Look at the pattern in the equivalent fractions below.

$$\frac{1}{4} \overset{\times 3}{=} \frac{3}{12} \qquad\qquad \frac{1}{3} \overset{\times 4}{=} \frac{4}{12}$$

So, to get an equivalent fraction, multiply the numerator and denominator by the same number.

We may also get equivalent fractions by dividing.

For example, $\frac{8}{10}$ can be written: $\frac{8 \div 2}{10 \div 2} = \frac{4}{5}$

$\frac{8}{10}$ and $\frac{4}{5}$ are equivalent fractions.

$\frac{4}{5}$ is in simplest form.

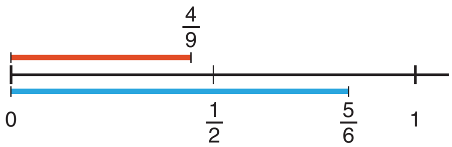
Example

Add: