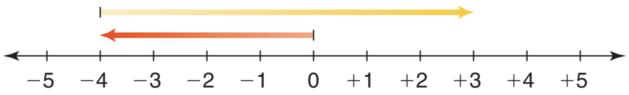
ii) You withdraw $5, then deposit $8.

iii) A deep sea diver descends 8 m, then ascends 6 m.

iv) A person drives a snowmobile 4 km east, then 7 km west.

v) A person gains 6 kg, then loses 10 kg.

8. a) Write the addition equation modelled by each number line.
b) Describe a situation that each number line could represent.

i)


ii)


9. **Assessment Focus** Is each statement always true, sometimes true, or never true?
Use a number line to support your answers.
a) The sum of two opposite integers is 0.
b) The sum of two positive integers is negative.
c) The sum of two negative integers is negative.
d) The sum of a negative integer and a positive integer is negative.

10. **Take It Further** Add.
a) $(+4) + (+3) + (-6)$
b) $(-2) + (-4) + (+1)$
c) $(-5) + (+3) + (-4)$
d) $(+6) + (-8) + (+2)$

11. **Take It Further** The temperature in Calgary, Alberta, was −2°C.
A Chinook came through and the temperature rose 15°C.
At nightfall, it fell 7°C. What was the final temperature?
Support your answer with a drawing.

Reflect

Compare adding on a number line to adding with coloured tiles.
Which method do you prefer?
When might you need to use a different method?

Mid-Unit Review

LESSON

2.1 **1.** Use coloured tiles to model each integer in two different ways. Draw the tiles.

a) -5 **b)** 0
c) $+8$ **d)** -1
e) $+3$ **f)** -7

2. Suppose you have 8 red tiles. How many yellow tiles would you need to model +3? How do you know?

2.2 **3.** What sum does each set of tiles model? How do you know you are correct? Write the addition equations.

a) 6 yellow tiles and 1 red tile
b) 5 yellow tiles and 7 red tiles
c) 4 yellow tiles and 4 red tiles

4. Use coloured tiles to add. Draw pictures of the tiles you used.

a) $(+4) + (-1)$ **b)** $(-3) + (-2)$
c) $(-5) + (+1)$ **d)** $(+6) + (+3)$
e) $(-4) + (-8)$ **f)** $(+4) + (+8)$

2.3 **5.** Use a number line to add. Write the addition equations.

a) $(+3) + (+2)$ **b)** $(-5) + (-1)$
c) $(-10) + (+8)$ **d)** $(+6) + (-5)$
e) $(-8) + (+8)$ **f)** $(-5) + (+12)$

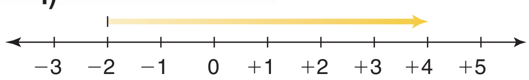
6. **a)** Add. $(+4) + (-5)$
b) Find 4 different pairs of integers that have the same sum as part a.

7. Write an addition equation for each situation.

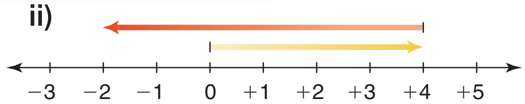
a) Puja earned \$50, and spent \$20. How much did Puja then have?
b) The temperature is 5°C, then drops 10°C. What is the final temperature?
c) The population of a city was 124 000, then it dropped by 4000 people. What was the population then?
d) A plane was cruising at an altitude of 12 000 m, then dropped 1200 m. What was the cruising altitude then?

8. **a)** Write the addition equation modelled by each number line.
b) Describe a situation that each number line could represent.

i)



ii)



9. Each integer below is written as the sum of consecutive integers.

$(+5) = (+2) + (+3)$
$(+6) = (+1) + (+2) + (+3)$

Write each of these integers as the sum of consecutive integers.

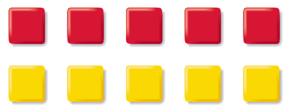
a) $+10$ **b)** 0 **c)** $+2$
d) $+7$ **e)** $+4$ **f)** $+8$

2.4 Subtracting Integers with Tiles

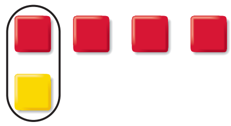
Focus Use coloured tiles to subtract integers.

To add integers, we combine groups of tiles.
To subtract integers, we do the reverse:
we remove tiles from a group.

Recall that equal numbers of red and yellow tiles model 0.
For example, $+5$ and -5 form 5 zero pairs,
and $(-5) + (+5) = 0$



Adding a zero pair to a set of tiles does not change its value.
For example, $(-3) + 0 = -3$



Explore

You will need coloured tiles.
Use tiles to subtract.
Add zero pairs when you need to.
Sketch the tiles you used in each case.

- $(+5) - (+3)$
- $(+5) - (-3)$
- $(-3) - (+5)$
- $(-3) - (-5)$

Reflect & Share

Compare your results with those of another pair of classmates.
Explain why you may have drawn different sets of tiles, yet both may be correct.
When you subtracted, how did you know how many tiles to use to model each integer? How did adding zero pairs help you?

Connect

To use tiles to subtract integers, we model the first integer,
then take away the number of tiles indicated by the second integer.

We can use tiles to subtract: $(+5) - (+9)$

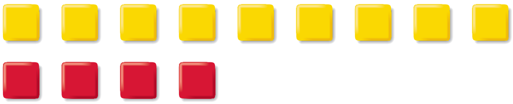
Model $+5$.

There are not enough tiles to take away $+9$.

To take away $+9$, we need 4 more yellow tiles.

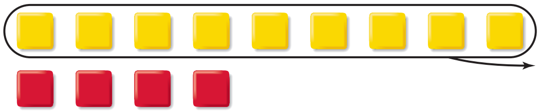
We add zero pairs without changing the value.

Add 4 yellow tiles and 4 red tiles. They represent 0.



By adding 0, the integer the tiles represent has not changed.

Now take away the 9 yellow tiles.



Since 4 red tiles remain, we write: $(+5) - (+9) = -4$

This is a subtraction equation.

Example

Use tiles to subtract.

a) $(-2) - (-6)$ **b)** $(-6) - (+2)$ **c)** $(+2) - (-6)$

A Solution

a) $(-2) - (-6)$

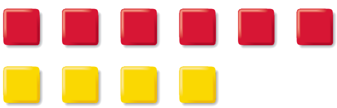
Model -2. 

There are not enough tiles to take away -6.

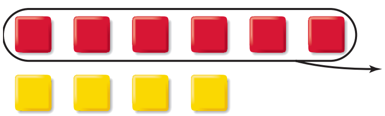
To take away -6, we need 4 more red tiles.

We add zero pairs without changing the value.

Add 4 red tiles and 4 yellow tiles.



Now take away 6 red tiles.



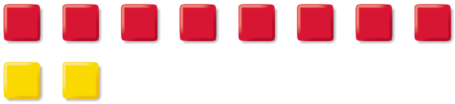
Since 4 yellow tiles remain, we write: $(-2) - (-6) = +4$

b) $(-6) - (+2)$

Model -6. 

There are no yellow tiles to take.
We need 2 yellow tiles to take away.

We add zero pairs.
Add 2 yellow tiles and 2 red tiles.



Now take away 2 yellow tiles.

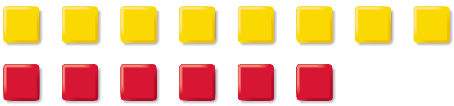
Since 8 red tiles remain, we write: $(-6) - (+2) = -8$

c) $(+2) - (-6)$

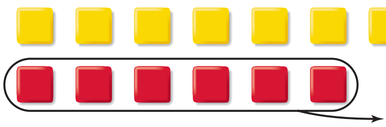
Model $+2$. 

There are no red tiles to take.
We need 6 red tiles to take away.

We add zero pairs.
Add 6 red tiles and 6 yellow tiles.



Now take away 6 red tiles.



Since 8 yellow tiles remain, we write: $(+2) - (-6) = +8$

Notice the results in the *Example*, parts b and c.
When we reverse the order in which we subtract two integers,
the answer is the opposite integer.

$(-6) - (+2) = -8$

$(+2) - (-6) = +8$

Practice

1. Use tiles to subtract. Draw pictures of the tiles you used.

a) $(+7) - (+4)$ **b)** $(-2) - (-2)$ **c)** $(-9) - (-6)$

d) $(+4) - (+2)$ **e)** $(-8) - (-1)$ **f)** $(+3) - (+3)$

2. Use tiles to subtract.

a) $(-1) - (-4)$ **b)** $(+3) - (+8)$ **c)** $(-4) - (-11)$

d) $(+7) - (+8)$ **e)** $(-4) - (-6)$ **f)** $(+1) - (+10)$

3. Subtract.

a) $(-4) - (-1)$ **b)** $(+8) - (+3)$ **c)** $(-11) - (-4)$

d) $(+8) - (+7)$ **e)** $(-6) - (-4)$ **f)** $(+10) - (+1)$

4. Subtract. Write the subtraction equations.

a) $(+4) - (-7)$ **b)** $(-2) - (+8)$ **c)** $(-9) - (+5)$

d) $(+6) - (-8)$ **e)** $(-3) - (+6)$ **f)** $(-5) - (+7)$

5. Subtract.

a) $(+4) - (+5)$ **b)** $(-3) - (+5)$ **c)** $(-4) - (+3)$

d) $(-1) - (-8)$ **e)** $(+8) - (-2)$ **f)** $(+4) - (-7)$

6. Use questions 1 to 5 as models.
Write 3 integer subtraction questions.
Trade questions with a classmate.
Solve your classmate's questions.

7. a) Use coloured tiles to subtract each pair of integers.

i) $(+3) - (+1)$ and $(+1) - (+3)$

ii) $(-3) - (-2)$ and $(-2) - (-3)$

iii) $(+4) - (-3)$ and $(-3) - (+4)$

b) What do you notice about each pair of questions in part a?

8. $(+5) - (-2) = +7$
Predict the value of $(-2) - (+5)$.
Explain your prediction, then check it.

9. **Assessment Focus** Use integers.
Write a subtraction question that would give each answer.
How many questions can you write each time?

a) $+2$ **b)** -3 **c)** $+5$ **d)** -6

10. Which expression in each pair has the greater value? Explain your reasoning.

a) i) $(+3) - (-1)$ ii) $(-3) - (+1)$

b) i) $(-4) - (-5)$ ii) $(+4) - (+5)$

11. Take It Further

a) Find two integers with a sum of -1 and a difference of $+5$.

b) Create and solve a similar integer question.

12. Take It Further Copy and complete.

a) $(+4) - \square = +3$

b) $(+3) - \square = -1$

c) $\square - (+1) = +4$

13. Take It Further Evaluate.

a) $(+4) + (+1) - (+3)$

b) $(+1) - (+2) - (-1)$

c) $(-3) - (+1) + (+4)$

d) $(-2) - (-4) + (-1)$

e) $(+2) - (+1) - (+4)$

f) $(+1) - (+2) + (+1)$

14. Take It Further Here is a magic square.

a) Subtract $+4$ from each entry. Is it still a magic square? Why?

b) Subtract -1 from each entry. Is it still a magic square? Why?

0	+5	−2
−1	+1	+3
+4	−3	+2

Reflect

Here are 4 types of subtraction questions:

- (negative integer) − (negative integer)
- (negative integer) − (positive integer)
- (positive integer) − (positive integer)
- (positive integer) − (negative integer)

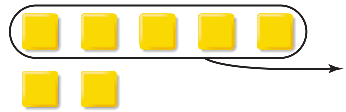
Write a question for each type of subtraction.
Show how you use tiles to solve each question.

2.5 Subtracting Integers on a Number Line

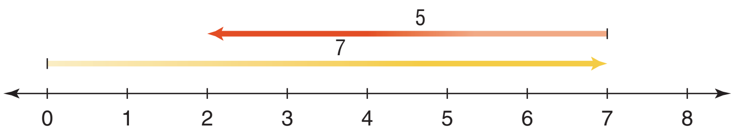
Focus Subtract integers using number lines.

Recall how to model the subtraction of whole numbers with coloured tiles.

$7 - 5 = 2$



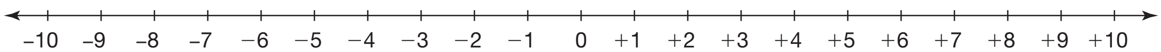
We can model this subtraction on a number line.



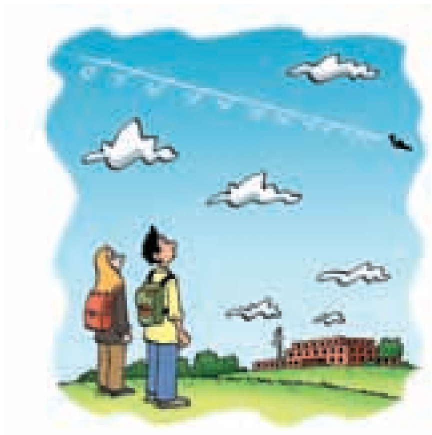
Subtraction is finding the difference. This number line shows how much more 7 is than 5.

Explore

You will need coloured tiles and copies of this number line.



Step 1 Use tiles to subtract.
Sketch the tiles you used each time.
$(+7) - (+2)$ $(-7) - (-2)$
$(+7) - (-2)$ $(-7) - (+2)$



Step 2 Model each subtraction done with tiles on a number line.

Step 3 Use any method. Add.
$(+7) + (-2)$ $(-7) + (+2)$
$(+7) + (+2)$ $(-7) + (-2)$

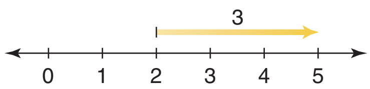
Step 4 Each expression in *Step 3* has a corresponding expression in *Step 1*.
What do you notice about the answers to corresponding expressions?
What patterns do you see in each subtraction and addition?
Check your pattern using other integers.

Reflect & Share

Compare your answers with those of another pair of classmates.
How can you use addition to subtract two integers?

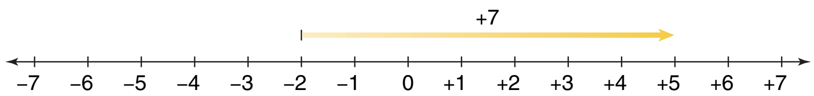
Connect

➤ To subtract two whole numbers, such as $5 - 2$,
we can think, "What do we add to 2 to get 5?"
We add 3 to 2 to get 5; so, $5 - 2 = 3$



> **We could also think: How much more is 5 than 2?**

➤ We can do the same to subtract two integers.
For example, to subtract: $(+5) - (-2)$
Think: "What do we add to -2 to get $+5$?"



We add $+7$ to -2 to get $+5$; so, $(+5) - (-2) = +7$
We also know that $(+5) + (+2) = +7$.
We can look at other subtraction equations and related addition equations.

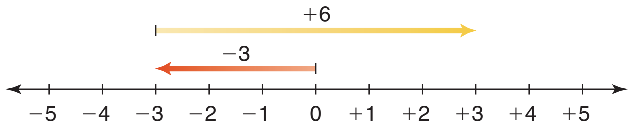
$(+9) - (+4) = +5$	$(+9) + (-4) = +5$
$(-9) - (-4) = -5$	$(-9) + (+4) = -5$
$(-9) - (+4) = -13$	$(-9) + (-4) = -13$
$(+9) - (-4) = +13$	$(+9) + (+4) = +13$

In each case, the result of subtracting an integer is the same as adding the opposite integer.
For example,

$(-9) \underbrace{- (+4)}_{\text{Subtract } +4.} = -13$ $\qquad$ $(-9) \underbrace{+ (-4)}_{\text{Add } -4.} = -13$

➤ To subtract an integer, we add the opposite integer.
For example, to subtract: $(-3) - (-6)$
Add the opposite: $(-3) + (+6)$

The opposite of -6 is $+6$.



So, $(-3) - (-6) = +3$

Example

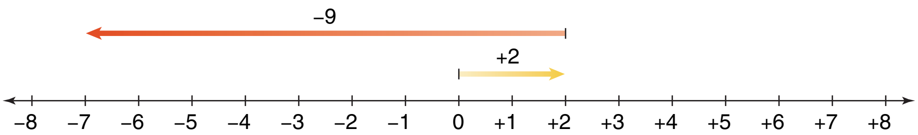
Subtract.

a) $(+2) - (+9)$ **b)** $(-2) - (+9)$

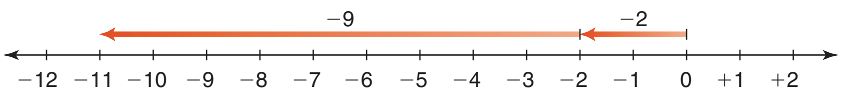
A Solution

Another Strategy
We could use coloured tiles.

a) To subtract: $(+2) - (+9)$
Add the opposite: $(+2) + (-9)$
Use a number line.
$(+2) + (-9) = -7$



b) To subtract: $(-2) - (+9)$
Add the opposite: $(-2) + (-9)$
Use a number line.
$(-2) + (-9) = -11$



Practice

1. Use a number line to subtract.
Use coloured tiles to check your answers.

a) $(+2) - (+1)$ **b)** $(+4) - (-3)$ **c)** $(-4) - (-1)$
d) $(-5) - (+2)$ **e)** $(-2) - (-6)$ **f)** $(-3) - (-7)$

2. a) Reverse the order of the integers in question 1, then subtract.
b) How are the answers different from those in question 1? Explain.

3. Use a number line to subtract. Write the subtraction equations.

a) $(+10) - (+5)$ **b)** $(+7) - (-3)$ **c)** $(-8) - (+6)$
d) $(-10) - (+5)$ **e)** $(-4) - (+4)$ **f)** $(-4) - (-4)$

4. Rewrite using addition to find each difference.

a) $(+6) - (+4)$ **b)** $(-5) - (+4)$ **c)** $(-2) - (-3)$
d) $(+4) - (-2)$ **e)** $(+1) - (+1)$ **f)** $(+1) - (-1)$

5. What is the difference in temperatures?
How can you subtract to find out?
a) A temperature 7°C above zero and a temperature 5°C below zero
b) A temperature 15°C below zero and a temperature 8°C below zero
c) A temperature 4°C below zero and a temperature 9°C above zero

6. What is the difference in golf scores?
How can you subtract to find out?
a) A golf score of 2 over par and a golf score of 6 under par
b) A golf score of 3 under par and a golf score of 8 under par
c) A golf score of 5 under par and a golf score of 4 over par

7. **a)** The table shows the average afternoon temperatures in January and April for four Canadian cities.
What is the rise in temperature from January to April for each city? Show your work.
b) Which city has the greatest difference in temperatures?
How do you know?

	City	January Temperature	April Temperature
i)	Calgary	−4°C	+13°C
ii)	Iqaluit	−22°C	−10°C
iii)	Toronto	−3°C	+12°C
iv)	Victoria	+7°C	+13°C

8. **Assessment Focus**

a) Subtract: $(-6) - (+11)$

b) Suppose we subtract the integers in the opposite order: $(+11) - (-6)$
How does the answer compare with the answer in part a?
Use number lines to explain.

c) How is $(+6) - (-11)$ different from $(-6) - (+11)$? Explain.

9. Show three ways that $+4$ can be written as the difference of two integers.

10. **Take It Further** Use patterns to subtract.

a) Subtract: $(+2) - (+5)$
Start the pattern with $(+6) - (+5) = +1$.

b) Subtract: $(+7) - (-3)$
Start the pattern with $(+7) - (+4) = +3$.

c) Subtract: $(-3) - (+7)$
Start the pattern with $(+8) - (+7) = +1$.

11. **Take It Further** Copy each integer pattern.
Write the next 4 terms.
What is the pattern rule?

a) $+6, +2, -2, \ldots$

b) $-3, -1, +1, \ldots$

c) $+5, +12, +19, \ldots$

d) $+1, 0, -1, \ldots$

12. **Take It Further** Evaluate.

a) $(+4) - (+2) - (+1)$

b) $(-2) - (+1) - (-4)$

c) $(-1) + (-2) - (+1)$

d) $(+5) - (+1) + (-2)$

e) $(+10) - (+3) - (-5)$

f) $(-7) - (+1) + (-3)$

Reflect

How is the subtraction of integers related to the addition of integers?
Use coloured tiles or a number line to show your thinking.

Writing to Reflect on Your Understanding

As you work through a math unit, you will come across many new ideas.

Sometimes it is hard to decide what you already know.
What you know can often help you understand the new ideas.

You can use a Homework Log to help you reflect on your understanding.

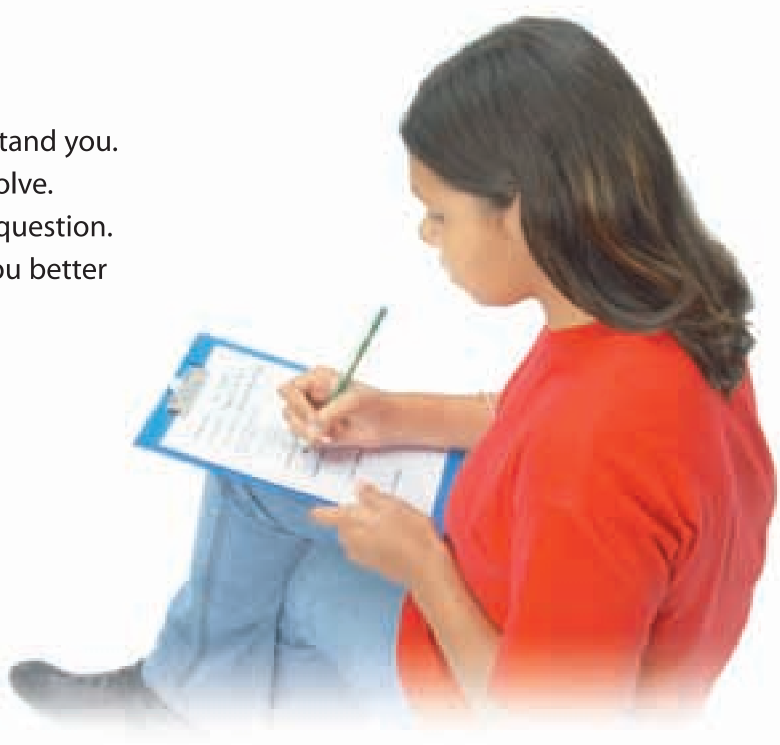
Using a Homework Log

As you work through your homework, ask yourself:

- What is the key idea?
- How difficult is the homework for me?
- Which questions am I able to do?
- Which questions do I need help with?
- What questions could I ask to help me with my homework?

Tips for Writing a Homework Log

- Write so that someone else can understand you.
- Write out a question that you cannot solve.
- Describe 3 ways you tried to solve the question.
- Write a question you can ask to help you better understand your homework.



Here is a sample Homework Log.

Name: Asad

Homework Log

The homework was . . . P26 #5–12

The key concept was . . . Subtracting integers on a number line

Overall, I'd rate the difficulty level of the homework as . . .

Easy 0 1 2 3 4 5 6 7 8 9 10 Hard

One question I had difficulty with but solved was . . .
What is the difference in temperatures?
A temperature 7°C above zero and a temperature 5°C below zero

7°C
0°C
-5°C
12°C

A question I couldn't solve was . . .
Evaluate: $(+5) - (+1) + (-2)$

To solve it I tried these things . . .
1. I used my calculator but I know I should be able to do it without one.
2. I tried to model it on a number line but I didn't know which way to draw the arrows.
3. I looked at the example in the book. It says to "add the opposite" but I don't know what that means.

Questions for experts . . .
What does "add the opposite" mean?
How do you take away a negative integer?

✓ Check

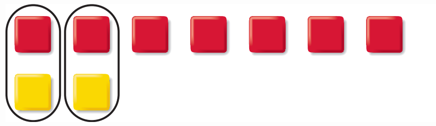
- Complete a Homework Log for your next homework assignment.
- Share your Homework Log with a classmate.
- Try to help each other with questions that you were unable to solve.

Unit Review

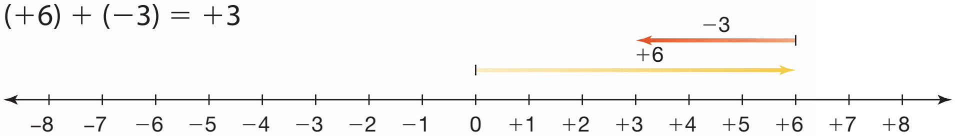
What Do I Need to Know?

☑ Adding Integers

- You can use tiles to add integers.
  $(-7) + (+2) = -5$



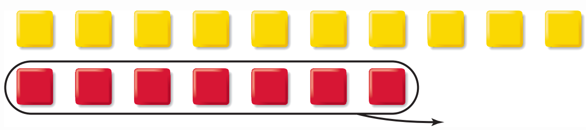
- You can use a number line to add integers.
  $(+6) + (-3) = +3$



☑ Subtracting Integers

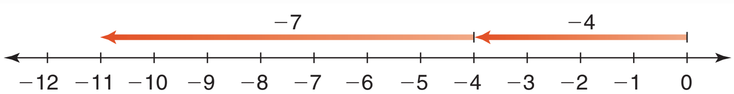
- You can use tiles to subtract integers: $(+3) - (-7)$
  We need enough red tiles to take away 7 of them.
  Model $+3$: 

  Since there are not enough tiles to take away -7, add 7 yellow tiles and 7 red tiles. Now take away 7 red tiles. There are 10 yellow tiles left.



$(+3) - (-7) = +10$

- You can also subtract by adding the opposite:
  $(-5) - (-8) = (-5) + (+8)$
  $= +3$
- You can use a number line to subtract integers.
  $(-4) - (+7)$
  Add the opposite: $(-4) + (-7)$
  Use a number line.
  $(-4) - (+7) = -11$



What Should I Be Able to Do?

LESSON

2.1

1. Suppose you have 17 red tiles. How many yellow tiles would you need to model:

a) −12? **b)** 0?

c) +20? **d)** −17?

How do you know?

2. Write the integer suggested by each of the following situations. Draw yellow or red tiles to model each integer. Explain your choice.

a) The temperature rises 8°C.

b) The price of 1 L of gas falls 5¢.

c) You deposit $12 in your bank account.

d) You take 7 steps backward.

e) The time is 9 s before take-off.

2.2

3. What sum does each set of tiles model?

a) 5 red tiles and 2 yellow tiles

b) 6 yellow tiles and 5 red tiles

c) 6 yellow tiles and 7 red tiles

d) 8 yellow tiles and 8 red tiles

4. Represent each sentence with integers, then find each sum.

a) The temperature was −6°C, then rose 4°C.

b) Surinder withdrew $25, then deposited $13.

c) A stock gained $15, then lost $23.

d) A submarine was 250 m below sea level, then ascended 80 m.

5. a) Find 4 pairs of integers that have the sum −5.

b) Find 4 pairs of integers that have the sum +4.

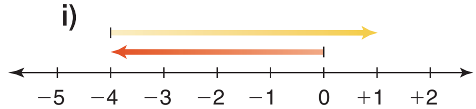
2.3

6. The temperature at 6 a.m. is −10°C. During the day, the temperature rises 17°C. What is the new temperature? Write an addition equation to represent this situation. Use a vertical number line to support your answer.

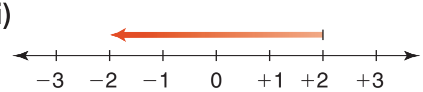
7. a) Write an addition equation modelled by each number line.

b) Describe a situation that each number line could represent.

i)



ii)



2.2
2.4

8. Use tiles to add or subtract.

a) (−1) + (+3)

b) (+3) + (−4)

c) (−2) − (+3)

d) (−1) − (−3)

LESSON

2.3
2.5

9. Use a number line to add or subtract.

a) $(-1) + (+3)$ **b)** $(+6) + (-4)$
c) $(-4) - (+6)$ **d)** $(-5) - (-3)$

10. When you add two positive integers, their sum is always a positive integer.
When you subtract two positive integers, is their difference always a positive integer? Explain.

11. a) What temperature is 7°C warmer than 2°C?
b) What temperature is 5°C warmer than -5°C?
c) What temperature is 8°C cooler than 2°C?
d) What temperature is 4°C cooler than -3°C?

2.4
2.5

12. Use tiles or a number line to subtract. Write the subtraction equations.

a) $(+4) - (+1)$ **b)** $(+5) - (-1)$
c) $(+2) - (-2)$ **d)** $(-4) - (+1)$
e) $(-6) - (-2)$ **f)** $(-10) - (-5)$
g) $(-4) - (-2)$ **h)** $(-5) - (-10)$

13. Subtract.

a) $(+7) - (+2)$ **b)** $(-7) - (+3)$
c) $(-4) - (-5)$ **d)** $(+3) - (+3)$
e) $(+3) - (-3)$ **f)** $(-3) - (-2)$

14. Use tiles or a number line.
Find the difference between:

a) a temperature of $+5$°C and -7°C
b) an elevation of -100 m and $+50$ m

15. What is the difference in heights?
How can you subtract to find out?

a) A water level of 2 m below sea level and a water level of 7 m above sea level
b) A balloon 25 m above ground and a balloon 11 m above ground

16. What is the difference in masses?
How can you subtract to find out?

a) A gain of 9 kg and a loss of 3 kg
b) A loss of 6 kg and a loss of 5 kg

17. We measure time in hours.
Suppose 12 noon is represented by the integer 0.

a) Which integer represents 1 p.m. the same day?
b) Which integer represents 10 a.m. the same day?
c) Find the difference between these times in 2 ways.
Show your work.

18. a) Find 5 pairs of integers with a difference of $+6$.
b) Find 5 pairs of integers with a difference of -3.

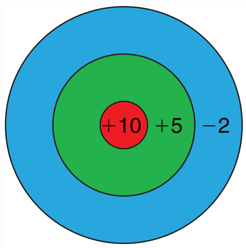
Practice Test

1. Evaluate. Use coloured tiles.
Record your work.
a) $(+5) + (-8)$ **b)** $(-3) - (+7)$ **c)** $(-9) + (-1)$
d) $(-4) + (+10)$ **e)** $(-6) - (-2)$ **f)** $(+12) - (-11)$

2. Evaluate. Use a number line.
Record your work.
a) $(+9) + (-1)$ **b)** $(-4) - (+11)$ **c)** $(-8) + (-3)$
d) $(+13) - (+6)$ **e)** $(-7) + (+9)$ **f)** $(-1) - (-5)$

3. Without calculating the sum, how can you tell if the sum of two integers will be:
a) zero? **b)** negative? **c)** positive?
Include examples in your explanations.

4. Here is a different type of dartboard.
A player throws 2 darts at the board.
His score is the sum of the integers in the areas his darts land.
Assume both darts hit the board.
a) How many different scores are possible?
b) Find each score.

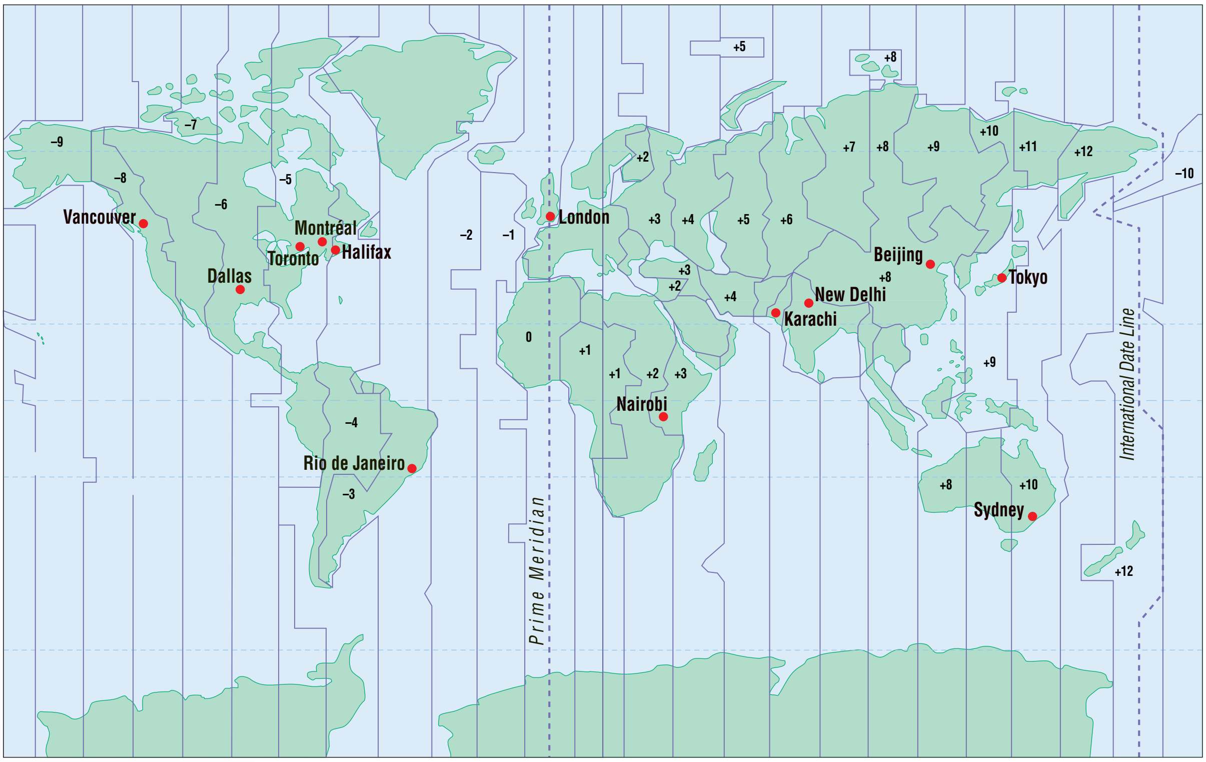


5. The lowest temperature possible is approximately −273°C.
The temperature at which water boils is 100°C.
What is the difference in these temperatures?

6. Place 3 integers in a row as shown.
$(+6)$ $(+4)$ (-3)
How many different answers can you get by putting addition and/or subtraction signs between the integers?
How do you know you have found all possible answers?
For example: $(+6) + (+4) - (-3)$
What if there were 4 integers in a row?

Unit Problem What Time Is It?

TIME ZONES



The map shows the world's time zones.
Greenwich, in London, England, is the reference point, or the zero for the time zones. Its time is called UTC, or Coordinated Universal Time. London, England, is also in this time zone.
The positive and negative integers on the map show the changes in time from UTC.

The 2008 Summer Olympics will be held in Beijing, China.

1. The local start times of some Olympic events are given.
Family members want to watch these events live, in Brandon (the same time zone as Dallas).
What time should they "tune in"? How do you know?

a) 200-m backstroke at 2:00 p.m.
b) 100-m dash at 7:00 p.m.
c) gymnastics at 11:00 p.m.
d) middleweight boxing at 8:00 a.m.

2. An event is broadcast live in Montreal at 9:00 p.m.
What time is it taking place in Beijing?
Show your work.

3. Two pen pals plan to meet in Beijing for the Olympics.
Atsuko lives in Tokyo, Japan.
She can get a direct flight to Beijing that takes 4 h.
Paula lives in Sydney, Australia, and her direct flight takes 13 h.
What time does each girl leave her country to arrive in Beijing at 6 p.m., Beijing time?

4. Olympic funding depends on money from North American television networks. What problems will the organizers of the Beijing Olympics encounter when they plan the times for events?

5. Make up your own problem about the time zone map.
Solve your problem. Show your work.

Show how you can use integers to solve each problem.

Check List

Your work should show:

- ✓ how you used integers to solve each problem
- ✓ the strategies you used to add and subtract integers
- ✓ correct calculations
- ✓ a clear explanation of how integers relate to time zones

Reflect on Your Learning

Suppose there were no negative integers.
Could we survive in a world without negative integers?
Explain.

UNIT 3

Fractions, Decimals, and Percents

Stores offer goods on sale to encourage you to spend money.
Look at these advertisements.
What is the sale price of each item in the picture using each advertisement?
How did you calculate the sale price?
Explain your strategy.

What You'll Learn

- Convert between fractions and terminating or repeating decimals.
- Compare and order fractions, decimals, and mixed numbers.
- Add, subtract, multiply, and divide decimals.
- Solve problems involving fractions, decimals, and percents.

Why It's Important

- You use fractions and decimals when you shop, measure, and work with a percent; and in sports, recipes, and business.
- You use percents when you shop, to find sale prices and to calculate taxes.

Key Words

- terminating decimal
- repeating decimal

3.1 Fractions to Decimals

Focus Use patterns to convert between decimals and fractions.

Numbers can be written in both fraction and decimal form.

For example, 3 can be written as $\frac{3}{1}$ and 3.0.

A fraction illustrates division;

that is, $\frac{1}{10}$ means $1 \div 10$.

Recall that $\frac{1}{10}$ is 0.1 in decimal form.

$\frac{3}{100}$ is 0.03 in decimal form.

$\frac{45}{1000}$ is 0.045 in decimal form.

Here are some more fractions and decimals you learned in earlier grades.

Fraction	$\frac{7}{10}$	$\frac{1}{100}$	$\frac{19}{100}$	$\frac{1}{1000}$	$\frac{23}{1000}$	$\frac{471}{1000}$
Decimal	0.7	0.01	0.19	0.001	0.023	0.471

Explore

You will need a calculator.

➤ Use a calculator.
Write each fraction as a decimal: $\frac{1}{11}, \frac{2}{11}, \frac{3}{11}, \frac{4}{11}$
What patterns do you see?
Use your patterns to predict the decimal forms of these fractions:
$\frac{5}{11}, \frac{6}{11}, \frac{7}{11}, \frac{8}{11}, \frac{9}{11}, \frac{10}{11}$
Use a calculator to check your predictions.

➤ Use a calculator.
Write each fraction as a decimal: $\frac{1}{9}, \frac{2}{9}, \frac{3}{9}$
What patterns do you see?
Use your patterns to predict the fraction form of these decimals:
0.777 777 777… 0.888 888 888…
Check your predictions.
What do you notice about the last digit in the calculator display?

Reflect & Share

Compare your patterns, decimals, and fractions with those of another pair of classmates. How did you use patterns to make predictions?

Connect

➤ Decimals, such as 0.1 and 0.25, are **terminating decimals**.
Each decimal has a definite number of decimal places.

➤ Decimals, such as 0.333 333 333…; 0.454 545 454…; 0.811 111 111…
are **repeating decimals**.
Some digits in each repeating decimal repeat forever.
We draw a bar over the digits that repeat.
For example, $\frac{4}{33} = 4 \div 33 = 0.121\ 212\ 121\ldots$, which is written as $0.\overline{12}$
$\frac{73}{90} = 73 \div 90 = 0.811\ 111\ 111\ldots$, which is written as $0.8\overline{1}$

➤ Patterns sometimes occur when we write fractions in decimal form.
For example,
$\frac{1}{99} = 0.\overline{01}$ $\frac{2}{99} = 0.\overline{02}$ $\frac{15}{99} = 0.\overline{15}$ $\frac{43}{99} = 0.\overline{43}$
For fractions with denominator 99, the digits in the numerator of the fraction are the repeating digits in the decimal.
We can use this pattern to make predictions.
To write $0.\overline{67}$ as a fraction, write the repeating digits, 67, as the numerator of a fraction with denominator 99.
$0.\overline{67} = \frac{67}{99}$
Similarly, $0.\overline{7} = 0.\overline{77} = \frac{77}{99} = \frac{7}{9}$

Example

a) Write each fraction as a decimal.
b) Sort the fractions as representing repeating or terminating decimals:
$\frac{13}{200}, \frac{1}{5}, \frac{11}{20}, \frac{3}{7}$

A Solution

a) Try to write each fraction with denominator 10, 100, or 1000.

$$\frac{13}{200} = \frac{13 \times 5}{200 \times 5} = \frac{65}{1000}, \text{ or } 0.065$$

$$\frac{1}{5} = \frac{1 \times 2}{5 \times 2} = \frac{2}{10}, \text{ or } 0.2$$

$$\frac{11}{20} \underset{\times 5}{\overset{\times 5}{=}} \frac{55}{100}, \text{ or } 0.55$$

$\frac{3}{7}$ cannot be written as a fraction with denominator 10, 100, or 1000.

Use a calculator.

$\frac{3}{7} = 3 \div 7 = 0.428\,571\,429$

This appears to be a terminating decimal.

We use long division to check.

Since we are dividing by 7, the remainders must be less than 7.

Since we get a remainder that occurred before, the division repeats.

So, $\frac{3}{7} = 0.\overline{428\,571}$

$$\begin{array}{r}
0.4285714 \\
7\overline{)\,3.00000000} \\
\underline{28} \\
20 \\
\underline{14} \\
60 \\
\underline{56} \\
40 \\
\underline{35} \\
50 \\
\underline{49} \\
10 \\
\underline{7} \\
30 \\
\underline{28} \\
20
\end{array}$$

The calculator rounds the decimal to fit the display:

$\frac{3}{7} = 0.428\,571\,428\,571\ldots$

This is the last digit in the display.

Since this digit is 5, the calculator adds 1 to the preceding digit.

So, the calculator displays an approximate decimal value:

$\frac{3}{7} \doteq 0.428\,571\,429$

b) Since 0.065, 0.2, and 0.55 terminate, $\frac{13}{200}$, $\frac{1}{5}$, and $\frac{11}{20}$ represent terminating decimals.
Since $0.\overline{428\,571}$ repeats, $\frac{3}{7}$ represents a repeating decimal.

Practice

Use a calculator when you need to.

1. **a)** Write each fraction as a decimal.

i) $\frac{2}{3}$ **ii)** $\frac{3}{4}$ **iii)** $\frac{4}{5}$ **iv)** $\frac{5}{6}$ **v)** $\frac{6}{7}$

b) Identify each decimal as terminating or repeating.

2. Write each decimal as a fraction.

a) 0.9 **b)** 0.26 **c)** 0.45 **d)** 0.01 **e)** 0.125

3. a) Write each fraction as a decimal.

i) $\frac{1}{27}$ ii) $\frac{2}{27}$ iii) $\frac{3}{27}$

b) Describe the pattern in your answers to part a.

c) Use your pattern to predict the decimal form of each fraction.

i) $\frac{4}{27}$ ii) $\frac{5}{27}$ iii) $\frac{8}{27}$

4. For each fraction, write an equivalent fraction with denominator 10, 100, or 1000.
Then, write the fraction as a decimal.

a) $\frac{2}{5}$ b) $\frac{1}{4}$ c) $\frac{13}{25}$ d) $\frac{19}{50}$ e) $\frac{37}{500}$

5. Write each decimal as a fraction in simplest form.

a) $0.\overline{6}$ b) $0.\overline{5}$ c) $0.\overline{41}$ d) $0.\overline{16}$

6. Write each fraction as a decimal.

a) $\frac{4}{7}$ b) $\frac{4}{9}$ c) $\frac{6}{11}$ d) $\frac{7}{13}$

7. Write $\frac{5}{17}$ as a decimal.
The calculator display is not long enough to show the repeating digits.
How could you find the repeating digits?

8. Write $\frac{1}{5}$ as a decimal.
Use this decimal to write each number below as a decimal.

a) $\frac{4}{5}$ b) $\frac{7}{5}$ c) $\frac{9}{5}$ d) $\frac{11}{5}$

9. a) Write each fraction as a decimal.

i) $\frac{1}{999}$ ii) $\frac{2}{999}$ iii) $\frac{54}{999}$ iv) $\frac{113}{999}$

b) Describe the pattern in your answers to part a.

c) Use your pattern to predict the fraction form of each decimal.

i) $0.\overline{004}$ ii) $0.\overline{089}$ iii) $0.\overline{201}$ iv) $0.\overline{326}$

10. Match each set of decimals and fractions.
Explain how you know.

a) $\frac{1}{3}, \frac{2}{3}, \frac{3}{3}, \frac{4}{3}, \frac{5}{3}$ i) 0.125, 0.25, 0.375, 0.5, 0.625

b) $\frac{1}{8}, \frac{2}{8}, \frac{3}{8}, \frac{4}{8}, \frac{5}{8}$ ii) $0.1\overline{6}$, $0.\overline{3}$, 0.5, $0.\overline{6}$, $0.8\overline{3}$

c) $\frac{1}{5}, \frac{2}{5}, \frac{3}{5}, \frac{4}{5}, \frac{5}{5}$ iii) $0.\overline{3}$, $0.\overline{6}$, 1.0, $1.\overline{3}$, $1.\overline{6}$

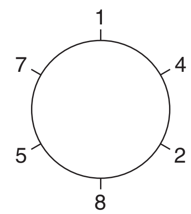
d) $\frac{1}{6}, \frac{2}{6}, \frac{3}{6}, \frac{4}{6}, \frac{5}{6}$ iv) 0.2, 0.4, 0.6, 0.8, 1.0

11. **Assessment Focus** Here is the Fibonacci sequence:
1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, …
We can write consecutive terms as fractions:
$\frac{1}{1}, \frac{2}{1}, \frac{3}{2}, \frac{5}{3}, \frac{8}{5}, \frac{13}{8}$, and so on

a) Write each fraction above as a decimal.
What do you notice about the trend in the decimals?

b) Continue to write consecutive terms as decimals.
Write about what you find out.

12. a) Write $\frac{1}{7}$ as a repeating decimal.
How many digits repeat?
These repeating digits are shown around the circle at the right.



b) Write the fractions $\frac{2}{7}, \frac{3}{7}, \frac{4}{7}, \frac{5}{7}$, and $\frac{6}{7}$ in decimal form.
What patterns do you see?
Explain how the circle of digits can help you write these fractions as decimals.

13. **Take It Further**

a) Write each fraction as a decimal.
Identify the decimals as repeating or terminating.
i) $\frac{7}{8}$ ii) $\frac{5}{18}$ iii) $\frac{3}{10}$ iv) $\frac{8}{27}$ v) $\frac{4}{25}$

b) Write the denominator of each fraction in part a as a product of prime factors.

c) What do you notice about the prime factors of the denominators of the terminating decimals? The repeating decimals?

d) Use your answers to part c.
Predict which of these fractions can be written as terminating decimals.
i) $\frac{7}{15}$ ii) $\frac{13}{40}$ iii) $\frac{5}{81}$ iv) $\frac{9}{16}$

A prime number has exactly two factors, itself and 1. We can write 12 as a product of prime factors: $2 \times 2 \times 3$

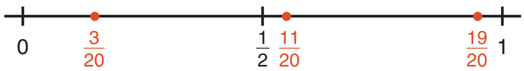
Reflect

Sometimes it is hard to figure out if a fraction can be written as a terminating decimal or a repeating decimal.
What can you do if you are stuck?

3.2 Comparing and Ordering Fractions and Decimals

Focus Use benchmarks, place value, and equivalent fractions to compare and order fractions and decimals.

Recall how to use the benchmarks 0, $\frac{1}{2}$, and 1 to compare fractions.
For example, $\frac{3}{20}$ is close to 0 because the numerator is much less than the denominator.
$\frac{11}{20}$ is close to $\frac{1}{2}$ because the numerator is about $\frac{1}{2}$ the denominator.
$\frac{19}{20}$ is close to 1 because the numerator and denominator are close in value.



Explore

Use any materials to help.
Dusan, Sasha, and Kimberley sold chocolate bars as a fund-raiser for their choir.
The bars were packaged in cartons, but sold individually.
Dusan sold $2\frac{2}{3}$ cartons. Sasha sold $\frac{5}{2}$ cartons. Kimberley sold 2.25 cartons.
Who sold the most chocolate bars?

Reflect & Share

Share your solution with another pair of classmates.
How did you decide which number was greatest?
Did you use any materials to help? How did they help?
Try to find a way to compare the numbers without using materials.

Connect

Any fraction greater than 1 can be written as a mixed number.
The benchmarks 0, $\frac{1}{2}$, and 1 can be used to compare the fraction parts of mixed numbers.

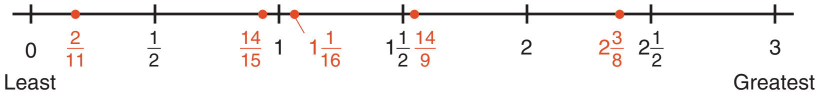
We can use benchmarks on a number line to order these numbers: $\frac{2}{11}$, $2\frac{3}{8}$, $1\frac{1}{16}$, $\frac{14}{9}$, $\frac{14}{15}$
$\frac{2}{11}$ is close to 0.
Since $\frac{3}{8}$ is close to $\frac{1}{2}$, but less than $\frac{1}{2}$,
$2\frac{3}{8}$ is close to $2\frac{1}{2}$, but less than $2\frac{1}{2}$.
$1\frac{1}{16}$ is close to 1, but greater than 1.

$\frac{14}{9} = \frac{9}{9} + \frac{5}{9} = 1\frac{5}{9}$

$1\frac{5}{9}$ is close to $1\frac{1}{2}$, but greater than $1\frac{1}{2}$.

$\frac{14}{15}$ is close to 1, but less than 1.

Place the fractions on a number line.



The numbers in order from greatest to least are: $2\frac{3}{8}, \frac{14}{9}, 1\frac{1}{16}, \frac{14}{15}, \frac{2}{11}$

We can also use equivalent fractions to order fractions.

Example

a) Write these numbers in order from least to greatest: $\frac{7}{8}, \frac{9}{8}, 1\frac{1}{4}, 0.75$

b) Write a fraction between $\frac{9}{8}$ and $1\frac{1}{4}$.

A Solution

a) Write equivalent fractions with like denominators, then compare the numerators.

First write the decimal as a fraction: $0.75 = \frac{75}{100} = \frac{3}{4}$

Compare: $\frac{7}{8}, \frac{9}{8}, 1\frac{1}{4}, \frac{3}{4}$

Since 8 is a multiple of 4, use 8 as a common denominator.

$$1\frac{1}{4} = \frac{4}{4} + \frac{1}{4}$$
$$= \frac{5}{4}$$

$$\frac{3}{4} \overset{\times 2}{\underset{\times 2}{=}} \frac{6}{8}$$

$$\frac{5}{4} \overset{\times 2}{\underset{\times 2}{=}} \frac{10}{8}$$

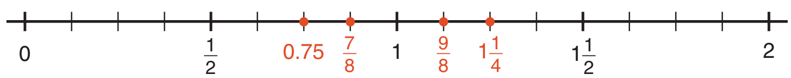
Each fraction now has denominator 8: $\frac{7}{8}, \frac{9}{8}, \frac{10}{8}, \frac{6}{8}$

Compare the numerators: $6 < 7 < 9 < 10$

So, $\frac{6}{8} < \frac{7}{8} < \frac{9}{8} < \frac{10}{8}$

So, $0.75 < \frac{7}{8} < \frac{9}{8} < 1\frac{1}{4}$

We can verify this order by placing the numbers on a number line.



b) Use the equivalent fraction for $1\frac{1}{4}$ with denominator 8 from part a: $\frac{10}{8}$

Find a fraction between $\frac{9}{8}$ and $\frac{10}{8}$.

The numerators are consecutive whole numbers. There are no whole numbers between 9 and 10. Multiply the numerator and denominator of both fractions by the same number to get equivalent fractions.

Choose 2:

$$\frac{9}{8} \underset{\times 2}{\overset{\times 2}{=}} \frac{18}{16} \qquad \frac{10}{8} \underset{\times 2}{\overset{\times 2}{=}} \frac{20}{16}$$

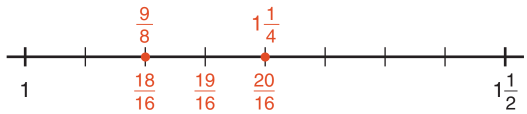
Look at the numerators.

19 is between 18 and 20,

so $\frac{19}{16}$ is between $\frac{18}{16}$ and $\frac{20}{16}$.

So, $\frac{19}{16}$, or $1\frac{3}{16}$, is between $\frac{9}{8}$ and $1\frac{1}{4}$.

We can verify this using a number line.



Another Solution

We can also use place value to order decimals.

a) Write each number as a decimal.

$\frac{7}{8} = 0.875$ $\qquad \frac{9}{8} = 1.125$ $\qquad 1\frac{1}{4} = 1.25$ $\qquad 0.75$

Write each decimal in a place-value chart.

Compare the ones.

Two numbers have 1 one and two numbers have 0 ones.

Ones	•	Tenths	Hundredths	Thousandths
0	•	8	7	5
1	•	1	2	5
1	•	2	5	0
0	•	7	5	0

Look at the decimals with 0 ones: 0.**8**75, 0.**7**50

Compare the tenths: 7 tenths is less than 8 tenths, so $0.750 < 0.875$

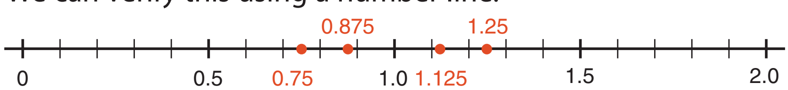
Look at the decimals with 1 one: 1.**1**25 and 1.**2**50

Compare the tenths: 1 tenth is less than 2 tenths, so $1.125 < 1.250$

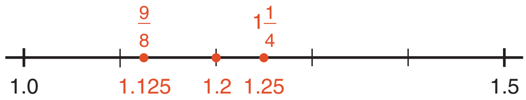
The numbers in order from least to greatest are: 0.750, 0.875, 1.125, 1.250

So, $0.75 < \frac{7}{8} < \frac{9}{8} < 1\frac{1}{4}$

We can verify this using a number line.



b) $\frac{9}{8} = 1.125$ $\qquad$ $1\frac{1}{4} = 1.25$



Use the number line above.

1.2 lies between 1.125 and 1.25.

Write 1.2 as a fraction.

1.2 is $1\frac{2}{10}$, or $1\frac{1}{5}$.

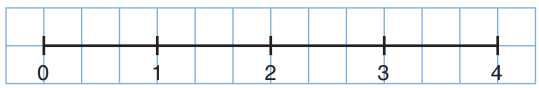
So, $1\frac{1}{5}$, or $\frac{6}{5}$, lies between $\frac{9}{8}$ and $1\frac{1}{4}$.

There are many other possible fractions between $\frac{9}{8}$ and $1\frac{1}{4}$.

Practice

1. Write 5 different fractions with like denominators.
Draw a number line, then order the fractions on the line.
Explain your strategy.

2. Use 1-cm grid paper.
Draw a 12-cm number line like the one shown.
Use the number line to order these numbers from greatest to least.
$2\frac{1}{2}, \frac{11}{3}, 2\frac{5}{6}$



3. Use benchmarks and a number line to order each set of numbers from least to greatest.

a) $\frac{7}{6}, \frac{15}{12}, 1\frac{2}{9}, 1$ $\qquad$ b) $1\frac{3}{4}, \frac{7}{3}, \frac{7}{6}, 2$ $\qquad$ c) $\frac{7}{4}, \frac{15}{10}, \frac{11}{5}, 2$ $\qquad$ d) $\frac{10}{4}, 2\frac{1}{3}, \frac{9}{2}, 3$

4. Use equivalent fractions.
Order each set of numbers from greatest to least.
Verify by writing each fraction as a decimal.

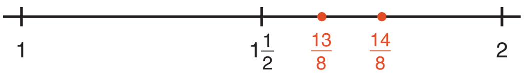
a) $3\frac{1}{2}, \frac{13}{4}, 3\frac{1}{8}$ $\qquad$ b) $\frac{5}{6}, \frac{2}{3}, 1\frac{1}{12}, \frac{9}{12}$ $\qquad$ c) $1\frac{2}{5}, \frac{4}{3}, \frac{3}{2}$

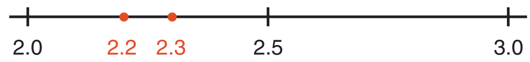
5. Use place value.
Order each set of numbers from least to greatest.
Verify by using a number line.

a) $\frac{7}{4}$, 1.6, $1\frac{4}{5}$, 1.25, 1 $\qquad$ b) $2\frac{5}{8}$, 1.875, $2\frac{3}{4}$, $\frac{5}{2}$, 2

6. a) Use any method. Order these numbers from greatest to least.
Explain the method you used.
$\frac{17}{5}$, 3.2, 2.8, $3\frac{1}{4}$, $\frac{21}{7}$, 2
b) Use a different method. Verify your answer in part a.

7. Find a number between the two numbers represented by each pair of dots.

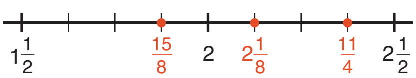
a) 

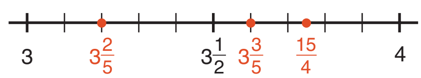
b) 

8. Find a number between each pair of numbers.

a) $\frac{5}{7}$, $\frac{6}{7}$ b) $1\frac{2}{5}$, $\frac{8}{5}$ c) 1.3, $1\frac{2}{5}$ d) 0.5, 0.6

9. Identify the number that has been placed incorrectly.
Explain how you know.

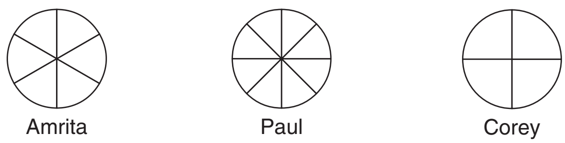
a) 

b) 

10. In each set, identify the number that is not in the correct order.
Show where it should go. Explain your work.

a) $\frac{29}{5}$, $6\frac{2}{10}$, 6.25, $6\frac{2}{20}$ b) $1\frac{7}{16}$, $1\frac{3}{8}$, $\frac{3}{2}$, 1.2, $\frac{3}{4}$

11. **Assessment Focus** Amrita, Paul, and Corey baked pizzas for the fund-raising sale.
The students cut their pizzas into different sized slices.



Amrita sold $\frac{11}{6}$ pizzas. Paul sold 1.875 pizzas. Corey sold $\frac{9}{4}$ pizzas.
a) Use a number line to order the numbers of pizzas sold from least to greatest.
b) Who sold the most pizzas? The fewest pizzas?
c) Use a different method. Verify your answers in part b.
d) Alison sold $2\frac{1}{5}$ pizzas. Where does this fraction fit in part a?

Reflect

Describe 3 ways to compare and order fractions and decimals.
Give an example of when you would use each method.
Which way do you prefer? Why?

3.3 Adding and Subtracting Decimals

Focus Add and subtract decimals to thousandths.

When you go to the theatre to see a movie, your attendance and how much you paid to see the movie are entered in a database.
Data are collected from theatres all across Canada and the United States.
Movie studios use these data to help predict how much money the movie will earn.

Explore

Shrek 2 was one of the highest-earning movies of 2004.
The table shows how much money *Shrek 2* earned in Canada and the United States for the first week it played in theatres.
Studios record the earnings in millions of US dollars.

Date	Earnings (US$ Millions)
Wednesday, May 19	11.786
Thursday, May 20	9.159
Friday, May 21	28.340
Saturday, May 22	44.797
Sunday, May 23	34.901
Monday, May 24	11.512
Tuesday, May 25	8.023

➤ Estimate first.
Then find the combined earnings on:
- the first 2 days
- Saturday and Sunday
- all 7 days

➤ Estimate first.
Then find the difference in earnings on:
- Thursday and Friday
- Saturday and Sunday
- Sunday and Monday
- the days with the greatest and the least earnings

Reflect & Share

Share your results with another pair of classmates.
Discuss the strategies you used to estimate and to find the sums and differences.
Why do you think the earnings on 3 of the days are so much higher? Explain.

Connect

When we add or subtract decimals, we estimate if we do not need an exact answer.
We also estimate to check the answer is reasonable.

Example

Ephram is a long-distance runner. His practice distances for 5 days last week are shown in the table.

a) How far did Ephram run in 5 days last week?

b) How much farther did Ephram run on Tuesday than on Thursday?

Day	Distance (km)
Monday	8.85
Tuesday	12.25
Wednesday	10.9
Thursday	9.65
Friday	14.4

A Solution

a) $8.85 + 12.25 + 10.9 + 9.65 + 14.4$

Use front-end estimation.
Add the whole-number part of each decimal.
Think: $8 + 12 + 10 + 9 + 14 = 53$
Ephram ran about 53 km.

Add. Write each number with the same number of decimal places.
Use zeros as placeholders: 8.85, 12.25, 10.90, 9.65, 14.40
Record the numbers without the decimal points.
Add as you would whole numbers.

$$\begin{array}{r} {}^{2\,3\,1} \\ 885 \\ 1225 \\ 1090 \\ 965 \\ +\ 1440 \\ \hline 5605 \end{array}$$

Since the estimate is 53 km, place the decimal point after the first 2 digits; that is, between the 6 and the 0.
Ephram ran 56.05 km.

b) Ephram ran 12.25 km on Tuesday and 9.65 km on Thursday.
Estimate.
$12.25 - 9.65$
Think: $12 - 9 = 3$
Ephram ran about 3 km farther on Tuesday.

Subtract. Align the numbers.
Subtract as you would whole numbers.

$$\begin{array}{r} {}^{11\ 12} \\ \cancel{1}\cancel{2}.\cancel{2}5 \\ -\ \ 9.65 \\ \hline 2.60 \end{array}$$

2.6 is close to the estimate 3, so the answer is reasonable.
Ephram ran 2.6 km farther on Tuesday than on Thursday.

Practice

1. Use front-end estimation to estimate each sum or difference.

a) $2.876 - 0.975$ b) $71.382 + 6.357$

c) $125.12 + 37.84$ d) $9.7 - 1.36$

2. The tallest building in the world is the Taipei 101 in Taiwan. Its height is 0.509 km. The tallest building in North America is the Sears Tower in Chicago, USA. Its height is 0.442 km. What is the difference in the heights of the buildings?

3. Four classes of students from Mackenzie School are planning a field trip. The total cost of the trip is \$1067.50.
To date, the classes have raised: \$192.18, \$212.05, \$231.24, \$183.77

a) How much money have the classes raised so far?

b) How much more money do the classes need to raise in total?
Show your work.

4. **Assessment Focus** A baker wants to make 3 different kinds of chocolate chip cookies. The recipes call for 2.75 kg, 4.4 kg, and 5.55 kg of chocolate chips. The baker has 10.5 kg of chocolate chips.

a) How many kilograms of chocolate chips does the baker need?
Estimate to check your answer is reasonable.

b) Does the baker have enough chocolate chips to make the cookies?
How do you know?

c) The baker wants to follow the recipes exactly.
If your answer to part b is no, how many more kilograms of chocolate chips are needed? If your answer to part b is yes, how many kilograms of chocolate chips will the baker have left over?

5. Estimate, then calculate, the sum below.
Explain how you estimated.
46.71 + 3.9 + 0.875

6. The Robb family and the Chan family have similar homes.
The Robb family sets its thermostat to 20°C during the winter months.
Its monthly heating bills were: \$171.23, \$134.35, and \$123.21
The Chan family used a programmable thermostat to lower the temperature at night, and during the day when the family was out.
The Chan family's monthly heating bills were: \$134.25, \$103.27, and \$98.66
a) How much money did each family pay to heat its home during the winter months?
b) How much more money did the Robb family pay?
Estimate to check your answer is reasonable.
c) What other things could a family do to reduce its heating costs?

7. Find two numbers with a difference of 151.297.

8. Use each of the digits from 0 to 7 once to make this addition true.
Find as many different answers as you can.

$$\begin{array}{r} \square.\square\square\square \\ +\ \square.\square\square\square \\ \hline 5.788 \end{array}$$

9. A student subtracted 0.373 from 4.81 and got the difference 0.108.
a) What mistake did the student make?
b) What is the correct answer?

10. Two 4-digit numbers were added. Their sum was 3.3.
What could the numbers have been?
Find as many different answers as you can. Show your work.

11. **Take It Further** Find each pattern rule. Explain how you found it.
a) 2.09, 2.13, 2.17, 2.21, …
b) 5.635, 5.385, 5.135, 4.885, …

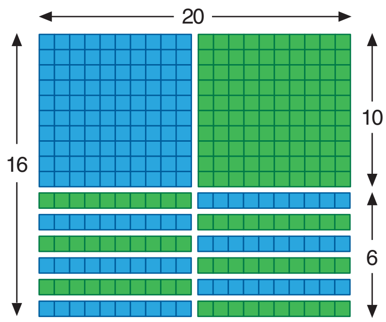
Reflect

How did your knowledge of estimation help you in this lesson?

3.4 Multiplying Decimals

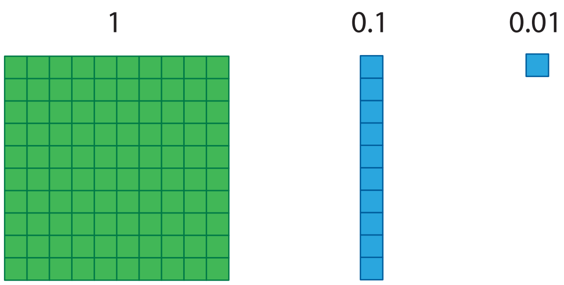
Focus Use Base Ten Blocks, paper and pencil, and calculators to multiply decimals.

Recall how to multiply 2 whole numbers using Base Ten Blocks.
This picture shows the product:
$20 \times 16 = 100 + 100 + 60 + 60$
$= 320$



We can also use Base Ten Blocks to multiply 2 decimals.

Let the flat represent 1, the rod represent 0.1, and the small cube represent 0.01.



Explore

You will need Base Ten Blocks and grid paper.
Use Base Ten Blocks to model a rectangular patio with area greater than 4 m^2 and less than 6 m^2.
Let the side length of the flat represent 1 m.
How many different patios can you model?
Record your designs on grid paper.

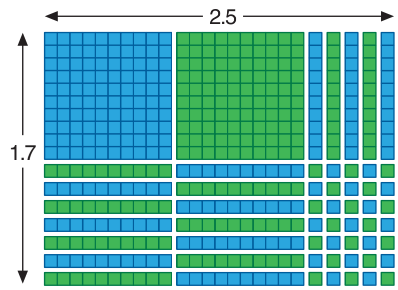
Reflect & Share

Compare your designs with those of another pair of classmates.
Did you have any designs the same? Explain.
Explain how your designs show the area of the patio.

Connect

A rectangular park measures 1.7 km by 2.5 km.
Here are 2 ways to find the area of the park.

➤ Use Base Ten Blocks.
Build a rectangle with length 2.5 and width 1.7.
Count the blocks in the rectangle.
There are 2 flats: $2 \times 1 = 2$
There are 19 rods: $19 \times 0.1 = 1.9$
There are 35 small cubes: $35 \times 0.01 = 0.35$
The total area is: $2 + 1.9 + 0.35 = 4.25$
The total area of the park is 4.25 km^2.



➤ Use the method for multiplying 2 whole numbers.
The area, in square kilometres, is 1.7×2.5.
Multiply: 17×25

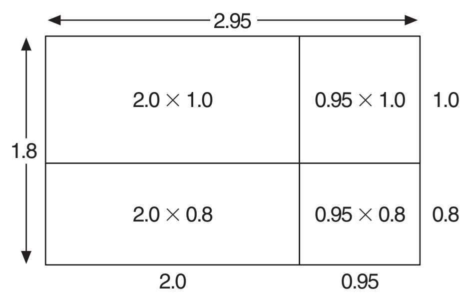
$$\begin{array}{r} 17 \\ \underline{\times\ 25} \\ 85 \\ \underline{340} \\ 425 \end{array}$$

1.7×2.5
Think: $1 \times 2 = 2$
So, 1.7×2.5 is about 2.
Place the decimal point between the 4 and the 2.

Using front-end estimation to place the decimal point, $1.7 \times 2.5 = 4.25$.
The area of the park is 4.25 km^2.

Example

At the Farmers' Market, 1 kg of grapes costs \$2.95.
How much would 1.8 kg of grapes cost?



A Solution

1 kg of grapes costs \$2.95.
So, 1.8 kg would cost: $\$2.95 \times 1.8$
Use a rectangle model.

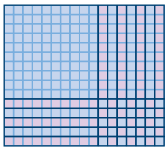
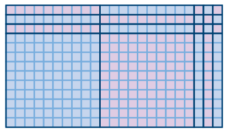
$$\begin{aligned} 2.95 \times 1.8 &= (2.0 \times 1.0) + (0.95 \times 1.0) + (2.0 \times 0.8) + (0.95 \times 0.8) \\ &= (2 \times 1) + (0.95 \times 1) + (2 \times 0.8) + (0.95 \times 0.8) \\ &= 2 + 0.95 + 1.6 + 0.76 \\ &= 5.31 \end{aligned}$$

1.8 kg of grapes would cost \$5.31.

Another Strategy
We could use a calculator to multiply.

Use a calculator when the multiplier has more than 2 digits.

Practice

1. Write the product that each picture represents.
   Each small square represents 0.01.
   a) 
   b) 

2. Use Base Ten Blocks to find each product.
   Record your work on grid paper.
   a) 2.6×1.5 b) 2.3×0.4 c) 0.8×0.7

3. Choose one part from question 2.
   Explain how the Base Ten Blocks show the product.

4. Multiply. Use a rectangle model.
   a) 4.2×3.7 b) 8.9×0.3 c) 0.6×0.9

5. A rectangular plot of land measures 30.5 m by 5.3 m.
   What is the area of the plot?
   Estimate to check your answer is reasonable.

6. Multiply. Describe any patterns you see.
   a) 8.36×10
   8.36×100
   8.36×1000
   $8.36 \times 10\,000$
   b) 8.36×0.1
   8.36×0.01
   8.36×0.001
   8.36×0.0001

7. **Assessment Focus** An area rug is rectangular.
   Its dimensions are 3.4 m by 2.7 m.
   Show different strategies you can use
   to find the area of the rug.
   Which strategy is best? Justify your answer.

8. Multiply.
   a) 2.7×4.786
   b) 12.52×13.923
   c) 0.986×1.352
   Explain how you can check your answers.

9. The fuel consumption estimates of Josie's car are:
City: 21.2 km/L Highway: 23.3 km/L
The car's gas tank holds 40.2 L of fuel.
a) How far could Josie drive on a full tank of gas on the highway before she runs out of fuel?
b) How far could she drive on a full tank of gas in the city?
What assumptions did you make?

10. Find the cost of each item at the Farmers' Market.
Which strategy will you use? Justify your choice.
a) 2.56 kg of apples at $0.95/kg
b) 10.5 kg of potatoes at $1.19/kg
c) 0.25 kg of herbs at $2.48/kg

11. The product of 2 decimals is 0.36.
What might the decimals be?
Find as many answers as you can.

12. a) Multiply 18×12.
b) Use only the result from part a and estimation.
Find each product.
i) 1.8×12 ii) 18×0.12 iii) 0.18×12 iv) 0.18×0.12
Explain your strategies.

13. **Take It Further**
a) Multiply.
i) 6.3×1.8 ii) 0.37×0.26 iii) 3.52×2.4 iv) 1.234×0.9
b) Look at the questions and products in part a.
What patterns do you see in the numbers of decimal places in the question and the product?
How could you use this pattern to place the decimal point in a product without estimating?
c) Multiply: 2.6×3.5
Does the pattern from part b hold true?
If your answer is no, explain why not.

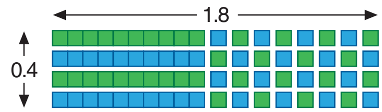
Reflect

When you multiply 2 decimals, how do you know where to place the decimal point in the product? Use examples to explain.

3.5 Dividing Decimals

Focus Use Base Ten Blocks, paper and pencil, and calculators to divide decimals.

Recall how you used Base Ten Blocks to multiply:



$1.8 \times 0.4 = 0.72$

Since multiplication and division are related, we can also use Base Ten Blocks to divide.

Which division sentences could you write for this diagram?

Explore

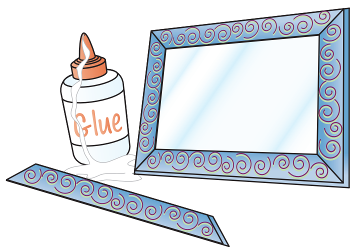
You will need Base Ten Blocks and grid paper.
Marius bought 1.44 m of ribbon for his craft project.
He needs to cut the ribbon into 0.6-m lengths.
How many 0.6-m lengths can he cut?
Use Base Ten Blocks to find out.
Record your work on grid paper.

Reflect & Share

Compare your solution with that of another pair of classmates.
What was your strategy?
How could you use division of whole numbers to check your answer?

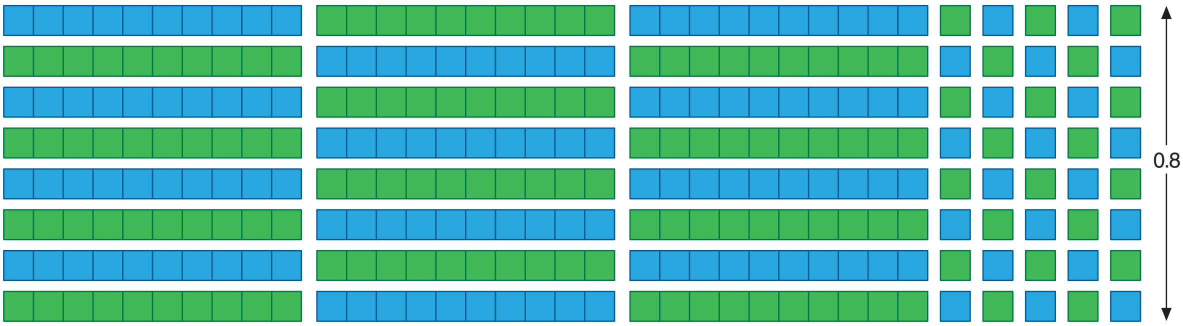
Connect

Jan bought 2.8 m of framing to make picture frames.
Each picture needs 0.8 m of frame.
How many frames can Jan make?
How much framing material is left over?



Use Base Ten Blocks to divide: $2.8 \div 0.8$

Make a rectangle with area 2.8 and width 0.8.



The length of the rectangle is 3.5.
So, Jan can make 3 frames.
3 frames use: 3×0.8 m $= 2.4$ m
So, the framing material left is: 2.8 m $-$ 2.4 m $=$ 0.4 m

Sometimes when we divide 2 decimals, the quotient is not a terminating decimal.
Then we can use paper and pencil.

Example

Divide: $52.1 \div 0.9$

A Solution

Estimate first: $52.1 \div 0.9$
Write each decimal to the nearest whole number, then divide.
$52 \div 1 = 52$
So, $52.1 \div 0.9$ is about 52.

52.1 is closer to 52.
0.9 is closer to 1.

Divide as you would whole numbers.
$521 \div 9$

$$
\begin{array}{r l}
5788 & \leftarrow \textbf{quotient} \\
\textbf{divisor} \rightarrow 9\overline{)52100} & \leftarrow \textbf{dividend} \\
\underline{45} & \\
71 & \\
\underline{63} & \\
80 & \\
\underline{72} & \\
80 & \\
\underline{72} & \\
8 &
\end{array}
$$

Divide until the quotient has 2 more digits than the estimate. Then we can write the quotient to the nearest tenth.

If the quotient is not exact, write zeros in the dividend, then continue to divide.

Since the estimate has 2 digits, divide until there are 4 digits in the quotient.

Since the estimate was 52, place the decimal point
so the quotient is close to 52: 52.1 ÷ 0.9 ≐ 57.88
In the question, the dividend and divisor were given to the nearest tenth.
So, we write the quotient to the nearest tenth.
52.1 ÷ 0.9 ≐ 57.90, or 57.9

57.88 is closer to 57.90 than to 57.80.

We can use a calculator when the divisor has more than 1 digit.

Practice

1. Use Base Ten Blocks to divide. Record your work on grid paper.
a) 0.8 ÷ 0.1 **b)** 1.2 ÷ 0.3 **c)** 2.7 ÷ 0.6 **d)** 2.2 ÷ 0.4

2. Divide. Describe any patterns you see.

a)	**b)**
124.5 ÷ 10	124.5 ÷ 0.1
124.5 ÷ 100	124.5 ÷ 0.01
124.5 ÷ 1000	124.5 ÷ 0.001
124.5 ÷ 10 000	124.5 ÷ 0.0001

3. Why do all these division statements have 6 as the answer?
a) 30 ÷ 5 **b)** 3.0 ÷ 0.5 **c)** 0.3 ÷ 0.05 **d)** 300 ÷ 50
Which one is easiest to calculate? Explain.

4. Estimate to choose the correct quotient for each division question.

Question	**Possible Quotients**		
a) 59.5 ÷ 5	119	11.9	1.19
b) 195.3 ÷ 0.2	9765	976.5	97.65
c) 31.32 ÷ 0.8	3915	391.5	39.15

5. Use paper and pencil to divide.
a) 1.5 ÷ 0.6 **b)** 2.24 ÷ 0.7 **c)** 1.28 ÷ 0.8 **d)** 2.16 ÷ 0.9

6. Divide. Write each quotient to the nearest tenth.
Use front-end estimation to check your answer is reasonable.
a) 8.36 ÷ 2.4 **b)** 1.98 ÷ 1.3 **c)** 27.82 ÷ 3.9 **d)** 130.4 ÷ 5.4

7. A toonie is approximately 0.2 cm thick.
How many toonies are in a stack of toonies 17.4 cm high?

8. The area of a large rectangular flowerbed is 22.32 m^2.
The width is 0.8 m. What is the length?

9. A 0.4-kg bag of oranges costs $1.34.

a) Estimate. About how much does 1 kg of oranges cost?

b) What is the actual cost of 1 kg of oranges?
How do you know your answer is reasonable?

c) Suppose you spent $10 on oranges.
What mass of oranges did you buy?

10. **Assessment Focus** Alex finds a remnant of landscaping fabric at a garden store.
The fabric is the standard width, with length 9.88 m.
Alex needs fourteen 0.8-m pieces for a garden patio.

a) How many 0.8-m pieces can Alex cut from the remnant?
What assumptions did you make?

b) Will Alex have all the fabric he needs? Why or why not?

c) If your answer to part b is no, how much more fabric does Alex need?

d) Alex redesigns his patio so that he needs fourteen 0.7-m pieces of fabric.
Will the remnant be enough fabric? Explain.

11. The quotient of two decimals is 0.12. What might the decimals be?
Write as many different possible decimal pairs as you can.

12. Last week, Alicia worked 37.5 h. She earned $346.88.
How much money did Alicia earn per hour?
Why is the answer different from the number in the calculator display?

13. The question 237 ÷ 7 does not have an exact quotient.
The first five digits of the quotient are 33857.
The decimal point has been omitted. Use only this information and estimation.
Write an approximate quotient for each question.
Justify each answer.

a) 237 ÷ 0.7 b) 2.37 ÷ 0.07 c) 23.7 ÷ 7 d) 2370 ÷ 70

Reflect

Talk to a partner. Tell how you can find 1.372 ÷ 0.7 by dividing by 7.
Why does this work?

3.6 Order of Operations with Decimals

Focus Use the order of operations to evaluate expressions.

Explore

How many different ways can you find the answer for this expression?

$6 \times 15.9 + 36.4 \div 4$

Show your work for each answer.

Reflect & Share

Compare your answers with those of another pair of classmates.
Which solution do you think is correct? Explain your reasoning.

Connect

To make sure everyone gets the same answer for a given expression, we add, subtract, multiply, and divide in this order:

- Do the operations in brackets first.
- Then divide and multiply, in order, from left to right.
- Then add and subtract, in order, from left to right.

When we find the answer to an expression, we *evaluate*.

We use the same order of operations for decimals as for whole numbers.

Example

Evaluate: $12.376 \div (4.75 + 1.2) + 2.45 \times 0.2 - 1.84$

Use a calculator when you need to.

A Solution

$12.376 \div (\underline{4.75 + 1.2}) + 2.45 \times 0.2 - 1.84$ **Calculate in brackets.**

$= \underline{12.376 \div 5.95} + \underline{2.45 \times 0.2} - 1.84$ **Multiply and divide from left to right.**

$= \underline{2.08 + 0.49} - 1.84$ **Add and subtract from left to right.**

$= 2.57 - 1.84$

$= 0.73$

Many calculators follow the order of operations.
To see whether your calculator does, enter: $12.4 \times 2.2 - 15.2 \div 4$
If your answer is 23.48, your calculator follows the order of operations.

Practice

1. Evaluate.
a) $4.6 + 5.1 - 3.2$ **b)** $8 - 3.6 \div 2$ **c)** $46.4 - 10.8 \times 3$ **d)** $85.6 \div 0.4 \times 7$

2. Evaluate.
a) $(46.78 - 23.58) \times 2.5$ **b)** $(98.5 + 7) \div 0.5$ **c)** $7.2 \div (2.4 - 1.8)$

3. Evaluate.
a) $9.8 - 3.2 \div 0.4 + 2.6$ **b)** $(9.8 - 3.2) \div (0.4 + 2.6)$
Explain why the answers are different.

4. Evaluate.
a) $1.35 + (5 \times 4.9 \div 0.07) - 2.7 \times 2.1$ **b)** $9.035 \times 5.2 - 4.32 \times 6.7$
c) $2.368 \div 0.016 + 16.575 \div 1.105$ **d)** $0.38 + 16.2 \times (2.1 + 4.7) + 21 \div 3.5$

5. **Assessment Focus** Ioana, Aida, and Norman got different answers for this problem: $12 \times (4.8 \div 0.3) - 3.64 \times 3.5$
Ioana's answer was 39.12, Aida's answer was 179.26, and Norman's answer was 659.26.
a) Which student had the correct answer? How do you know?
b) Show and explain how the other two students got their answers. Where did they go wrong?

6. Evaluate. Show all steps:
$0.38 + 16.2 \times (2.1 - 1.2) + 21 \div 0.8$

7. **Take It Further** Use at least 4 of the numbers 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8 and 0.9, and any operations or brackets to make each whole number from 1 to 5.

Reflect

Why do we need to agree on an order of operations?

Mid-Unit Review

LESSON

Use a calculator when you need to.

3.1 **1.** a) Write each fraction as a decimal.

i) $\frac{1}{33}$ ii) $\frac{2}{33}$ iii) $\frac{3}{33}$

b) Describe the pattern in your answers to part a.

c) Use your pattern to predict the fraction form of each decimal.

i) $0.\overline{15}$ ii) $0.\overline{36}$

2. Write each fraction as a decimal. Identify the decimals as repeating or terminating.

a) $\frac{1}{8}$ b) $\frac{3}{5}$

c) $\frac{2}{3}$ d) $\frac{7}{13}$

3. Write each decimal as a fraction.

a) 0.2 b) $0.\overline{8}$

c) 0.005 d) $0.\overline{23}$

3.2 **4.** Order each set of numbers from least to greatest. Use a different method for each part.

a) $2\frac{1}{4}, \frac{11}{6}, \frac{8}{3}, 2$ b) $3.5, \frac{23}{8}, 1\frac{3}{4}$

c) $1.75, \frac{13}{10}, \frac{9}{5}, 1\frac{3}{5}, 1$

5. Find a number between each pair of numbers. Which strategy did you use each time?

a) $\frac{4}{3}, \frac{5}{3}$ b) $2\frac{3}{8}, \frac{5}{2}$ c) $1.4, \frac{8}{5}$

3.3 **6.** Use front-end estimation to place the decimal point in each answer.

a) $32.47 - 6.75 = 2\ 5\ 7\ 2$

b) $118.234 + 19.287 = 1\ 3\ 7\ 5\ 2\ 1$

c) $17.9 - 0.8 = 1\ 7\ 1$

7. Winsome is being trained as a guide dog for a blind person.
At birth, she had a mass of 0.475 kg.
At 6 weeks, her mass was 4.06 kg.
From 6 weeks to 12 weeks, she gained 5.19 kg.

a) By how much did Winsome's mass change from birth to 6 weeks?

b) What was her mass at 12 weeks?

3.4 **8.** Estimate to place the decimal point in each product.
Show your estimation strategy.

a) $9.3 \times 0.8 = 7\ 4\ 4$

b) $3.62 \times 1.3 = 4\ 7\ 0\ 6$

c) $11.25 \times 5.24 = 5\ 8\ 9\ 5$

9. A rectangular park has dimensions 2.84 km by 3.5 km.
What is the area of the park?

3.5 **10.** When you divide 15.4 by 2, the quotient is 7.7.
When you divide 1.54 by 0.2, the quotient is 7.7.
Explain why the quotients are the same.

3.6 **11.** Evaluate.

a) $5.9 + 3.7 \times 2.8$

b) $12.625 \times (1.873 + 2.127)$

c) $2.1 \div 0.75 + 6.38 \times 2.45$

3.7 Relating Fractions, Decimals, and Percents

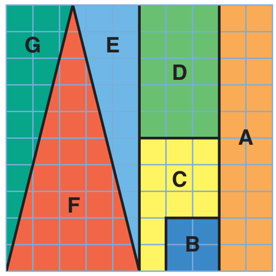
Focus Relate percent to fractions and decimals.

We see uses of percent everywhere.

What do you know from looking at each picture?
Recall that percent means per hundred.
49% is $\frac{49}{100} = 0.49$

Explore

Your teacher will give you a large copy of this puzzle.
Describe each puzzle piece as a percent, then as a fraction and a decimal of the whole puzzle.

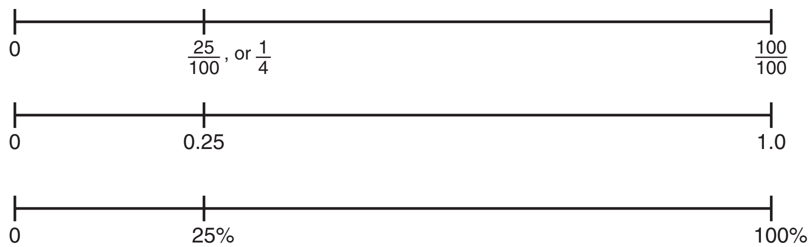


Reflect & Share

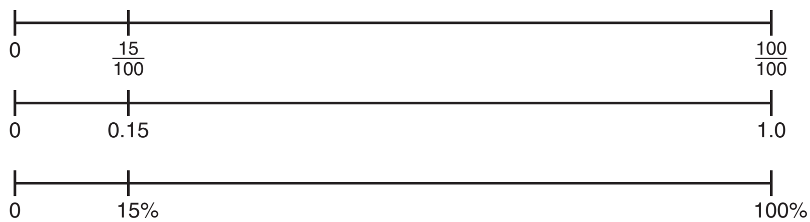
Compare your answers with those of another pair of classmates.
If the answers are different, how do you know which are correct?

Connect

➤ We can use number lines to show how percents relate to fractions and decimals.
For example:
$25\% = \frac{25}{100} = 0.25$

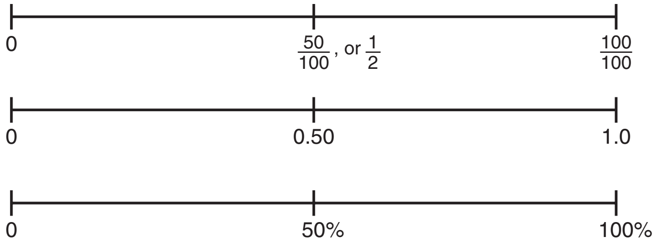


➤ Conversely, a decimal can be written as a percent:
$0.15 = \frac{15}{100} = 15\%$



➤ To write a fraction as a percent, write the equivalent fraction with denominator 100.
For example:

$$\frac{1}{2} \underset{\times 50}{\overset{\times 50}{=}} \frac{50}{100} = 50\%$$



Example

a) Write each percent as a fraction and as a decimal.
   i) 75%   ii) 9%

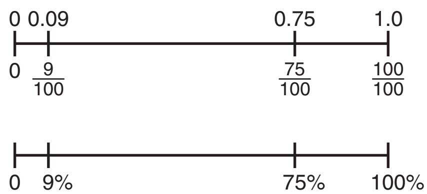
b) Write each fraction as a percent and as a decimal.
   i) $\frac{2}{5}$ ii) $\frac{7}{20}$

Draw number lines to show how the numbers are related.

A Solution

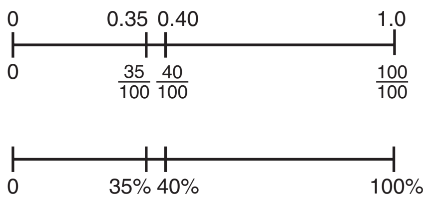
a) i) $75\% = \frac{75}{100} = 0.75$

ii) $9\% = \frac{9}{100} = 0.09$



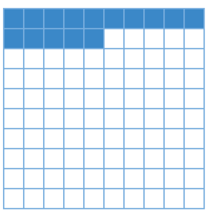
b) i) $\frac{2}{5} \underset{\times 20}{\overset{\times 20}{=}} \frac{40}{100} = 40\% = 0.40$

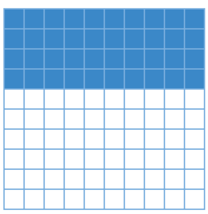
ii) $\frac{7}{20} \underset{\times 5}{\overset{\times 5}{=}} \frac{35}{100} = 35\% = 0.35$

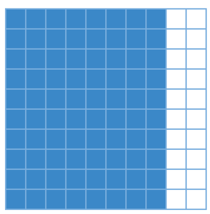


Practice

1. What percent of each hundred chart is shaded?
   Write each percent as a fraction and as a decimal.

a)


b)


c)


2. Write each percent as a fraction and a decimal.
Sketch number lines to show how the numbers are related.
a) 2% b) 9% c) 28% d) 95%

3. Write each fraction as a decimal and a percent.
a) $\frac{2}{10}$ b) $\frac{3}{50}$ c) $\frac{4}{25}$ d) $\frac{13}{20}$ e) $\frac{4}{5}$

4. Fred had 8 out of 10 on a test. Janet had 82% on the test.
Who did better? How do you know?

5. **Assessment Focus** You will need a sheet of paper and coloured pencils.
Divide the paper into these 4 sections.
- 1 blue section that is $\frac{1}{2}$ of the page
- 1 red section that is 10% of the page
- 1 yellow section that is 25% of the page
- 1 green section to fill the remaining space.

Explain how you did this.
What percent of the page is the green section?
How do you know?

6. **Take It Further** Suppose each pattern is continued on a hundred chart.
The numbers in each pattern are coloured red.
For each pattern, what percent of the numbers on the chart are red?
Explain your strategy for each pattern.
a) 4, 8, 12, 16, 20, … b) 1, 3, 5, 7, … c) 2, 4, 8, 16, … d) 1, 3, 7, 13, …

Reflect

Suppose you know your mark out of 20 on an English test.
Tell how you could write the mark as a decimal and a percent.

3.8 Solving Percent Problems

Focus Solve problems involving percents to 100%.

When shopping, it is often useful to be able to calculate a percent, to find the sale price, the final price, or to decide which of two offers is the better deal.

Explore

A jacket originally cost \$48.00.
It is on sale for 25% off.
What is the sale price of the jacket?
How much is saved by buying the jacket on sale?
Find several ways to solve this problem.

Reflect & Share

Compare strategies with those of another pair of classmates.
Which strategy would you use if the sale was 45% off? Explain your choice.

Connect

A paperback novel originally cost \$7.99.
It is on sale for 15% off.
To find how much you save, calculate 15% of \$7.99.

$15\% = \frac{15}{100} = 0.15$

So, 15% of $\$7.99 = \frac{15}{100}$ of 7.99

$= 0.15 \times 7.99$

Use a calculator.

$0.15 \times 7.99 = 1.1985$

So, $0.15 \times \$7.99 = \$1.1985$

\$1.1985 to the nearest cent is \$1.20.

You save \$1.20 by buying the book on sale.

We can show this on a number line.

Estimate to check if the answer is reasonable.
15% is about 20%, which is $\frac{1}{5}$.
$7.99 is about $10.00.
So, 0.15×7.99 is about $\frac{1}{5}$ of 10, which is 2.
This is close to the calculated amount, so the answer is reasonable.

Example

Sandi works at Fancies Flowers on Saturdays.
The owner pays Sandi 3% of all money she takes in on a day.
Last Saturday, Sandi took in $1200.00.
How much money did Sandi earn last Saturday?
Illustrate the answer on a number line.

A Solution

Another Strategy
We could find 1% of $1200.00, then multiply by 3.

Sandi took in $1200.00.
We want to find 3% of $1200.00.
3% is $\frac{3}{100} = 0.03$
So, 3% of $1200 $= 0.03 \times 1200$
Ignore the decimal point and multiply as whole numbers.

$$\begin{array}{r} 1200 \\ \times \quad 3 \\ \hline 3600 \end{array}$$

Estimate to place the decimal point.
$1200 is about $1000.
1% of $1000 is $10.
So, 3% of $1000 is: $10 × 3 = $30

So, $0.03 \times$ $1200 = $36.00
Sandi earned $36.00 last Saturday.
Show this on a number line.

Practice

1. Calculate.
a) 10% of 30 **b)** 20% of 50 **c)** 18% of 36 **d)** 67% of 112

2. The regular price of a radio is $60.00.
Find the sale price before taxes when the radio is on sale for:
a) 25% off **b)** 30% off **c)** 40% off

3. Find the sale price before taxes of each item.
   a) coat: 55% off \$90 b) shoes: 45% off \$40 c) sweater: 30% off \$50

4. Find the tip left by each customer at a restaurant.
   a) Denis: 15% of \$24.20 b) Molly: 20% of \$56.50 c) Tudor: 10% of \$32.70

5. The Goods and Services tax (GST) is currently 6%.
   For each item below:
   i) Find the GST.
   ii) Find the cost of the item including GST.
   a) bicycle: \$129.00 b) DVD: \$24.99 c) skateboard: \$42.97

6. There are 641 First Nations bands in Canada.
   About 30% of these bands are in British Columbia.
   About how many bands are in British Columbia?
   Sketch a number line to show your answer.

7. **Assessment Focus** A clothing store runs this advertisement in a local paper.
   "Our entire stock up to 60% off"
   a) What does "up to 60% off" mean?
   b) Which items in the advertisement have been reduced by 60%?
   c) Suppose all items are reduced by 60%. Explain the changes you would make to the sale prices.

Item	Regular Price	Sale Price
Sweaters	\$49.99	\$34.99
Ski Jackets	\$149.99	\$112.49
Scarves	\$29.99	\$12.00
Leather Gloves	\$69.99	\$38.49
Hats	\$24.99	\$10.00

8. **Take It Further** Marissa and Jarod plan to purchase DVD players with a regular price of \$199.99.
   The DVD players are on sale for 25% off.
   Marissa starts by calculating 25% of \$199.99.
   Jarod calculates 75% of \$199.99.
   a) Show how Marissa uses her calculation to find the sale price.
   b) How does Jarod find the sale price? Show his work.
   c) Do both methods result in the same sale price? Explain.

Reflect

How does a good understanding of percents help you outside the classroom? Give an example.

Sports Trainer

Sports trainers use scientific research and scientific techniques to maximize an athlete's performance. An athlete may be measured for percent body fat, or percent of either fast- or slow-twitch muscle fibre.

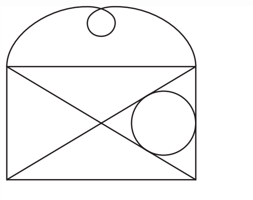
A trainer may recommend the athlete eat pre-event meals that contain a certain percent of carbohydrate, or choose a "sports drink" that contains a high percent of certain minerals. The trainer creates and monitors exercise routines. These enable the athlete to attain a certain percent of maximum heart rate, speed, or power.

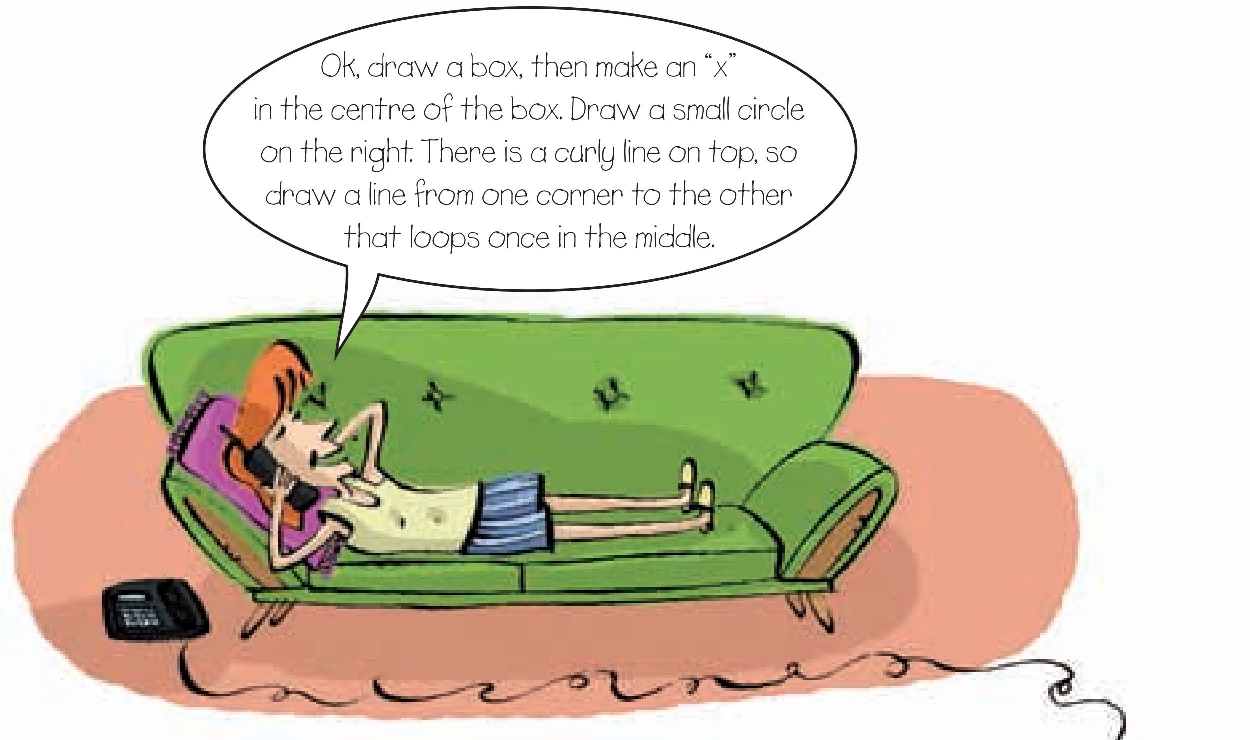
Most sports drinks contain minerals. Research shows that the most effective sports drink has a magnesium to calcium ratio of 1:2. The body absorbs about 87% of magnesium in a drink, and about 44% of calcium in a drink. One serving of a particular sports drink contains about 96 mg of calcium and 48 mg of magnesium. About how many milligrams of each mineral will the body absorb?

Writing Instructions

Life is full of instructions.
If you have ever filled out a form, assembled a desk, or followed directions to someone's house, you know the importance of good instructions.

Rhonda describes this shape to Rashad. She asks Rashad to sketch it.

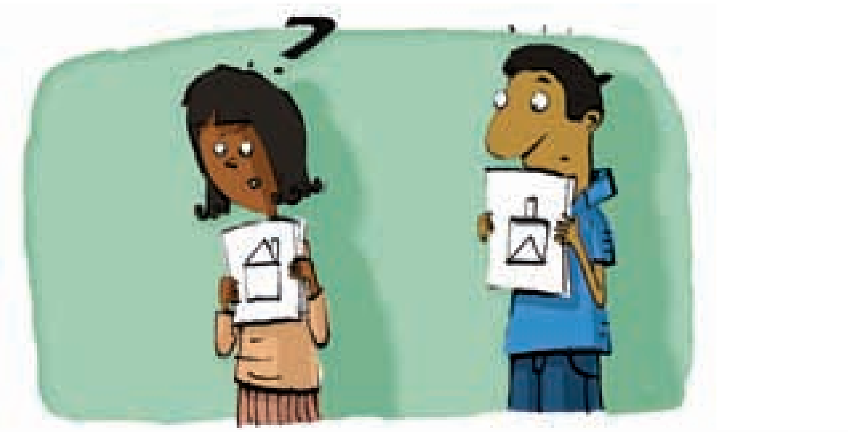




- Follow Rhonda's instructions.
  Draw 3 different shapes that are different from the original figure.
- Describe how to improve Rhonda's instructions.
- Rewrite Rhonda's instructions using your suggestions.

Instructions for Drawing

- Draw a shape.
  Write instructions that someone else could follow to draw it.
- Trade instructions with a classmate.
  Follow your classmate's instructions to draw the shape.
- Compare your shape with the original shape.
  Are the shapes the same?
  If not, suggest ways to improve your classmate's instructions.



Instructions for Calculations

- Your friend has forgotten his calculator at school.
  Write instructions that your friend could follow
  to find 87×0.17 without a calculator.
- Trade instructions with a classmate.
  Follow your classmate's instructions
  to find the product.
- Use a calculator to find the product.
  Are the products the same?
  If not, suggest ways to improve
  your classmate's instructions.

Unit Review

What Do I Need to Know?

☑ Here are some fractions and decimals you should know.

$\frac{1}{2} = 0.5$	$\frac{1}{3} = 0.\overline{3}$	$\frac{1}{4} = 0.25$
$\frac{1}{5} = 0.2$	$\frac{1}{8} = 0.125$	$\frac{1}{10} = 0.1$
$\frac{1}{20} = 0.05$	$\frac{1}{100} = 0.01$	$\frac{1}{1000} = 0.001$

☑ Percent is the number of parts per hundred.
A percent can be written as a fraction and as a decimal.

☑ Here are some fractions, decimals, and percents that you should also know.

$\frac{1}{4} = 0.25 = 25\%$	$\frac{1}{2} = 0.5 = 50\%$	$\frac{3}{4} = 0.75 = 75\%$
$\frac{1}{20} = 0.05 = 5\%$	$\frac{1}{10} = 0.1 = 10\%$	$\frac{1}{5} = 0.2 = 20\%$

☑ The order of operations with whole numbers applies to decimals.
- Do the operations in brackets first.
- Then divide and multiply, in order, from left to right.
- Then add and subtract, in order, from left to right.

Math Link

Your World

When you buy a Canada Savings Bond (CSB), you are lending the Canadian government money. The government pays you for borrowing your money. It pays a percent of what you invested. In 2006, if you invested money for 1 year, the government paid you 2% of the amount you invested. Suppose you bought a \$2000 CSB in 2006. How much will the government pay you for 1 year? What if you bought a \$2500 CSB?

What Should I Be Able to Do?

LESSON

3.1

1. Write each fraction as a decimal. Identify each decimal as terminating or repeating.

a) $\frac{3}{5}$ b) $\frac{5}{6}$ c) $\frac{3}{8}$ d) $\frac{3}{20}$

2. Write each decimal as a fraction or a mixed number in simplest form.

a) 0.55 b) $1.\overline{3}$

c) 0.8 d) $0.\overline{07}$

3.2

3. a) Use any method. Order these numbers from least to greatest. Explain the method you used.

$\frac{5}{4}, 1\frac{1}{16}, \frac{3}{6}, 1.1, \frac{5}{8}$

b) Use a different method to order the numbers, to verify your answer in part a.

4. In each ordered set, identify the number that has been placed incorrectly. Explain how you know.

a) $2\frac{1}{3}, 2.25, \frac{17}{6}, 2\frac{11}{12}$

b) $\frac{3}{5}, \frac{9}{10}, \frac{21}{20}, 1\frac{3}{15}, 1.1$

3.3

5. Two decimals have a sum of 3.41. What might the decimals be? Find as many answers as you can.

6. Asafa Powell of Jamaica holds the men's world record for the 100-m sprint, with a time of 9.77 s. Florence Griffith Joyner of the United States holds the women's world record, with a time of 10.49 s. What is the difference in their times?

3.4

7. Kiah works at the library after school. She earns $7.65/h. She usually works 15.5 h a week.

a) What does Kiah earn in a week? Use estimation to check your answer.

b) One week Kiah only works one-half the hours she usually works. What are her earnings that week?

8. Lok needs 1.2 m of fabric to make a tote bag. He finds two fabrics he likes. One fabric costs $7.59/m and the other fabric costs $6.29/m. How much money will Lok save if he buys the less expensive fabric?

3.5

9. Estimate.
Which quotients are:
i) greater than 100?
ii) less than 50?
Calculate the quotients that are less than 50.

a) 259.8 ÷ 1.65
b) 35.2 ÷ 0.2
c) 175.08 ÷ 0.8
d) 93.8 ÷ 22.4
e) 162.24 ÷ 31.2
f) 883.3 ÷ 36.5

LESSON

10. The area of a rectangle is 3.75 m^2.
Its length is 0.6 m.
What is the width of the rectangle?

3.6 11. Evaluate.
Use the order of operations.
a) $8.11 + 6.75 \times 5.6 - 2.12$
b) $3.78 \times 2.25 - 4.028 \div 1.52$

12. a) Simplify.
i) $1.2 + 2.8 \times 2.1 + 3.6$
ii) $1.2 \times 2.8 + 2.1 \times 3.6$
iii) $1.2 \times (2.8 + 2.1) + 3.6$
iv) $1.2 + 2.8 + 2.1 \times 3.6$
b) All the expressions in part a have the same numbers and operations. Why are the answers different?

3.7 13. Write each percent as a fraction and as a decimal.
Sketch number lines to illustrate.
a) 80% b) 12%
c) 2% d) 63%

14. Write each fraction as a decimal and as a percent.
Sketch number lines to illustrate.
a) $\frac{14}{25}$ b) $\frac{19}{20}$
c) $\frac{7}{50}$ d) $\frac{1}{5}$

3.8 15. There are 35 students in a Grade 7 class. On one day, 20% of the students were at a sports meet.
How many students were in class?

16. Find the sale price before taxes of each item.
a) video game: 15% off \$39
b) lacrosse stick: 25% off \$29
c) fishing rod: 30% off \$45

17. A souvenir Olympic hat sells for \$29.99.
a) Russell lives in Newfoundland where there is a sales tax of 14%. Calculate the final cost of the hat in Newfoundland.
b) Jenna lives in Alberta where the GST tax is 6%.
Calculate the final cost of the hat in Alberta.
c) What is the difference between the final costs of the hat in Newfoundland and Alberta?

18. Madeleine received good service in a restaurant. She left the waitress a tip of 20%. Madeleine's bill was \$32.75. How much tip did the waitress receive? Show your work. Draw a number line to illustrate your answer.

Practice Test

1. Write each decimal as a fraction in simplest form and each fraction as a decimal.

a) 0.004 **b)** 0.64 **c)** $0.\overline{3}$ **d)** $\frac{51}{200}$ **e)** $\frac{3}{4}$

2. Ryan earns \$18.00 a day walking dogs.
He walked dogs 5 days last week.

a) How much money did Ryan earn last week?

Ryan is saving to buy inline skating equipment.
The skates cost \$59.95.
A helmet costs \$22.90.
A set of elbow, knee, and wrist guards costs \$24.95.

b) Does Ryan have enough money to buy the equipment? Show your work.

c) If your answer to part b is no, how much more money does Ryan need? What assumptions did you make?

3. Maria stated that $1\frac{5}{6}$ is between 1.8 and $\frac{13}{7}$.
Do you agree?
Give reasons for your answer.

4. Evaluate.

a) $3.8 + 5.1 \times 6.4 - 1.7$

b) $3.54 \div 0.3 + (2.58 \times 1.5)$

5. Last spring, 40 cats were adopted from the local animal shelter.
This spring, the number of cats adopted dropped by 35%.
How many cats were adopted this spring?
Draw a number line to show your answer.

6. The regular price of a pair of shoes is \$78.00.
The shoes are on sale for 25% off.

a) What is the sale price of the shoes?

b) How much money is saved by buying the shoes on sale?

c) The GST is 6%. How much GST would be added to the sale price?

d) What is the final price of the shoes?

Unit Problem Shopping with Coupons

Part 1

Tanya and Marcus used the store's coupon below while shopping at *Savings for U*. They purchased:

a) Find the total cost of the items before sales tax.

b) Find the total cost of the items after they used the coupon.

c) Find the total cost of the items including GST of 6%.

Part 2

Winnie used the Scratch'n'Save coupon to buy:

The clerk scratched the coupon to reveal 20%.

a) Find the total cost of the items before sales tax.

b) Find the total cost of the items after the scratch coupon was used.

c) Find the total cost of the items including GST of 6%.

Part 3

Marty wants to purchase the following items.

i) 

ii) 

iii) 

iv) 

Use each discount coupon on page 124. The scratch coupon shows 20%.
Calculate the cost of each item, with each coupon.
Which coupon offers the better deal on each item?

Check List

Your work should show:

- ✓ all calculations in detail
- ✓ that you can use percents to solve real-life problems
- ✓ clear explanations and conclusions, and how you reached your conclusions

Reflect on Your Learning

Look back at the goals under *What You'll Learn*.
Which goals were easiest for you to achieve? Why do you think so?
Which were more challenging? What strategies did you use to meet these goals?

Units 1–3 Cumulative Review

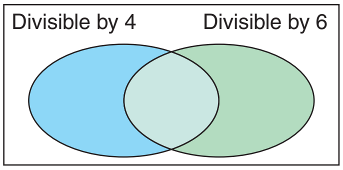
UNIT

1

1. Copy this Venn diagram.
Sort these numbers.

320	264	762	4926
2660	1293	488	504

How did you know where to put each number?



2. Suppose you have 40 strawberries. You must share the strawberries equally with everyone at the picnic table. How many strawberries will each person get, in each case?

a) There are 8 people at the table.
b) There are 5 people at the table.
c) There is no one at the table. Explain your answer.

3. Write an algebraic expression for each phrase.

a) a number divided by twelve
b) eleven added to a number
c) eight less than a number

4. **a)** Describe the patterns in this table.
b) Use the patterns to extend the table 3 more rows.
c) Use algebra. Write a relation that describes how the output is related to the input.

Input x	Output
1	4
2	6
3	8
4	10

5. Identify the numerical coefficient, the variable, and the constant term in each algebraic expression.

a) $3s + 2$ **b)** $7p$
c) $c + 8$ **d)** $11w + 9$

6. The cost to park a car is $5 for the first hour, plus $3 for each additional half hour.

a) Write a relation to show how the total cost is related to the number of additional half hours.
b) Copy and complete this table.

Number of Additional Half Hours	Cost ($)
0	
1	
2	
3	
4	

c) Draw a graph to show the relation. Describe the graph.
d) Use the graph to answer these questions.
 i) Tanya parked for 6 additional half hours. What was her total cost?
 ii) Uton paid $29 to park his car. How long was he parked?

7. Draw pictures to represent the steps you took to solve each equation.

a) $3x = 15$
b) $x + 9 = 11$

UNIT

2

8. **a)** Suppose you have 8 yellow tiles, and use all of them. How many red tiles would you need to model −3? How do you know?

b) Suppose you have 5 red tiles and 5 yellow tiles. How many ways can you find to model −3 with tiles?

9. Use coloured tiles to represent each sum. Find each sum. Sketch the tiles you used.

a) $(-7) + (+7)$ **b)** $(-7) + (+5)$

c) $(-7) + (-5)$ **d)** $(+7) + (-5)$

10. Use a number line.
For each sentence below:

a) Write each number as an integer.

b) Write an addition equation.
Explain your answer in words.

i) You deposit \$10, then withdraw \$5.

ii) A balloon rises 25 m, then falls 10 m.

iii) You ride the elevator down 9 floors, then up 12 floors.

11. What is the difference in altitudes? How can you subtract to find out?

a) An altitude of 80 m above sea level and an altitude of 35 m below sea level

b) An altitude of 65 m below sea level and an altitude of 10 m above sea level

12. Add or subtract.

a) $(+5) + (-9)$ **b)** $(-1) + (-5)$

c) $(+2) - (-8)$ **d)** $(-9) - (-3)$

3

13. **a)** Write each fraction as a decimal.

i) $\frac{1}{33}$ **ii)** $\frac{2}{33}$ **iii)** $\frac{3}{33}$

b) Describe the pattern in your answers to part a.

c) Use your pattern to predict the fraction form of each decimal.

i) $0.\overline{15}$ **ii)** $0.\overline{24}$ **iii)** $0.\overline{30}$

14. **a)** Use any method. Order these numbers from greatest to least.
$\frac{21}{4}$, 4.9, $5\frac{1}{3}$, $\frac{24}{5}$, 5.3

b) Use a different method.
Verify your answer in part a.

15. The tallest woman on record was 2.483 m tall. The shortest woman on record was 0.61 m tall. What is the difference in their heights?

16. Multiply. Draw a diagram to show each product.

a) 2.3×3.4 **b)** 1.8×2.2

c) 4.1×3.7 **d)** 1.7×2.9

17. Nuri has 10.875 L of water.
He pours 0.5 L into each of several plastic bottles.

a) How many bottles can Nuri fill?

b) How much water is left over?

18. The Goods and Services Tax (GST) is currently 6%. For each item below:

i) Find the GST.

ii) Find the cost of the item including GST.

a) snowshoes that cost \$129.99

b) a CD that costs \$17.98

UNIT 4 Circles and Area

Look at these pictures.
What shapes do you see?

- What is a circle?
- Where do you see circles?
- What do you know about a circle?
- What might be useful to know about a circle?
  A parallelogram?
  A triangle?

What You'll Learn

- Investigate and explain the relationships among the radius, diameter, and circumference of a circle.
- Determine the sum of the central angles of a circle.
- Construct circles and solve problems involving circles.
- Develop formulas to find the areas of a parallelogram, a triangle, and a circle.
- Draw, label, and interpret circle graphs.

Why It's Important

- The ability to measure circles, triangles, and parallelograms is an important skill.
  These shapes are used in design, architecture, and construction.

Key Words

- radius, radii
- diameter
- circumference
- π
- irrational number
- base
- height
- circle graph
- sector
- legend
- percent circle
- central angle
- sector angle
- pie chart

4.1 Investigating Circles

Focus Measure radius and diameter and discover their relationship.

Explore

You will need circular objects, a compass, and a ruler.

➤ Use a compass. Draw a large circle. Use a ruler. Draw a line segment that joins two points on the circle. Measure the line segment. Label the line segment with its length. Draw and measure other segments that join two points on the circle. Find the longest segment in the circle. How many other segments can you draw with this length? Repeat the activity for other circles.

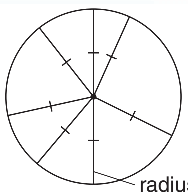
➤ Trace a circular object. Cut out the circle. How many ways can you find the centre of the circle? Measure the distance from the centre to the circle. Measure the distance across the circle, through its centre. Record the measurements in a table. Repeat the activity with other circular objects. What pattern do you see in your results?

Reflect & Share

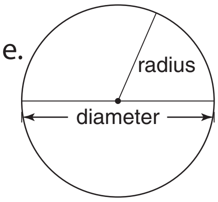
Compare your results with those of another pair of classmates.
Where is the longest segment in any circle?
What relationship did you find between the distance across a circle through its centre, and the distance from the centre to the circle?

Connect

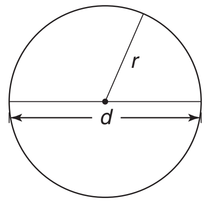
All points on a circle are the same distance from the centre of the circle. This distance is the **radius** of the circle.



The longest line segment in any circle is the **diameter** of the circle.
The diameter passes through the centre of the circle.
The radius is one-half the length of the diameter.
The diameter is two times the length of the radius.



Let r represent the radius, and d the diameter.
Then the relationship between the radius and diameter of a circle is:
$r = d \div 2$, which can be written as $r = \frac{d}{2}$
And, $d = 2r$



The plural of *radius* is *radii*; that is, one radius, two or more radii.

Example

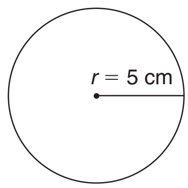
Use a compass. Construct a circle with:

a) radius 5 cm **b)** diameter 10 cm

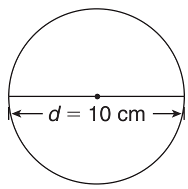
What do you notice about the circles you constructed?

A Solution

a) Draw a line segment with length 5 cm.
Place the compass point at one end.
Place the pencil point at the other end.
Draw a circle.



b) Draw a line segment with length 10 cm.
Use a ruler to find its midpoint.
Place the compass point at the midpoint.
Place the pencil point at one end of the segment.
Draw a circle.



The two circles are congruent.
A circle with radius 5 cm has diameter 10 cm.

Recall that congruent shapes are identical.

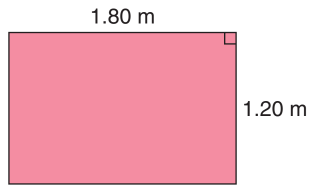
Practice

1. Use a compass.
Draw a circle with each radius.
a) 6 cm **b)** 8 cm
Label the radius, then find the diameter.

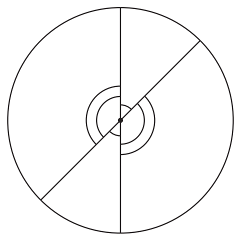
2. Draw a circle with each radius without using a compass.
   a) 7 cm b) 4 cm
   Label the radius, then find the diameter.
   Explain the method you used to draw the circles.
   What are the disadvantages of not using a compass?

3. a) A circle has diameter 3.8 cm. What is the radius?
   b) A circle has radius 7.5 cm. What is the diameter?

4. A circular tabletop is to be cut from a rectangular piece of wood that measures 1.20 m by 1.80 m.
   What is the radius of the largest tabletop that could be cut?
   Justify your answer. Include a sketch.



5. a) Use a compass. Draw a circle. Draw 2 different diameters.
   b) Use a protractor. Measure the angles at the centre of the circle.
   c) Find the sum of the angles.
   d) Repeat parts a to c for 3 different circles.
   What do you notice about the sum of the angles in each circle?



6. A glass has a circular base with radius 3.5 cm.
   A rectangular tray has dimensions 40 cm by 25 cm.
   How many glasses will fit on the tray?
   What assumptions did you make?

7. **Assessment Focus** Your teacher will give you a large copy of this logo.
   Find the radius and diameter of each circle in this logo. Show your work.

This is the logo for the Aboriginal Health Department of the Vancouver Island Health Authority.

8. **Take It Further** A circular area of grass needs watering.
   A rotating sprinkler is to be placed at the centre of the circle.
   Explain how you would locate the centre of the circle.
   Include a diagram in your explanation.

Reflect

How are the diameter and radius of a circle related?
Include examples in your explanation.

4.2 Circumference of a Circle

Focus Investigate the relationship between the circumference and diameter of a circle.

Explore

You will need 3 circular objects of different sizes, string, and a ruler.

➤ Each of you chooses one of the objects.
Use string to measure the distance around it.
Measure the radius and diameter of the object.
Record these measures.

➤ Repeat the activity until each of you has measured all 3 objects.
Compare your results.
If your measures are the same, record them in a table.
If your measures for any object are different, measure again to check.
When you agree upon the measures, record them in the table.

Object	Distance Around (cm)	Radius (cm)	Diameter (cm)
Can			

➤ What patterns do you see in the table?
How is the diameter related to the distance around?
How is the radius related to the distance around?

➤ For each object, calculate:
- distance around ÷ diameter
- distance around ÷ radius

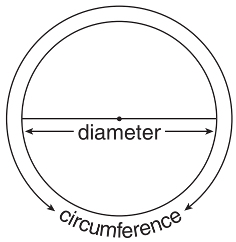
What do you notice?
Does the size of the circle affect your answers? Explain.

Reflect & Share

Compare your results with those of another group.
Suppose you know the distance around a circle.
How can you find its diameter?

Connect

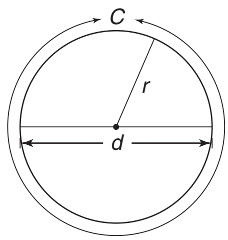
The distance around a circle is its **circumference**.
For any circle, the circumference, C, divided by the diameter, d, is approximately 3.
Circumference ÷ diameter $\doteq 3$, or $\frac{C}{d} \doteq 3$



The circumference of a circle is also the perimeter of the circle.

For any circle, the ratio $\frac{C}{d} = \pi$
The symbol π is a Greek letter that we read as "pi."
$\pi = 3.141\,592\,653\,589\ldots$, or $\pi \doteq 3.14$

π is a decimal that never repeats and never terminates.
π cannot be written as a fraction.
For this reason, we call π an **irrational number**.



So, the circumference is π multiplied by d.
We write: $C = \pi d$

Since the diameter is twice the radius,
the circumference is also π multiplied by $2r$.
We write: $C = \pi \times 2r$, or $C = 2\pi r$

When we know the radius or diameter of a circle, we can use one of the formulas above to find the circumference of the circle.

The face of a toonie has radius 1.4 cm.

- To find the diameter of the face:
  The diameter $d = 2r$, where r is the radius
  Substitute: $r = 1.4$
  $$d = 2 \times 1.4$$
  $$= 2.8$$
  The diameter is 2.8 cm.

The circumference is a length, so its units are units of length such as centimetres, metres, or millimetres.

- To find the circumference of the face:

$C = \pi d$	OR	$C = 2\pi r$
Substitute: $d = 2.8$		Substitute: $r = 1.4$
$C = \pi \times 2.8$		$C = 2 \times \pi \times 1.4$
$\doteq 8.796$		$\doteq 8.796$
$\doteq 8.8$		$\doteq 8.8$

The circumference is 8.8 cm, to one decimal place.

Use the π key on your calculator. If the calculator does not have a π key, use 3.14 instead.

- We can estimate to check if the answer is reasonable.
The circumference is approximately 3 times the diameter:
$3 \times 2.8 \text{ cm} \doteq 3 \times 3 \text{ cm}$
$= 9 \text{ cm}$
The circumference is approximately 9 cm.
The calculated answer is 8.8 cm, so this answer is reasonable.

When we know the circumference, we can use a formula to find the diameter.
Use the formula $C = \pi d$.
To isolate d, divide each side by π.

$$\frac{C}{\pi} = \frac{\pi d}{\pi}$$

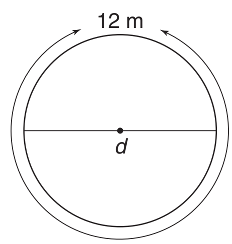
$$\frac{C}{\pi} = d$$

So, $d = \frac{C}{\pi}$

Example

An above-ground circular swimming pool has circumference 12 m.
Calculate the diameter and radius of the pool.
Give the answers to two decimal places.
Estimate to check the answers are reasonable.

A Solution



The diameter is: $d = \frac{C}{\pi}$
Substitute: $C = 12$

$d = \frac{12}{\pi}$

$= 3.8197\ldots$

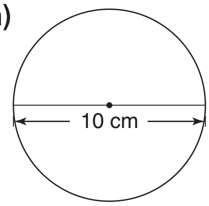
Use a calculator.
Do not clear your calculator.

The radius is $\frac{1}{2}$ the diameter, or $r = d \div 2$.
Divide the number in the calculator display by 2.
$r \doteq 1.9099$
The diameter is 3.82 m to two decimal places.
The radius is 1.91 m to two decimal places.

Since the circumference is approximately 3 times the diameter,
the diameter is about $\frac{1}{3}$ the circumference.
One-third of 12 m is 4 m. So, the diameter is about 4 m.
The radius is $\frac{1}{2}$ the diameter. One-half of 4 m is 2 m.
So, the radius of the pool is about 2 m.
Since the calculated answers are close to the estimates, the answers are reasonable.

Practice

1. Calculate the circumference of each circle.
Give the answers to two decimal places.
Estimate to check the answers are reasonable.

a)


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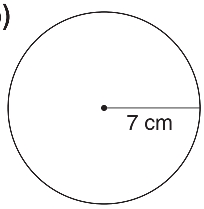
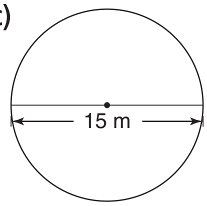
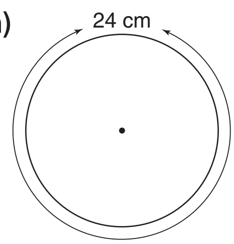
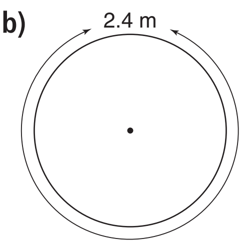
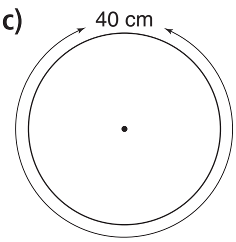


c)


2. Calculate the diameter and radius of each circle.
Give the answers to two decimal places.
Estimate to check the answers are reasonable.

a)


b)


c)


3. When you estimate to check the circumference, you use 3 instead of π.
Is the estimated circumference greater than or less than the actual circumference?
Why do you think so?

4. A circular garden has diameter 2.4 m.

a) The garden is to be enclosed with plastic edging.
How much edging is needed?

b) The edging costs $4.53/m.
What is the cost to edge the garden?

5. a) Suppose you double the diameter of a circle.
What happens to the circumference?
b) Suppose you triple the diameter of a circle.
What happens to the circumference?
Show your work.

6. A carpenter is making a circular tabletop with circumference 4.5 m.
What is the radius of the tabletop in centimetres?

Recall: 1 m = 100 cm

7. Can you draw a circle with circumference 33 cm?
If you can, draw the circle and explain how you know its circumference is correct.
If you cannot, explain why it is not possible.

8. **Assessment Focus** A bicycle tire has a spot of wet paint on it.
The radius of the tire is 46 cm.
Every time the wheel turns, the paint marks the ground.
a) What pattern will the paint make on the ground as the bicycle moves?
b) How far will the bicycle have travelled between two consecutive paint marks on the ground?
c) Assume the paint continues to mark the ground.
How many times will the paint mark the ground when the bicycle travels 1 km?
Show your work.

9. **Take It Further** Suppose a metal ring could be placed around Earth at the equator.
a) The radius of Earth is 6378.1 km. How long is the metal ring?
b) Suppose the length of the metal ring is increased by 1 km.
Would you be able to crawl under the ring, walk under the ring, or drive a school bus under the ring?
Explain how you know.

Reflect

What is π?
How is it related to the circumference, diameter, and radius of a circle?

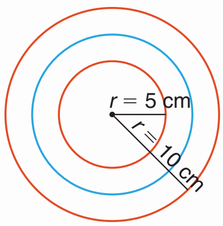
Mid-Unit Review

LESSON

4.1

1. **a)** Use a compass.
Draw a circle with radius 3 cm.
b) Do not use a compass.
Draw a circle with radius 7 cm.
The circle should have the same centre as the circle in part a.

2. Two circles have the same centre. Their radii are 5 cm and 10 cm. Another circle lies between these circles. Give two possible diameters for this circle.



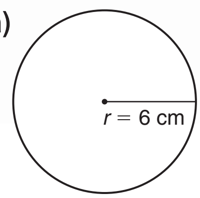
3. Find the radius of a circle with each diameter.
a) 7.8 cm **b)** 8.2 cm
c) 10 cm **d)** 25 cm

4. Is it possible to draw two different circles with the same radius and diameter? Why or why not?

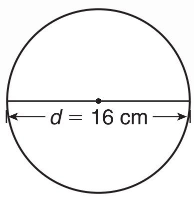
4.2

5. Calculate the circumference of each circle. Give the answers to two decimal places. Estimate to check your answers are reasonable.

a)



b)



6. **a)** Calculate the circumference of each object.
i) A wheelchair wheel with diameter 66 cm
ii) A tire with radius 37 cm
iii) A hula-hoop with diameter 60 cm
b) Which object has the greatest circumference? How could you tell without calculating the circumference of each object?

7. Suppose the circumference of a circular pond is 76.6 m. What is its diameter?

8. Find the radius of a circle with each circumference. Give your answers to one decimal place.
a) 256 cm **b)** 113 cm **c)** 45 cm

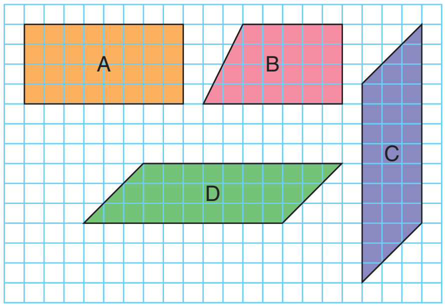
9. An auger is used to drill a hole in the ice, for ice fishing. The diameter of the hole is 25 cm. What is the circumference of the hole?

4.3 Area of a Parallelogram

Focus Develop a formula to find the area of a parallelogram.

Which of these shapes are parallelograms?
How do you know?

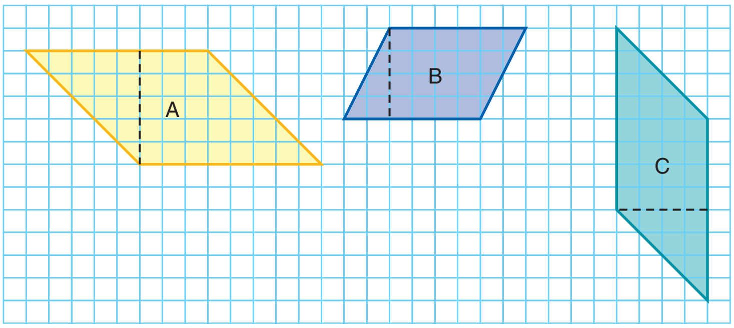
How are Shapes C and D alike?
How are they different?



Explore

You will need scissors and 1-cm grid paper.

➤ Copy Parallelogram A on grid paper.
Estimate, then find, the area of the parallelogram.



➤ Cut out the parallelogram.
Then, cut along the broken line segment.

➤ Arrange the two pieces to form a rectangle.
What is the area of the rectangle?
How does the area of the rectangle compare to the area of the parallelogram?

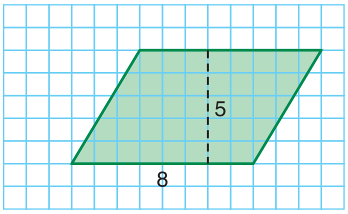
➤ Repeat the activity for Parallelograms B and C.

Reflect & Share

Share your work with another pair of classmates.
Can every parallelogram be changed into a rectangle by cutting and moving one piece? Explain.
Work together to write a rule for finding the area of a parallelogram.

Connect

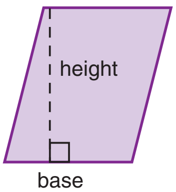
To estimate the area of this parallelogram, count the whole squares and the part squares that are one-half or greater.



There are:
- 33 whole squares
- 8 part squares that are one-half or greater

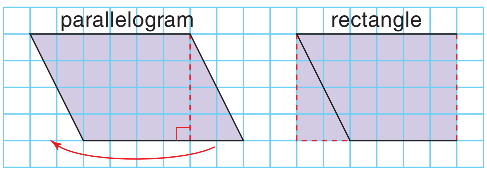
The area of this parallelogram is about 41 square units.

Any side of a parallelogram is a **base** of the parallelogram.
The **height** of a parallelogram is the length of a line segment that joins parallel sides and is perpendicular to the base.

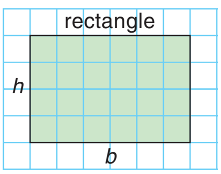


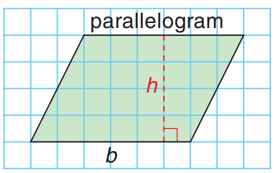
Recall that both a rectangle and a square are parallelograms.

Any parallelogram that is not a rectangle can be "cut" and rearranged to form a rectangle. Here is one way to do this.



The parallelogram and the rectangle have the same area.
The area of a parallelogram is equal to the area of a rectangle with the same height and base.
To find the area of a parallelogram, multiply the base by the height.





b represents the base.
h represents the height.

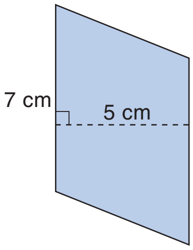
Area of rectangle:
$A = bh$

Area of parallelogram:
$A = bh$

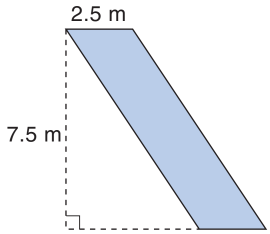
Example

Calculate the area of each parallelogram.

a)



b)



The height can be drawn outside the parallelogram.

A Solution

The area of a parallelogram is given by the formula $A = bh$.

a) $A = bh$

Substitute: $b = 7$ and $h = 5$

$A = 7 \times 5$

$= 35$

The area of the parallelogram is 35 cm^2.

b) $A = bh$

Substitute: $b = 2.5$ and $h = 7.5$

$A = 2.5 \times 7.5$

$= 18.75$

The area of the parallelogram is 18.75 m^2.

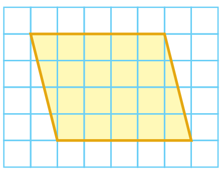
Practice

1. i) Copy each parallelogram on 1-cm grid paper.

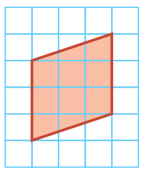
ii) Show how the parallelogram can be rearranged to form a rectangle.

iii) Estimate, then find, the area of each parallelogram.

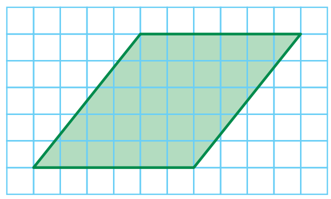
a)



b)

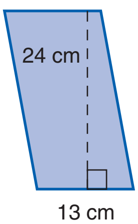


c)

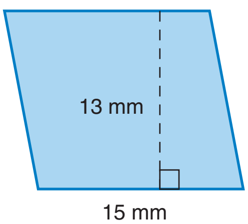


2. Find the area of each parallelogram.

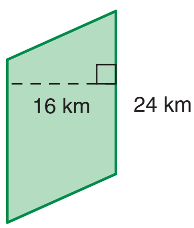
a)



b)



c)

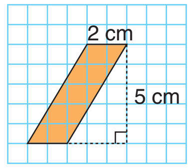


3. a) On 1-cm grid paper, draw 3 different parallelograms with base 3 cm and height 7 cm.

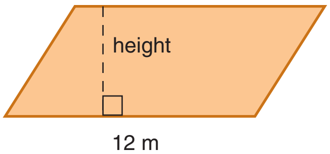
b) Find the area of each parallelogram you drew in part a. What do you notice?

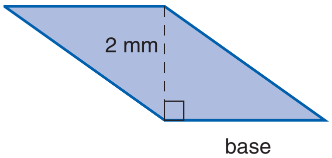
4. Repeat question 3. This time, you choose the base and height. Are your conclusions the same as in question 3? Why or why not?

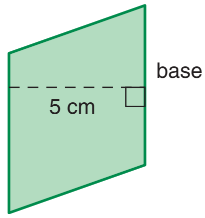
5. Copy this parallelogram on 1-cm grid paper.
   a) Show how this parallelogram could be rearranged to form a rectangle.
   b) Find the area of the parallelogram.



6. Use the given area to find the base or the height of each parallelogram.
   a) Area = 60 m^2
   b) Area = 6 mm^2
   c) Area = 30 cm^2

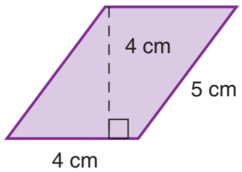




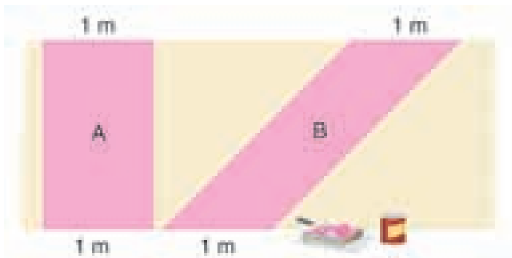


7. On 1-cm grid paper, draw as many different parallelograms as you can with each area.
   a) 10 cm^2
   b) 18 cm^2
   c) 28 cm^2

8. A student says the area of this parallelogram is 20 cm^2. Explain the student's error.



9. **Assessment Focus** Sasha is buying paint for a design on a wall. Here is part of the design. Sasha says Shape B will need more paint than Shape A. Do you agree? Why or why not?



10. **Take It Further** A restaurant owner built a patio in front of his store to attract more customers.
   a) What is the area of the patio?
   b) What is the total area of the patio and gardens?
   c) How can you find the area of the gardens? Show your work.

Reflect

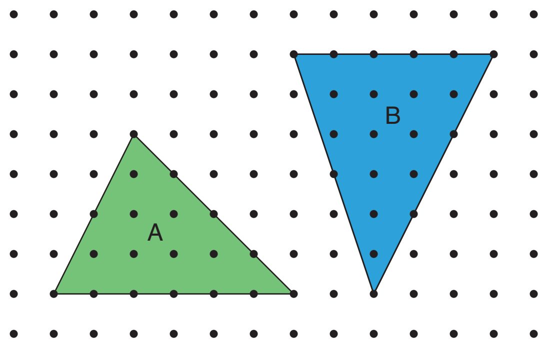
How can you use what you know about rectangles to help you find the area of a parallelogram? Use an example to explain.

4.4 Area of a Triangle

Focus Develop and use a formula to find the area of a triangle.

Explore

You will need a geoboard, geobands, and dot paper.



➤ Make Triangle A on a geoboard.
Add a second geoband to Triangle A to make a parallelogram with the same base and height.
This is called a *related* parallelogram.
Make as many different parallelograms as you can.
How does the area of the parallelogram compare to the area of Triangle A each time?
Record your work on dot paper.

➤ Repeat the activity with Triangle B.

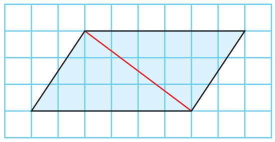
➤ What is the area of Triangle A? Triangle B?
What strategy did you use to find the areas?

Reflect & Share

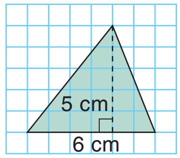
Share the different parallelograms you made with another pair of classmates.
Discuss the strategies you used to find the area of each triangle.
How did you use what you know about a parallelogram to find the area of a triangle?
Work together to write a rule for finding the area of a triangle.

Connect

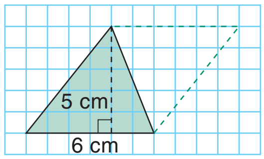
When we draw a diagonal in a parallelogram, we make two congruent triangles. Congruent triangles have the same area. The area of the two congruent triangles is equal to the area of the parallelogram that contains them.
So, the area of one triangle is $\frac{1}{2}$ the area of the parallelogram.



To find the area of this triangle:



Complete a parallelogram on one side of the triangle.



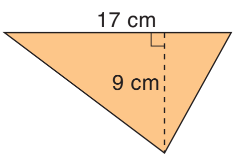
The area of the parallelogram is:
A = base $\times$ height, or $A = bh$
So, $A = 6 \times 5$
$= 30$
The area of the parallelogram is 30 cm^2.
So, the area of the triangle is: $\frac{1}{2}$ of 30 cm^2 = 15 cm^2

We can write a formula for the area of a triangle.
The area of a parallelogram is:
A = base $\times$ height
So, the area of a triangle is:
A = one-half of base $\times$ height
$A = bh \div 2$, which can be written as $A = \frac{bh}{2}$

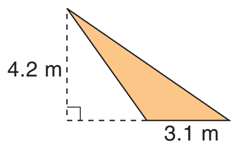
Example

Find the area of each triangle.

a)



b)



For an obtuse triangle, the height might be drawn outside the triangle.

A Solution

a) $A = \frac{bh}{2}$

Substitute: $b = 17$ and $h = 9$

$A = \frac{17 \times 9}{2}$

$= \frac{153}{2}$

$= 76.5$

The area is 76.5 cm^2.

b) $A = \frac{bh}{2}$

Substitute: $b = 3.1$ and $h = 4.2$

$A = \frac{3.1 \times 4.2}{2}$

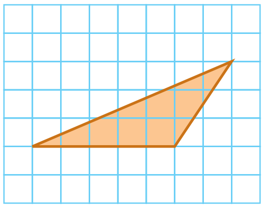
$= \frac{13.02}{2}$

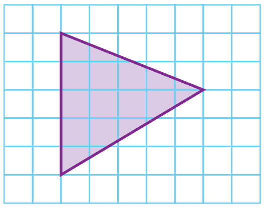
$= 6.51$

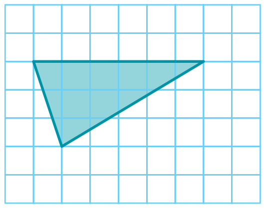
The area is 6.51 m^2.

Practice

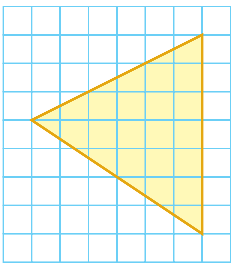
1. Copy each triangle on 1-cm grid paper. Draw a related parallelogram.

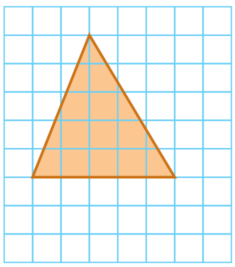
a) 

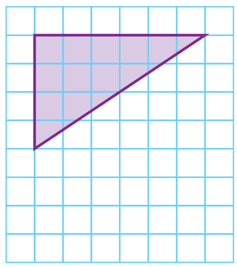
b) 

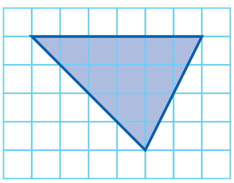
c) 

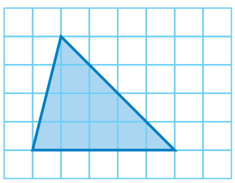
2. Each triangle is drawn on 1-cm grid paper.
Find the area of each triangle. Use a geoboard if you can.

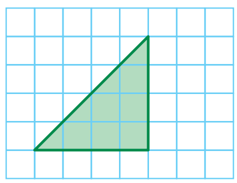
a) 

b) 

c) 

d) 

e) 

f) 

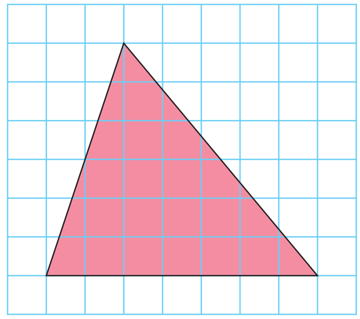
3. Draw two right triangles on 1-cm grid paper.

a) Record the base and the height of each triangle.

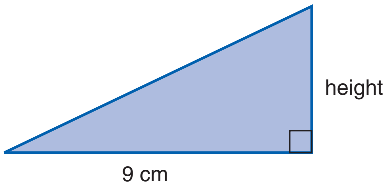
b) What do you notice about the height of a right triangle?

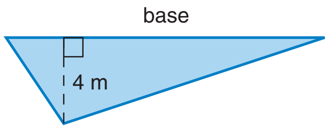
c) Find the area of each triangle you drew.

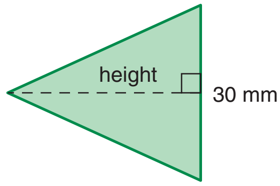
4. a) Find the area of this triangle.
   b) Use 1-cm grid paper.
   How many different parallelograms can you draw that have the same base and the same height as this triangle?
   Sketch each parallelogram.
   c) Find the area of each parallelogram.
   What do you notice?



5. Use the given area to find the base or height of each triangle.
   How could you check your answers?
   a) Area $= 18\text{ cm}^2$
   b) Area $= 32\text{ m}^2$
   c) Area $= 480\text{ mm}^2$







6. Use 1-cm grid paper.
   a) Draw 3 different triangles with each base and height.
      i) base: 1 cm; height: 12 cm
      ii) base: 2 cm; height: 6 cm
      iii) base: 3 cm; height: 4 cm
   b) Find the area of each triangle you drew in part a.
   What do you notice?

7. On 1-cm grid paper, draw two different triangles with each area below.
   Label the base and height each time.
   How do you know these measures are correct?
   a) 14 cm^2
   b) 10 cm^2
   c) 8 cm^2

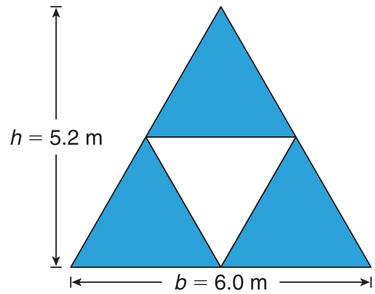
8. a) Draw any triangle on grid paper.
   What happens to the area of the triangle in each case?
      i) the base is doubled
      ii) both the height and the base are doubled
      iii) both the height and the base are tripled
   b) What could you do to the triangle you drew in part a to triple its area?
   Explain why this would triple the area.

9. Assessment Focus

This triangle is made from 4 congruent triangles.
Three triangles are to be painted blue.
The fourth triangle is not to be painted.

a) What is the area that is to be painted?
Show your work.

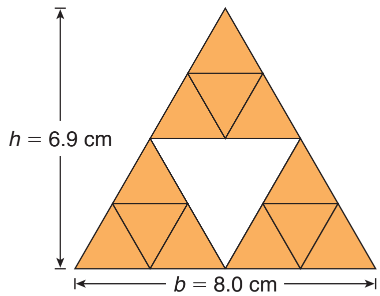
b) The paint is sold in 1-L cans.
One litre of paint covers 5.5 m^2.
How many cans of paint are needed?
What assumptions did you make?



The height is approximate.

10. Look at the diagram to the right.

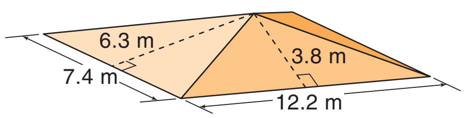
a) How many triangles do you see?
b) How are the triangles related?
c) How many parallelograms do you see?
d) Find the area of the large triangle.
e) Find the area of one medium-sized triangle.
f) Find the area of one small triangle.
g) Find the area of a parallelogram of your choice.

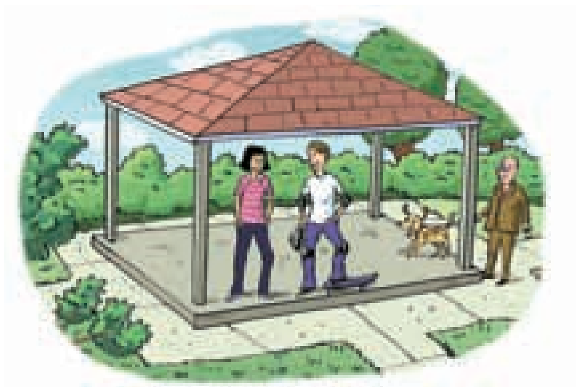


The height is approximate.

11. Take It Further

A local park has a pavilion to provide shelter.
The pavilion has a roof the shape of a rectangular pyramid.





a) What is the total area of all four parts of the roof?

b) One sheet of plywood is 240 cm by 120 cm.
What is the least number of sheets of plywood needed to cover the roof?
Explain how you got your answer.

Reflect

What do you know about finding the area of a triangle?

4.5 Area of a Circle

Focus Develop and use a formula to find the area of a circle.

Explore

You will need one set of fraction circles, masking tape, and a ruler.

➤ Each of you chooses one circle from the set of fraction circles. The circle you choose should have an even number of sectors, and at least 4 sectors.

➤ Each of you cuts 3 strips of masking tape:
- 2 short strips
- 1 strip at least 15 cm long

Use the short strips to fasten the long strip face up on the table.

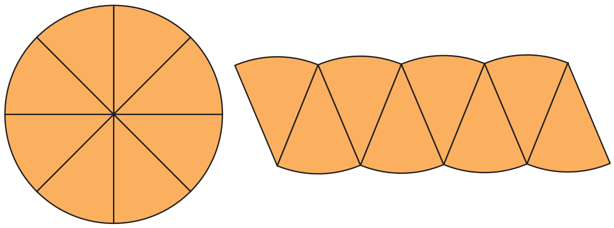
➤ Arrange all your circle sectors on the tape to approximate a parallelogram.
Trace your parallelogram, then use a ruler to make the horizontal sides straight.
Calculate the area of the parallelogram. Estimate the area of the circle.
How does the area of the parallelogram compare to the area of the circle?

Reflect & Share

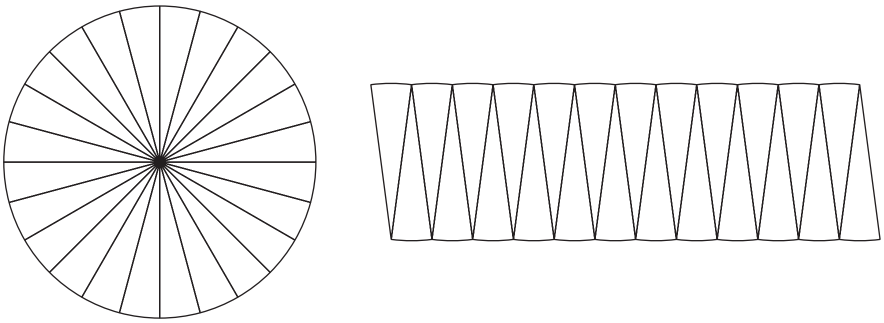
Compare your measure of the area of the circle with the measures of your group members.
Which area do you think is closest to the area of the circle? Why?
How could you improve your estimate for the area?
Which circle measure best represents the height of the parallelogram?
The base? Work together to write a formula for the area of a circle.

Connect

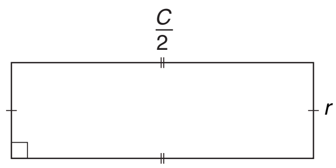
Suppose a circle was cut into 8 congruent sectors.
The 8 sectors were then arranged to approximate a parallelogram.



The more congruent sectors we use, the closer the area of the parallelogram is to the area of the circle.
Here is a circle cut into 24 congruent sectors.
The 24 sectors were then arranged to approximate a parallelogram.



The greater the number of sectors, the more the shape looks like a rectangle.
The sum of the two longer sides of the rectangle is equal to the circumference, C.
So, each longer side, or the base of the rectangle, is one-half the circumference of the circle, or $\frac{C}{2}$.

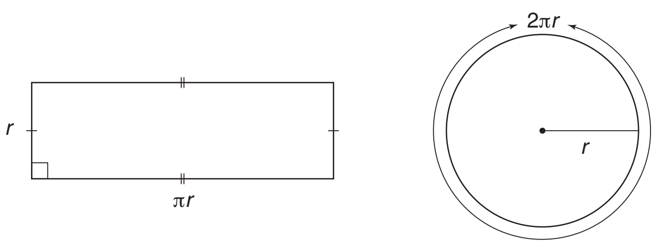


But $C = 2\pi r$

So, the base of the rectangle $= \frac{2\pi r}{2}$

$= \pi r$

Each of the two shorter sides is equal to the radius, r.

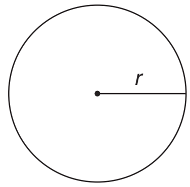


The area of a rectangle is: base $\times$ height
The base is πr. The height is r.
So, the area of the rectangle is: $\pi r \times r = \pi r^2$

When a number or variable is multiplied by itself we write: $7 \times 7 = 7^2$
$r \times r = r^2$

Since the rectangle is made from all sectors of the circle, the rectangle and the circle have the same area.
So, the area, A, of the circle with radius r is $A = \pi r^2$.

We can use this formula to find the area of any circle when we know its radius.



$A = \pi r^2$

Example

The face of a dime has diameter 1.8 cm.

a) Calculate the area.
Give the answer to two decimal places.

b) Estimate to check the answer is reasonable.

A Solution

The diameter of the face of a dime is 1.8 cm.
So, its radius is: $\frac{1.8\text{ cm}}{2} = 0.9$ cm

1.8 cm

a) Use the formula: $A = \pi r^2$
Substitute: $r = 0.9$
$A = \pi \times 0.9^2$
Use a calculator.
$A \doteq 2.544\ 69$
The area of the face of the dime is 2.54 cm^2 to two decimal places.

If your calculator does not have an x^2 key, key in 0.9×0.9 instead.

b) Recall that $\pi \doteq 3$.
So, the area of the face of the dime is about $3r^2$.

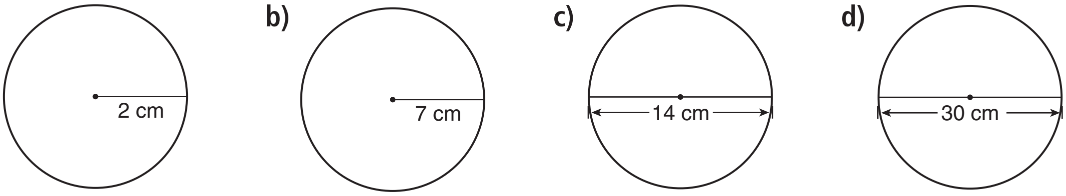
$$r \doteq 1$$
$$\text{So, } r^2 = 1$$
$$\text{and } 3r^2 = 3 \times 1$$
$$= 3$$

The area of the face of the dime is approximately 3 cm^2.
Since the calculated area, 2.54 cm^2, is close to 3 cm^2, the answer is reasonable.

Practice

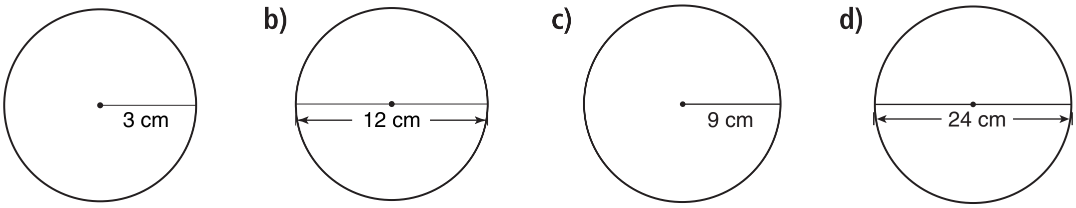
1. Calculate the area of each circle.
Estimate to check your answers are reasonable.

a) b) c) d)



2. Calculate the area of each circle. Give your answers to two decimal places.
Estimate to check your answers are reasonable.

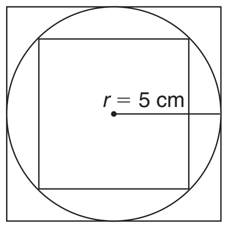
a) b) c) d)



3. Use the results of questions 1 and 2. What happens to the area in each case?
   a) You double the radius of a circle.
   b) You triple the radius of a circle.
   c) You quadruple the radius of a circle.

   Justify your answers.

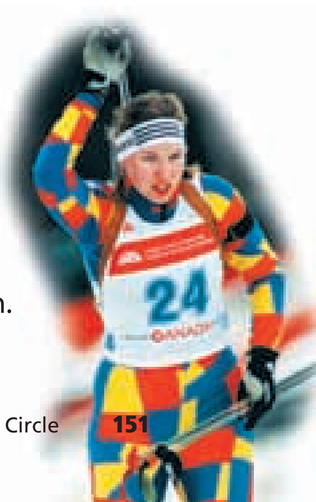
4. **Assessment Focus** Use 1-cm grid paper.
Draw a circle with radius 5 cm.
Draw a square outside the circle that just encloses the circle.
Draw a square inside the circle
so that its vertices lie on the circle.
Measure the sides of the squares.



   a) How can you use the areas of the two squares to estimate the area of the circle?
   b) Check your estimate in part a by calculating the area of the circle.
   c) Repeat the activity for circles with different radii.
      Record your results. Show your work.

5. In the biathlon, athletes shoot at targets. Find the area of each target.
   a) The target for the athlete who is standing is a circle with diameter 11.5 cm.
   b) The target for the athlete who is lying down is a circle with diameter 4.5 cm.

   Give the answers to the nearest square centimetre.



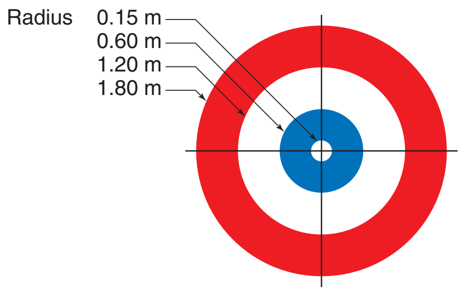
6. In curling, the target area is a bull's eye with 4 concentric circles.

a) Calculate the area of the smallest circle.

b) When a smaller circle overlaps a larger circle, a ring is formed.
Calculate the area of each ring on the target area.

Give your answers to 4 decimal places.

Concentric circles have the same centre.



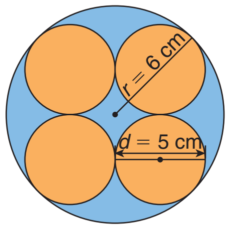
7. **Take It Further**

A circle with radius 6 cm contains 4 small circles.
Each small circle has diameter 5 cm.
Each small circle touches two other small circles and the large circle.

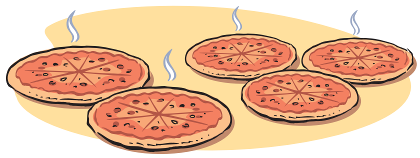
a) Find the area of the large circle.

b) Find the area of one small circle.

c) Find the area of the region that is shaded yellow.



8. **Take It Further** A large pizza has diameter 35 cm.
Two large pizzas cost $19.99.
A medium pizza has diameter 30 cm.
Three medium pizzas cost $24.99.
Which is the better deal: 2 large pizzas or 3 medium pizzas?
Justify your answer.



Math Link

Agriculture: Crop Circles

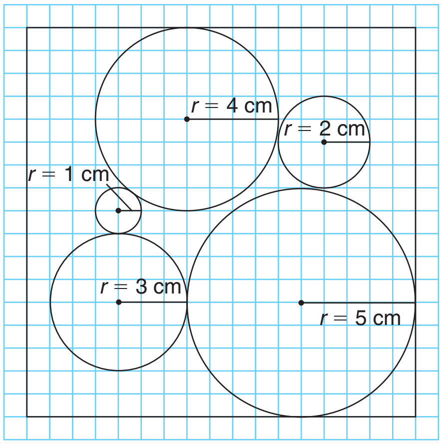
In Red Deer, Alberta, on September 17, 2001, a crop circle formation was discovered that contained 7 circles. The circle shown has diameter about 10 m. This circle destroyed some wheat crop. What area of wheat crop was lost in this crop circle?

Reflect

You have learned two formulas for measurements of a circle.
How do you remember which formula to use for the area of a circle?

Packing Circles

These circles are packed in a square.
In this game, you will pack circles in other shapes.



YOU WILL NEED

2 sheets of circles
scissors
ruler
compass
calculator

NUMBER OF PLAYERS

2

GOAL OF THE GAME

Construct the circle, triangle, and parallelogram with the lesser area.

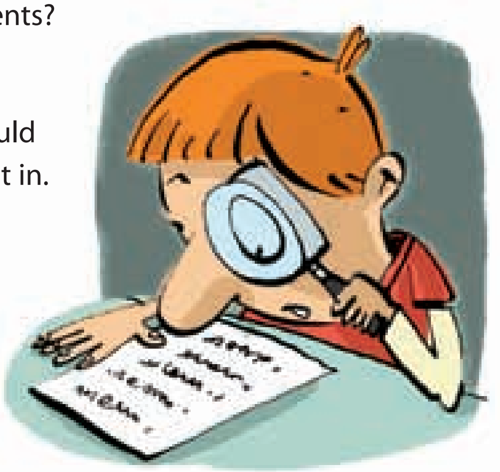
What strategies did you use to pack your circles to construct the shape with the lesser area?

HOW TO PLAY THE GAME:

1. Each player cuts out one sheet of circles.
2. Each player arranges his 5 circles so they are packed tightly together.
3. Use a compass. Draw a circle that encloses these circles.
4. Find the area of the enclosing circle.
   The player whose circle has the lesser area scores 2 points.
5. Pack the circles again.
   This time draw the parallelogram that encloses the circles.
   Find the area of the parallelogram.
   The player whose parallelogram has the lesser area scores 2 points.
6. Repeat *Step 5*. This time use a triangle to enclose the circles.
7. The player with the higher score wins.

Reading for Accuracy—Checking Your Work

Do you make careless mistakes on homework assignments? On quizzes or tests?



To help improve your accuracy and test scores, you should get in the habit of checking your work before handing it in.

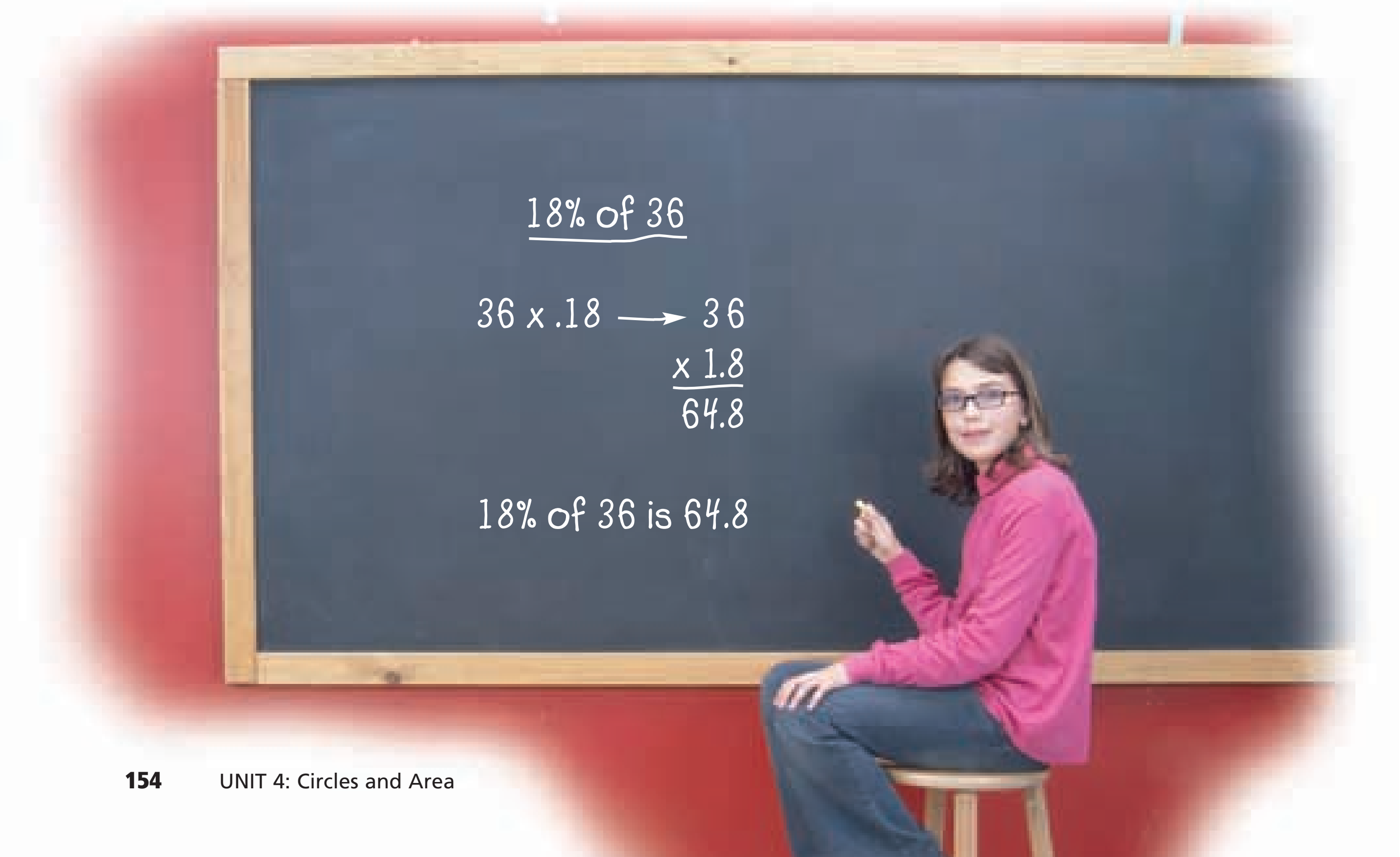
The three most common types of mistakes are:
- copying errors
- notation errors
- calculation errors

Copying Errors

A copying error occurs when you copy the question or numbers in the question incorrectly.

Find the copying error in this solution.
How can you tell that the solution is not reasonable? Explain.

1. Find 18% of 36.



Notation Errors

A notation error occurs when you use a math symbol incorrectly.

Find the notation error in this solution.

2. Evaluate: (–8) + (+3) – (+2)

Evaluate $(-8) + (+3) - (+2)$

$(-8) + (+3) - (+2) = (-5) - (+2)$

$= (+5) + (+2)$

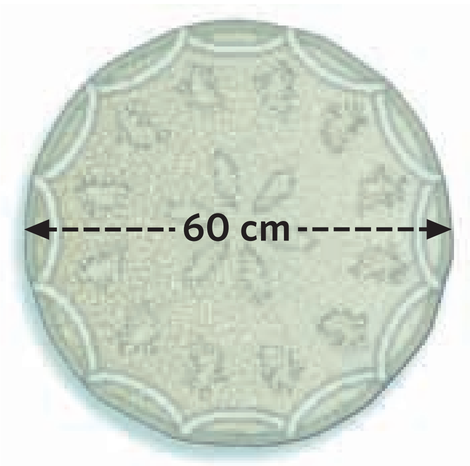
$= (+7)$

Calculation Errors

A calculation error occurs when you make a mistake in your calculations.

Find the calculation error in this solution.
How can you tell that the solution is not reasonable? Explain.

3. A circular mat has diameter 60 cm.
What is the area of the mat?



The diameter of the mat is 60 cm.
So, its radius is: 60 cm/2 = 30 cm
Use the formula $A = \pi r^2$
Substitute: $r = 30$
$A = \pi \times 30^2$
$= \pi \times 9000$
$\doteq 28\ 274$
The area of the mat is about 28 274 cm^2.

4. Correct the error you found in each solution to find the correct answer.
Show your work.

4.6 Interpreting Circle Graphs

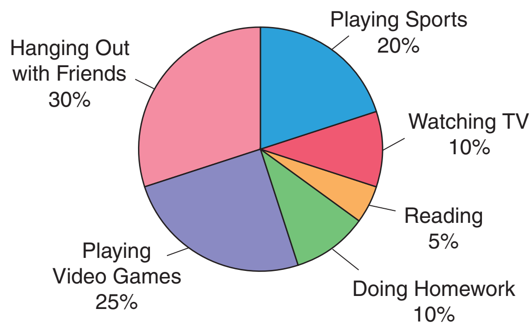
Focus Interpret circle graphs to solve problems.

We can apply what we have learned about circles to interpret a new type of graph.

Explore

Sixty Grade 7 students at l'école Orléans were surveyed to find out their favourite after-school activity. The results are shown on the circle graph.

After-School Activities



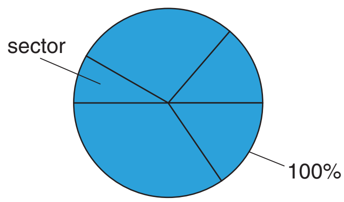
Which activity is most popular? Least popular? How do you know this from looking at the graph? How many students prefer each type of after-school activity? Which activity is the favourite for about $\frac{1}{3}$ of the students? Why do you think so? Write 3 more things you know from looking at the graph.

Reflect & Share

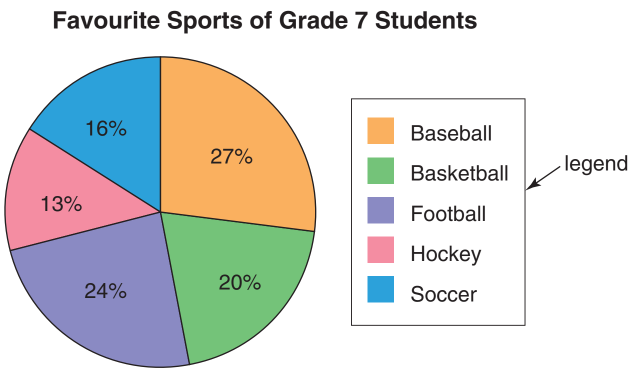
Compare your answers with those of another pair of classmates. What do you notice about the sum of the percents? Explain.

Connect

In a **circle graph**, data are shown as parts of one whole. Each **sector** of a circle graph represents a percent of the whole circle. The whole circle represents 100%.



A circle graph has a title.
Each sector is labelled with a category and a percent.
A circle graph compares the number in each category to the total number.
That is, a fraction of the circle represents the same fraction of the total.
Sometimes, a circle graph has a **legend** that shows what category each sector represents.
In this case, only the percents are shown on the graph.

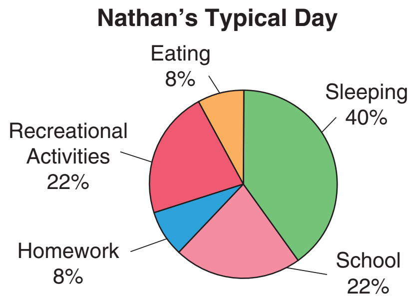


Example

This graph shows Nathan's typical day.

a) Which activity does Nathan do about $\frac{1}{4}$ of the time?

b) About how many hours does Nathan spend on each activity?
Check that the answers are reasonable.



A Solution

a) Each of the sectors for "School" and "Recreational Activities" is about $\frac{1}{4}$ of the graph. 22% is close to 25%, which is $\frac{1}{4}$.
So, Nathan is in school about $\frac{1}{4}$ of the day.
He also participates in recreational activities about $\frac{1}{4}$ of the day.

b) ➤ From the circle graph, Nathan spends 40% of his day sleeping.
There are 24 h in a day.
Find 40% of 24.
$40\% = \frac{40}{100} = 0.4$
Multiply: $0.4 \times 24 = 9.6$
Nathan spends about 10 h sleeping.

9.6 is closer to 10 than to 9.

➤ Nathan spends 22% of his day in school.

Find 22% of 24.

$22\% = \frac{22}{100} = 0.22$

Multiply: $0.22 \times 24 = 5.28$

5.28 is closer to 5 than to 6.

Nathan spends about 5 h in school.

Nathan also spends about 5 h doing recreational activities.

➤ Nathan spends 8% of his day doing homework.

Find 8% of 24.

$8\% = \frac{8}{100} = 0.08$

Multiply: 0.08×24

Multiply as you would whole numbers.

$$\begin{array}{r} 24 \\ \times 8 \\ \hline 192 \end{array}$$

Estimate to place the decimal point.

$0.1 \times 24 = 2.4$

So, $0.08 \times 24 = 1.92$

1.92 is closer to 2 than to 1.

Nathan spends about 2 h doing homework.

Nathan also spends about 2 h eating.

The total number of hours spent on all activities should be 24, the number of hours in a day:

$9.6 + 5.28 + 5.28 + 1.92 + 1.92 = 24$

So, the answers are reasonable.

Add the exact times, *not* the approximate times.

Practice

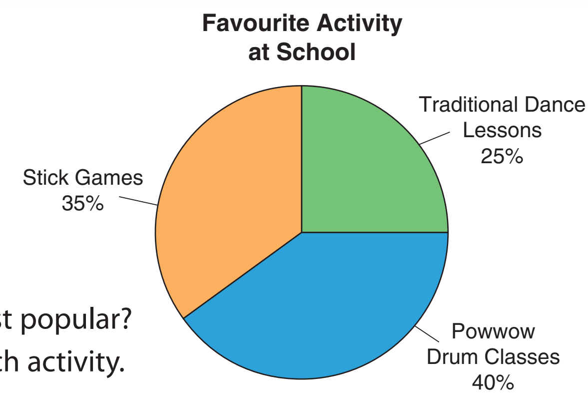
1. This circle graph shows the most popular activities in a First Nations school.
There are 500 students in the school.
All students voted.

a) Which activity did about $\frac{1}{4}$ of the students choose?
How can you tell by looking at the graph?

b) Which activity is the most popular? The least popular?

c) Find the number of students who chose each activity.

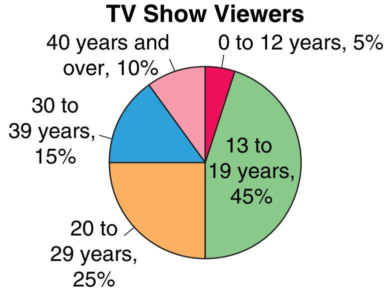
d) How can you check your answers to part c?



2. This circle graph shows the ages of viewers of a TV show.
One week, approximately 250 000 viewers tuned in.

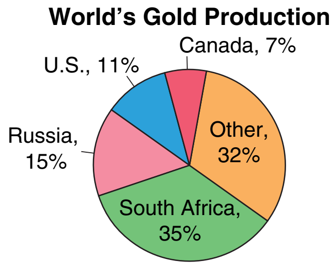
a) Which two age groups together make up $\frac{1}{2}$ of the viewers?

b) How many viewers were in each age group?
i) 13 to 19 ii) 20 to 29 iii) 40 and over



3. This graph shows the world's gold production for a particular year.
In this year, the world's gold production was approximately 2300 t.
About how much gold would have been produced in each country?

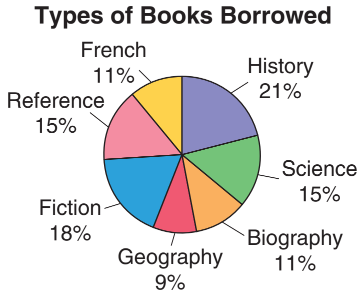
a) Canada b) South Africa



4. The school library budget to buy new books is $5000.
The librarian has this circle graph to show the types of books students borrowed in one year.

a) How much money should be spent on each type of book? How do you know?

b) Explain how you can check your answers in part a.



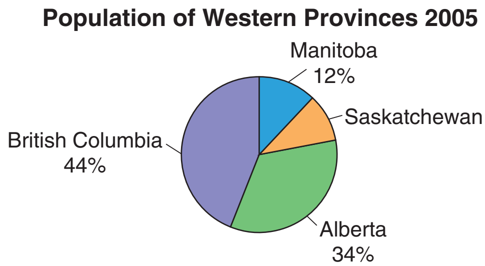
5. **Assessment Focus** This circle graph shows the populations of the 4 Western Canadian provinces in 2005.
The percent for Saskatchewan is not shown.

a) What percent of the population lived in Saskatchewan? How do you know?

b) List the provinces in order from least to greatest population.
How did the circle graph help you do this?

c) In 2005, the total population of the Western provinces was about 9 683 000 people.
Calculate the population of each province, to the nearest thousand.

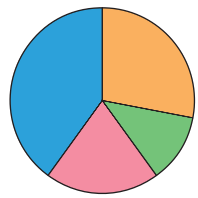
d) What else do you know from looking at the circle graph? Write as much as you can.



6. Gaston collected data about the favourite season of his classmates.

Classmates' Favourite Season

Season	Autumn	Winter	Spring	Summer
Number of Students	7	3	5	10



He recorded the results in a circle graph.
The graph is not complete.

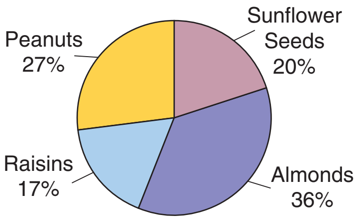
a) How many students were surveyed?

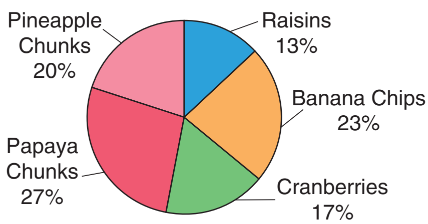
b) Write the number of students who chose each season as a fraction of the total number of students, then as a percent.

c) Explain how you can check your answers to part b.

d) Sketch the graph. Label each sector with its name and percent. How did you do this?

7. These circle graphs show the percent of ingredients in two 150-g samples of different snack mixes.





a) For each snack mix, calculate the mass, in grams, of each ingredient.

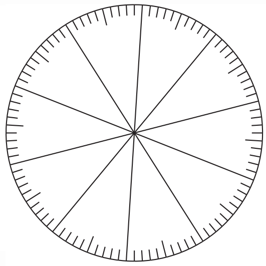
b) About what mass of raisins would you expect to find in a 300-g sample of each mix? What assumptions did you make?

Reflect

Search newspapers, magazines, and the Internet to find examples of circle graphs.
Cut out or print the graphs.
How are they the same? How are they different?
Why were circle graphs used to display these data?

4.7 Drawing Circle Graphs

Focus Construct circle graphs to display data.



This is a **percent circle**.
The circle is divided into 100 congruent parts.
Each part is 1% of the whole circle.
You can draw a circle graph on a percent circle.

Explore

Your teacher will give you a percent circle.
Students in a Grade 7 class were asked how many siblings they have.
Here are the results.

0 Siblings	1 Sibling	2 Siblings	More than 2 Siblings
3	13	8	1

Write each number of students as a fraction of the total number.
Then write the fraction as a percent.

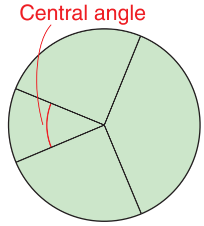
Use the percent circle.
Draw a circle graph to display the data.
Write 2 questions you can answer by looking at the graph.

Reflect & Share

Trade questions with another pair of classmates.
Use your graph to answer your classmates' questions.
Compare graphs. If they are different, try to find out why.
How did you use fractions and percents to draw a circle graph?

Connect

Recall that a circle graph shows how parts of a set of data compare with the whole set.
Each piece of data is written as a fraction of the whole.
Each fraction is then written as a percent.
Sectors of a percent circle are coloured to represent these percents.
The sum of the **central angles** is 360°.
A central angle is also called a **sector angle**.



Example

All the students in two Grade 7 classes were asked how they get to school each day.
Here are the results: 9 rode their bikes, 11 walked, 17 rode the bus, and 13 were driven by car.
Construct a circle graph to illustrate these data.

A Solution

➤ For each type of transport:
Write the number of students as a fraction of 50, the total number of students.
Then write each fraction as a decimal and as a percent.

Bike: $\frac{9}{50} = \frac{18}{100} = 0.18 = 18\%$ Walk: $\frac{11}{50} = \frac{22}{100} = 0.22 = 22\%$

Bus: $\frac{17}{50} = \frac{34}{100} = 0.34 = 34\%$ Car: $\frac{13}{50} = \frac{26}{100} = 0.26 = 26\%$

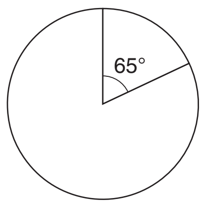
The circle represents all the types of transport.
To check, add the percents.
The sum should be 100%.
$18\% + 22\% + 34\% + 26\% = 100\%$

Another Strategy
We could use a percent circle to graph these data.

➤ To find the sector angle for each type of transport, multiply each decimal by 360°.
Write each angle to the nearest degree, when necessary.
Bike 18%: $0.18 \times 360° = 64.8° \doteq 65°$
Walk 22%: $0.22 \times 360° = 79.2° \doteq 79°$
Bus 34%: $0.34 \times 360° = 122.4° \doteq 122°$
Car 26%: $0.26 \times 360° = 93.6° \doteq 94°$

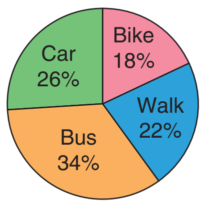
Check:
$64.8° + 79.2° + 122.4° + 93.6° = 360°$

➤ Construct a circle.
Use a protractor to construct each sector angle.
Start with the smallest angle.
Draw a radius. Measure 65°.



Start the next sector where the previous sector finished.
Label each sector with its name and percent.
Write a title for the graph.

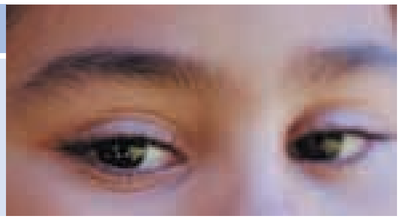
How Students Get to School



Practice

1. The table shows the number of Grade 7 students with each eye colour at Northern Public School.

Eye Colour	Number of Students
Blue	12
Brown	24
Green	8
Grey	6



a) Find the total number of students.
b) Write the number of students with each eye colour as a fraction of the total number of students.
c) Write each fraction as a percent.
d) Draw a circle graph to represent these data.

2. In a telephone survey, 400 people voted for their favourite radio station.
a) How many people chose EASY2?
b) Write the number of people who voted for each station as a fraction of the total number who voted.
Then write each fraction as a percent.
c) Draw a circle graph to display the results of the survey.

Radio Station	Votes
MAJIC99	88
EASY2	?
ROCK1	120
HITS2	100

3. **Assessment Focus** This table shows the method of transport used by U.S. residents entering Canada in one year.
a) How many U.S. residents visited Canada that year?
b) What fraction of U.S. residents entered Canada by boat?
c) What percent of U.S. residents entered Canada by plane?
d) Display the data in a circle graph.
e) What else do you know from the table or circle graph?
Write as much as you can.

United States Residents Entering Canada

Method of Transport	Number
Automobile	32 000 000
Plane	4 000 000
Train	400 000
Bus	1 600 000
Boat	1 200 000
Other	800 000

4. Can the data in each table below be displayed in a circle graph? Explain.

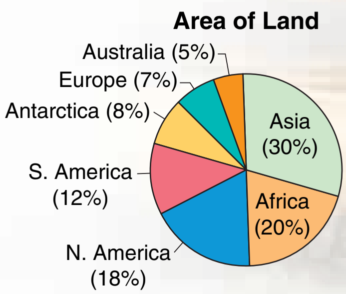
a)

Educational Attainment of Canadians	
0 to 8 years of elementary school	10%
Some secondary school	17%
Graduated from high school	20%
Some post-secondary education	9%
Post-secondary certificate or diploma	28%
University degree	16%

b)

Canadian Households with These Conveniences	
Automobile	64%
Cell phone	42%
Dishwasher	51%
Internet	42%

5. **Take It Further** This circle graph shows the percent of land occupied by each continent. The area of North America is approximately 220 million km^2. Use the percents in the circle graph. Find the approximate area of each of the other continents, to the nearest million square kilometres.



Reflect

When is it most appropriate to show data using a circle graph?
When is it not appropriate?

Using a Spreadsheet to Create Circle Graphs

Focus Display data on a circle graph using spreadsheets.

Spreadsheet software can be used to record, then graph, data. This table shows how Stacy budgets her money each month.

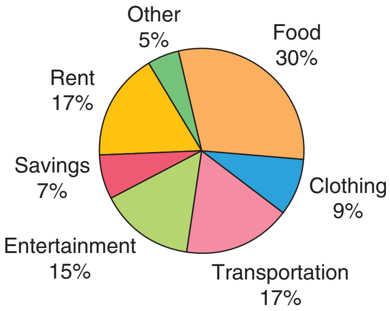
Stacy's Monthly Budget

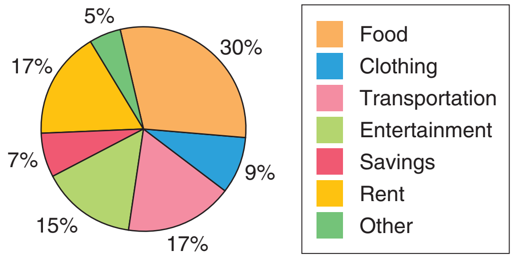
Category	Amount ($)
Food	160
Clothing	47
Transportation	92
Entertainment	78
Savings	35
Rent	87
Other	28

Enter the data into rows and columns of a spreadsheet. Highlight the data. Do not include the column heads.

	A	B
1	Category	Amount ($)
2	Food	160
3	Clothing	47
4	Transportation	92
5	Entertainment	78
6	Savings	35
7	Rent	87
8	Other	28

Click the graph/chart icon. In most spreadsheet programs, circle graphs are called **pie charts**. Select *pie chart*. Investigate different ways of labelling the graph. Your graph should look similar to one of the graphs on the following page.





This circle graph shows a legend at the right. The legend shows what category each sector represents.

These data are from *Statistics Canada*.

1. a) Use a spreadsheet.
Create a circle graph to display these data.

b) Write 3 questions about your graph.
Answer your questions.

c) Compare your questions with those of a classmate.
What else do you know from the table or the graph?

Population by Province and Territory, October 2005

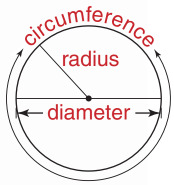
Region	Population
Newfoundland and Labrador	515 591
Prince Edward Island	138 278
Nova Scotia	938 116
New Brunswick	751 726
Quebec	7 616 645
Ontario	12 589 823
Manitoba	1 178 109
Saskatchewan	992 995
Alberta	3 281 296
British Columbia	4 271 210
Yukon Territories	31 235
Northwest Territories	42 965
Nunavut	30 133

Unit Review

What Do I Need to Know?

Measurements in a Circle

The distance from the centre to a point on the circle is the *radius*. The distance across the circle, through the centre, is the *diameter*.
The distance around the circle is the *circumference*.



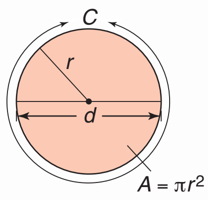
Circle Relationships

In a circle, let the radius be r, the diameter d, the circumference C, and the area A.
Then, $d = 2r$

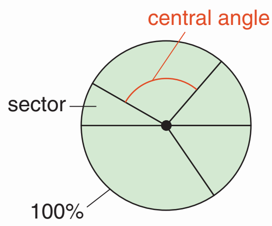
$$\frac{d}{2} = r$$

$$C = 2\pi r, \text{ or } C = \pi d$$

$$A = \pi r^2$$

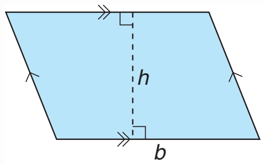


π is an irrational number that is approximately 3.14.
The sum of the central angles of a circle is 360°.

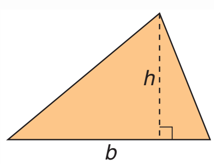


Area Formulas

Parallelogram: $A = bh$
where b is the base and
h is the height



Triangle: $A = \frac{bh}{2}$ or
$A = bh \div 2$
where b is the base and h is the height



Circle Graphs

In a circle graph, data are shown as parts of one whole.
The data are reported as a percent of the total, and the sum of the percents is 100%. The sum of the sector angles is 360°.

What Should I Be Able to Do?

LESSON

4.1

1. Draw a large circle without using a compass.
Explain how to find the radius and diameter of the circle you have drawn.

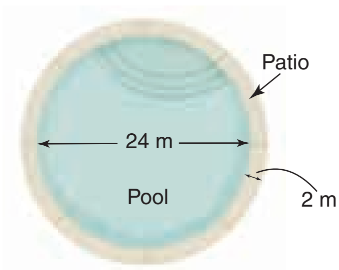
2. Find the radius of a circle with each diameter.
a) 12 cm **b)** 20 cm **c)** 7 cm

3. Find the diameter of a circle with each radius.
a) 15 cm **b)** 22 cm **c)** 4.2 cm

4.2

4. The circumference of a large crater is about 219 m.
What is the radius of the crater?

5. A circular pool has a circular concrete patio around it.
a) What is the circumference of the pool?
b) What is the combined radius of the pool and patio?
c) What is the circumference of the outside edge of the patio?



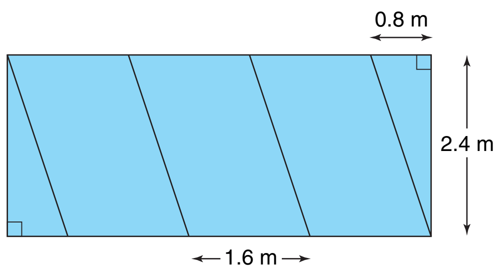
6. Mitra and Mel have different MP3 players.
The circular control dial on each player is a different size.
Calculate the circumference of the dial on each MP3 player.
a) Mitra's dial: diameter 30 mm
b) Mel's dial: radius 21 mm
c) Whose dial has the greater circumference? Explain.

4.3

7. On 0.5-cm grid paper, draw 3 different parallelograms with area 24 cm^2. What is the base and height of each parallelogram?

4.3
4.4

8. **a)** The window below consists of 5 pieces of glass. Each piece that is a parallelogram has base 1.6 m. What is the area of one parallelogram?

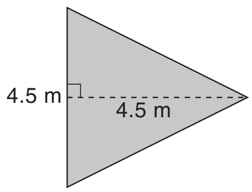


b) The base of each triangle in the window above is 0.8 m.
i) What is the area of one triangle?
ii) What is the area of the window?
Explain how you found the area.

LESSON

9. On 0.5-cm grid paper, draw 3 different triangles with area 12 cm^2.
   a) What is the base and height of each triangle?
   b) How are the triangles related to the parallelograms in question 7?

10. Po Ling is planning to pour a concrete patio beside her house. It has the shape of a triangle. The contractor charges $125.00 for each square metre of patio.



What will the contractor charge for the patio?

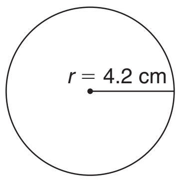
4.5 11. A goat is tied to an 8-m rope in a field.
   a) What area of the field can the goat graze?
   b) What is the circumference of the area in part a?

12. Choose a radius. Draw a circle. Suppose you divide the radius by 2.
   a) What happens to the circumference?
   b) Explain what happens to the area.

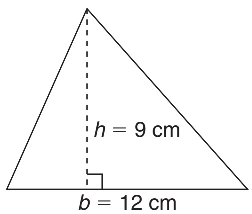
13. The diameter of a circular mirror is 28.5 cm. What is the area of the mirror? Give the answer to two decimal places.

14. Suppose you were to paint inside each shape below. Which shape would require the most paint? How did you find out?

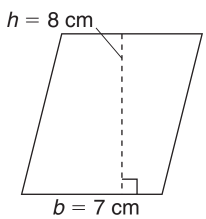
a)



b)



c)



LESSON

4.6

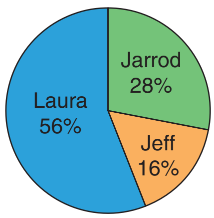
15. The results of the student council election are displayed on a circle graph. Five hundred students voted. The student with the most votes was named president.

a) Which student was named president? How do you know?

b) How many votes did each candidate receive?

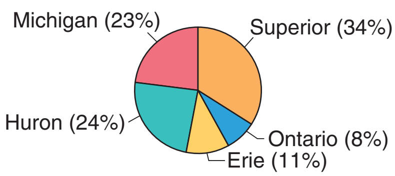
c) Write 2 other things you know from the graph.

Student Council Election Results



16. This circle graph shows the surface areas of the Great Lakes.

Areas of the Great Lakes



a) Which lake has a surface area about $\frac{1}{4}$ of the total area?

b) Explain why Lake Superior has that name.

c) The total area of the Great Lakes is about 244 000 km^2. Find the surface area of Lake Erie.

4.7

17. This table shows the approximate chemical and mineral composition of the human body.

Component	Percent
Water	62
Protein	17
Fat	15
Nitrogen	3
Calcium	2
Other	1

a) Draw a circle graph to display these data.

b) Jensen has mass 60 kg. About how many kilograms of Jensen's mass is water?

18. Here are the top 10 point scorers on the 2006 Canadian Women's Olympic Hockey Team. The table shows each player's province of birth.

Manitoba	Saskatchewan
Botterill	Wickenheiser
Quebec	**Ontario**
Ouellette	Apps
Goyette	Campbell
Vaillancourt	Hefford
	Piper
	Weatherston

a) What percent was born in each province?

b) Draw a circle graph to display the data in part a.

c) Why do you think more of these players come from Ontario than from any other province?

Practice Test

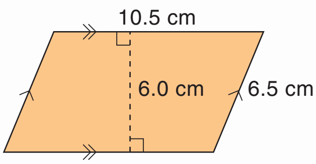
1. Draw a circle. Measure its radius.
   Calculate its diameter, circumference, and area.

2. The circular frame of this dream catcher has diameter 10 cm.
   a) How much wire is needed to make the outside frame?
   b) What is the area enclosed by the frame of this dream catcher?

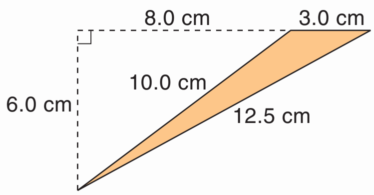
3. A circle is divided into 8 sectors.
   What is the sum of the central angles of the circle? Justify your answer.

4. Find the area of each shape. Explain your strategy.

   a)



   b)



5. a) How many different triangles and parallelograms can you sketch with area 20 cm^2?
   Have you sketched all possible shapes? Explain.
   b) Can you draw a circle with area 20 cm^2?
   If your answer is yes, explain how you would do it.
   If your answer is no, explain why you cannot do it.

6. The table shows the type of land cover in Canada, as a percent of the total area.
   a) Draw a circle graph.
   b) Did you need to know the area of Canada to draw the circle graph? Explain.
   c) Write 3 things you know from looking at the graph.

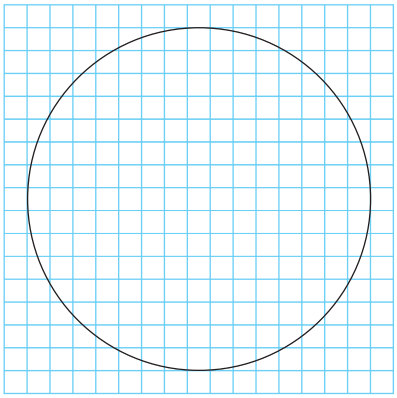
Type of Land Cover in Canada	
Forest and taiga	45%
Tundra	23%
Wetlands	12%
Fresh water	8%
Cropland and rangeland	8%
Ice and snow	3%
Human use	1%

Unit Problem Designing a Water Park

An anonymous donor gave a large sum of money to the Parks and Recreation Department.
The money is to be used to build a large circular water park.
Your task is to design the water park.

The water park has radius 30 m.

The side length of each square on this grid represents 4 m.



You must include the following features:

2 Wading Pools:

Each wading pool is triangular.
The pools do not have the same dimensions.
Each pool has area 24 m^2.

3 Geysers:

A geyser is circular.
Each geyser sprays water out of the ground,
and soaks a circular area with diameter 5 m or 10 m.

2 Wet Barricades:

A barricade has the shape of a parallelogram.
A row of nozzles in the barricade shoots water vertically.
The water falls within the area of the barricade.

4 Time-out Benches:

Each bench is shaped like a parallelogram.
It must be in the park.

At Least 1 Special Feature:

This feature will distinguish your park from other parks.
This feature can be a combination of any of the shapes you learned in this unit.
Give the dimensions of each special feature.
Explain why you included each feature in the park.

Your teacher will give you a grid to draw your design.
You may use plastic shapes or cutouts to help you plan your park.
Complete the design.
Colour the design to show the different features.

Design your park so that a person can walk through the middle of the park without getting wet.
What area of the park will get wet?

Check List

Your work should show:

- ✓ the area of each different shape you used
- ✓ a diagram of your design on grid paper
- ✓ an explanation of how you created the design
- ✓ how you calculated the area of the park that gets wet

Reflect on Your Learning

You have learned to measure different shapes.
When do you think you might use this knowledge outside the classroom?

Investigation

Digital Roots

Work with a partner.

The **digital root** of a number is the result of adding the digits of the number until a single-digit number is reached.

For example, the digital root of 27 is: $2 + 7 = 9$
To find the digital root of 168:
Add the digits: $1 + 6 + 8 = 15$
Since 15 is not a single-digit number, add the digits: $1 + 5 = 6$
Since 6 is a single-digit number, the digital root of 168 is 6.

A digital root can also be found for the product of a multiplication fact.
For the multiplication fact, 8×4:
$8 \times 4 = 32$
Add the digits in the product: $3 + 2 = 5$
Since 5 is a single-digit number, the digital root of 8×4 is 5.

You will explore the digital roots of the products in a multiplication table, then display the patterns you find.
As you complete the *Investigation*, include all your work in a report that you will hand in.

Materials:
- multiplication chart
- compass
- protractor
- ruler

Part 1

➤ Use a blank 12×12 multiplication chart.
Find each product.
Find the digital root of each product.
Record each digital root in the chart.
For example, for the product $4 \times 4 = 16$,
the digital root is $1 + 6 = 7$.

×	1	2	3	4	…
1					
2					
3					
4				7	
⋮					

➤ Describe the patterns in the completed chart.
Did you need to calculate the digital root of each product?
Did you use patterns to help you complete the table?
Justify the method you used to complete the chart.

➤ Look down each column. What does each column represent?

Part 2

- ➤ Use a compass to draw 12 circles.
  Use a protractor to mark 9 equally spaced points on each circle.
  Label these points in order, clockwise, from 1 to 9.
  Use the first circle.
  Look at the first two digital roots in the 1st column of your chart.
  Find these numbers on the circle.
  Use a ruler to join these numbers with a line segment.
  Continue to draw line segments to join points that match the digital roots in the 1st column.
  What shape have you drawn?

- ➤ Repeat this activity for each remaining column.
  Label each circle with the number at the top of the column.

- ➤ Which circles have the same shape?
  Which circle has a unique shape?
  What is unique about the shape?
  Why do some columns have the same pattern of digital roots?
  Explain.

Take It Further

- ➤ Investigate if similar patterns occur in each case:
  - Digital roots of larger 2-digit numbers, such as 85 to 99
  - Digital roots of 3-digit numbers, such as 255 to 269

  Write a report on what you find out.

UNIT 5

Operations with Fractions

Many newspapers and magazines sell advertising space. Why would a company pay for an advertisement?

Selling advertising space is a good way to raise funds. Students at Anishwabe School plan to sell advertising space in their yearbook.

How can fractions be used in advertising space?

What You'll Learn

- Add and subtract fractions using models, pictures, and symbols.
- Add and subtract mixed numbers.
- Solve problems involving the addition and subtraction of fractions and mixed numbers.

Why It's Important

- You use fractions when you read gauges, shop, measure, and work with percents and decimals, and in sports, recipes, and business.

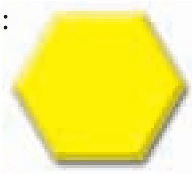
Key Words

- fraction strips
- simplest form
- related denominators
- unrelated denominators
- common denominator
- unit fraction

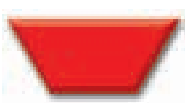
5.1 Using Models to Add Fractions

Focus Use Pattern Blocks and fraction circles to add fractions.

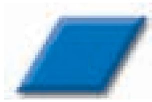
Let the yellow hexagon represent 1:



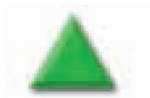
Then the red trapezoid represents $\frac{1}{2}$:



the blue rhombus represents $\frac{1}{3}$:



and the green triangle represents $\frac{1}{6}$:



Explore

Use Pattern Blocks.

Bakana trains for cross-country one hour a day. Here is her schedule:
Run for $\frac{1}{3}$ of the time, walk for $\frac{1}{6}$ of the time,
then run for the rest of the time.
How long does Bakana run altogether?
What fraction of the hour is this?

- Use fractions to write an addition equation to show how Bakana spent her hour.
- Bakana never runs for the whole hour.
  Write another possible schedule for Bakana.
  Write an addition equation for the schedule.
- Trade schedules with another pair of classmates.
  Write an addition equation for your classmates' schedule.

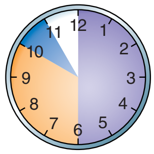
Reflect & Share

For the same schedule, compare equations with another pair of classmates.
Were the equations the same? How can you tell?
When are Pattern Blocks a good model for adding fractions?
When are Pattern Blocks not a good model?

Connect

There are many models that help us add fractions.

- We could use clocks to model halves, thirds, fourths, sixths, and twelfths.



$\frac{1}{2} + \frac{1}{3} + \frac{1}{12} = \frac{11}{12}$

  Circle models are useful when the fractions are less than 1.

- The example below uses fraction circles to add fractions.

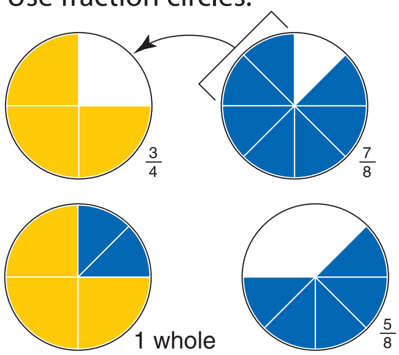
Example

Zack and Ronny each bought a small pizza.
Zack ate $\frac{3}{4}$ of his pizza and Ronny ate $\frac{7}{8}$ of his.
How much pizza did Zack and Ronny eat together?

A Solution

Add: $\frac{3}{4} + \frac{7}{8}$

Use fraction circles.



Use eighths to fill the circle for $\frac{3}{4}$.
Two-eighths fill the circle.

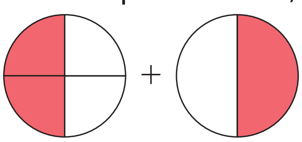
1 whole and 5 eighths equals $1\frac{5}{8}$.
So, $\frac{3}{4} + \frac{7}{8} = 1\frac{5}{8}$

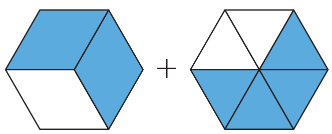
Together, Zack and Ronny ate $1\frac{5}{8}$ pizzas.

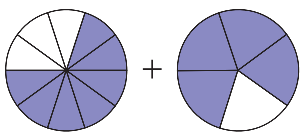
Practice

Use Pattern Blocks or fraction circles.

1. Model each picture. Then, find each sum.

a) 

b) 

c) 

2. Use a model to show each sum. Sketch the model.
Write an addition equation for each picture.

a) $\frac{7}{8} + \frac{1}{2}$ b) $\frac{3}{10} + \frac{2}{5}$ c) $\frac{2}{3} + \frac{1}{2}$ d) $\frac{2}{3} + \frac{5}{6}$

e) $\frac{3}{6} + \frac{1}{12}$ f) $\frac{1}{4} + \frac{2}{8}$ g) $\frac{1}{3} + \frac{1}{2}$ h) $\frac{1}{2} + \frac{4}{10}$

3. Simon spends $\frac{1}{6}$ h practising the whistle flute each day.
He also spends $\frac{1}{3}$ h practising the drums.
How much time does Simon spend each day practising these instruments?
Show how you found your solution.

4. a) Add.

i) $\frac{1}{5} + \frac{1}{5}$ ii) $\frac{2}{3} + \frac{1}{3}$ iii) $\frac{4}{10} + \frac{3}{10}$ iv) $\frac{1}{6} + \frac{3}{6}$

b) Look at your work in part a. How did you find your solutions?
How else could you add fractions with like denominators?

5. Is each sum greater than 1 or less than 1? How can you tell?

a) $\frac{1}{4} + \frac{2}{4}$ b) $\frac{2}{5} + \frac{7}{5}$ c) $\frac{3}{4} + \frac{1}{4}$ d) $\frac{1}{10} + \frac{3}{10}$

6. **Assessment Focus** Bella added 2 fractions. Their sum was $\frac{5}{6}$.
Which 2 fractions might Bella have added?
Find as many pairs of fractions as you can.
Show your work.

7. Asani's family had bannock with their dinner.
The bannock was cut into 8 equal pieces.
Asani ate 1 piece, her brother ate 2 pieces,
and her mother ate 3 pieces.

a) What fraction of the bannock did Asani eat?
Her brother? Her mother?

b) What fraction of the bannock was eaten?
What fraction was left?

Reflect

Which fractions can you add using Pattern Blocks? Fraction circles?
Give an example of fractions for which you cannot
use these models to add.

5.2 Using Other Models to Add Fractions

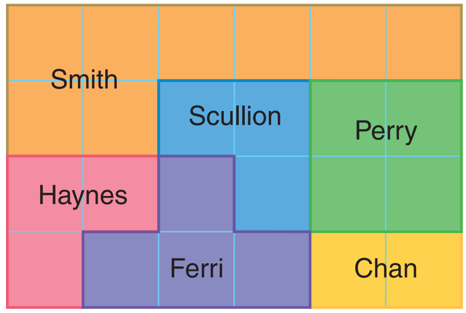
Focus Use fraction strips and number lines to add fractions.

We can use an area model to show fractions of one whole.

Explore

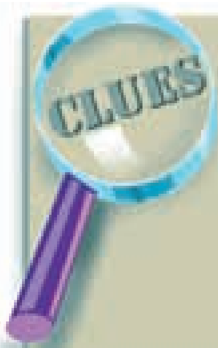
Your teacher will give you a copy of the map.
The map shows a section of land owned by 6 people.

- What fraction of land did each person own?
  What strategies did you use to find out?



Three people sold land to the other 3 people.

- Use the clues below to draw the new map.
- Write addition equations, such as $\frac{1}{2} + \frac{1}{4} = \frac{3}{4}$, to keep track of the land sales.



1. When all the sales were finished, four people owned all the land — Smith, Perry, Chan, and Haynes.
2. Smith now owns $\frac{1}{2}$ of the land.
3. Perry kept $\frac{1}{2}$ of her land, and sold the other half.
4. Chan bought land from two other people. He now owns $\frac{1}{4}$ of the land.
5. Haynes now owns the same amount of land as Perry started with.

Reflect & Share

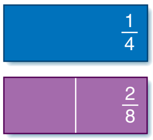
Did you find any equivalent fractions? How do you know they are equivalent?
Which clues helped you most to draw the new map? Explain how they helped.

Connect

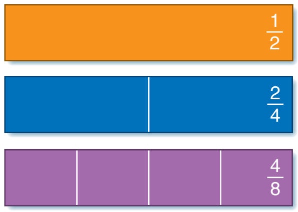
You can model fractions with strips of paper called **fraction strips**.

Here are more fraction strips and some equivalent fractions they show.

> **Recall that equivalent fractions show the same amount.**



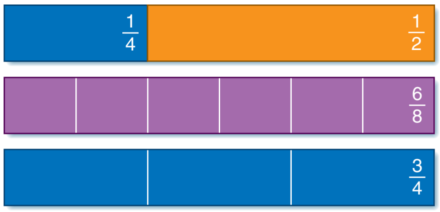
$\frac{1}{4} = \frac{2}{8}$



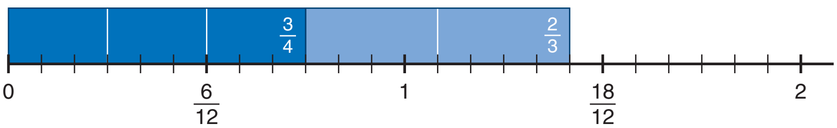
$\frac{1}{2} = \frac{2}{4} = \frac{4}{8}$

> **This strip represents 1 whole.**

To add $\frac{1}{4} + \frac{1}{2}$, align the strips for $\frac{1}{4}$ and $\frac{1}{2}$.
Find a single strip that has the same length as the two strips.
There are 2 single strips: $\frac{6}{8}$ and $\frac{3}{4}$
So, $\frac{1}{4} + \frac{1}{2} = \frac{6}{8}$
And, $\frac{1}{4} + \frac{1}{2} = \frac{3}{4}$
$\frac{3}{4}$ and $\frac{6}{8}$ are equivalent fractions.
The fraction $\frac{3}{4}$ is in simplest form.
A fraction is in **simplest form** when the numerator and denominator have no common factors other than 1.



When the sum is greater than 1, we could use fraction strips and a number line.



$\frac{3}{4} + \frac{2}{3} = \frac{17}{12}$

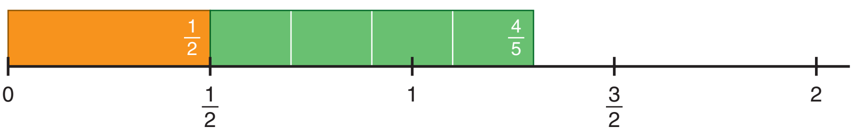
Example

Add. $\frac{1}{2} + \frac{4}{5}$

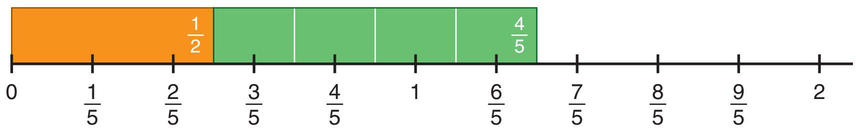
A Solution

$\frac{1}{2} + \frac{4}{5}$

Place both strips end-to-end on the halves line.
The right end of the $\frac{4}{5}$-strip does not line up with a fraction on the halves line.

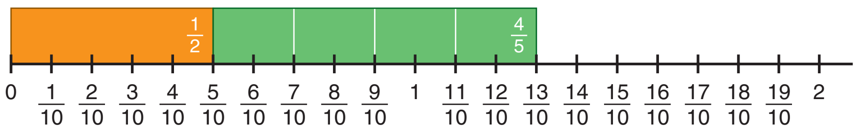


Place both strips on the fifths line.



The right end of the $\frac{4}{5}$-strip does not line up with a fraction on the fifths line. Find a line on which to place both strips so the end of the $\frac{4}{5}$-strip lines up with a fraction.

Another Strategy
We could add these fractions using fraction circles.



The end of the $\frac{4}{5}$-strip lines up with a fraction on the tenths line.
The strips end at $\frac{13}{10}$. So, $\frac{1}{2} + \frac{4}{5} = \frac{13}{10}$

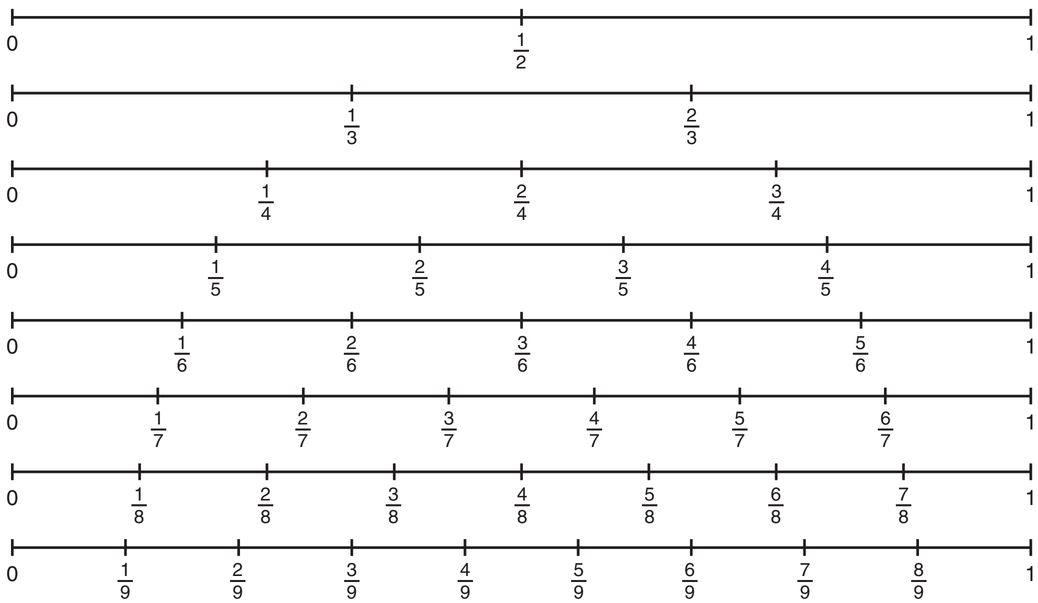
Practice

Use fraction strips and number lines.

1. Use the number lines below. List all fractions equivalent to:

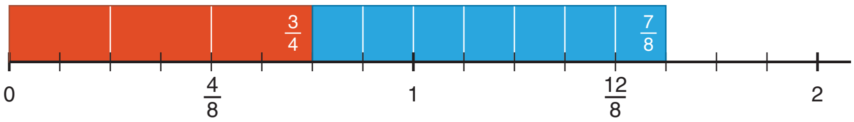
a) $\frac{1}{2}$ b) $\frac{1}{4}$ c) $\frac{2}{3}$

Use a ruler to align the fractions if it helps.

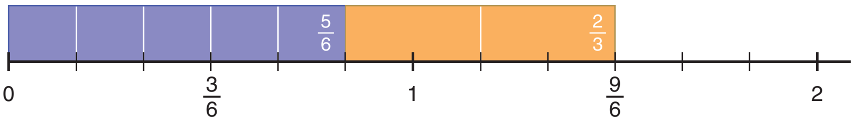


2. Write an addition equation for each picture.

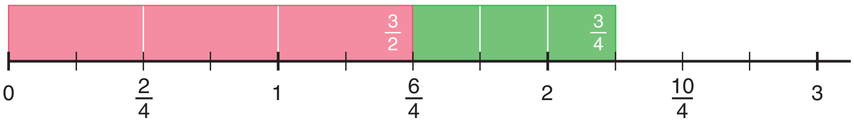
a)



b)



c)



3. Use your answers to question 2.

a) Look at the denominators in each part, and the number line you used to get the answer. What patterns do you see?

b) The denominators in each part of question 2 are **related denominators**. Why do you think they have this name?

4. Add.

a) $\frac{1}{3} + \frac{5}{6}$ b) $\frac{7}{12} + \frac{1}{3}$ c) $\frac{3}{5} + \frac{1}{10}$ d) $\frac{1}{6} + \frac{1}{12}$

5. Add.

a) $\frac{1}{3} + \frac{1}{2}$ b) $\frac{3}{4} + \frac{5}{6}$ c) $\frac{3}{5} + \frac{1}{2}$ d) $\frac{2}{3} + \frac{1}{5}$

6. Look at your answers to question 5.

a) Look at the denominators in each part, and the number line you used to get the answer. What patterns do you see?

b) The denominators in each part of question 5 are called **unrelated denominators**. Why do you think they have this name?

c) When you add 2 fractions with unrelated denominators, how do you decide which number line to use?

7. Add.

a) $\frac{1}{3} + \frac{2}{7}$ b) $\frac{3}{4} + \frac{2}{9}$ c) $\frac{4}{5} + \frac{5}{8}$ d) $\frac{2}{5} + \frac{3}{7}$

8. Abey and Anoki are eating chocolate bars.
The bars are the same size.
Abey has $\frac{3}{4}$ left. Anoki has $\frac{5}{6}$ left.
How much chocolate is left altogether? Show your work.

9. **Assessment Focus** Use any of the digits 1, 2, 3, 4, 5, 6 only once. Copy and complete. Replace each □ with a digit.

$$\frac{\square}{\square} + \frac{\square}{\square}$$

a) Find as many sums as you can that are between 1 and 2.
b) Find the least sum that is greater than 1.
Show your work.

10. Find 2 fractions with a sum of $\frac{3}{2}$. Try to do this as many ways as you can. Record each way you find.

11. **Take It Further** A jug holds 2 cups of liquid. A recipe for punch is $\frac{1}{2}$ cup of orange juice, $\frac{1}{4}$ cup of raspberry juice, $\frac{3}{8}$ cup of grapefruit juice, and $\frac{5}{8}$ cup of lemonade.
Is the jug big enough for the punch? Explain how you know.

12. **Take It Further** A pitcher of juice is half empty.
After $\frac{1}{2}$ cup of juice is added, the pitcher is $\frac{3}{4}$ full.
How much juice does the pitcher hold when it is full?
Show your thinking.

Math Link

Music
Musical notes are named for fractions.
The type of note shows a musician how long to play the note. In math, two halves make a whole — in music, two half notes make a whole note!

whole note	half note	quarter note	eighth note	sixteenth note

Reflect

What do you now know about adding fractions that you did not know at the beginning of the lesson?

5.3 Using Symbols to Add Fractions

Focus Use common denominators to add fractions.

In Lessons 5.1 and 5.2, you used models to add fractions.
You may not always have suitable models.

You need a strategy you can use to add fractions without using a model.

Explore

Copy these diagrams.

$$\frac{\square}{\square} + \frac{\square}{\square} = \qquad\qquad \frac{\square}{\square} + \frac{\square}{\square} =$$

greatest sum — least sum

Use the digits 1, 2, 4, and 8 to make the greatest sum and the least sum.
In each case, use each digit once.

Reflect & Share

Share your results with another pair of classmates.
Did you have the same answers?
If not, which is the greatest sum? The least sum?
What strategies did you use to add?

Connect

We can use equivalent fractions to add $\frac{1}{4} + \frac{1}{3}$.
Use equivalent fractions that have like denominators.
12 is a multiple of 3 and 4.
12 is a **common denominator**.

$\frac{1}{4} = \frac{3}{12}$ and $\frac{1}{3} = \frac{4}{12}$

So, $\frac{1}{4} + \frac{1}{3} = \frac{3}{12} + \frac{4}{12}$

$= \frac{7}{12}$

Both fractions are written with like denominators.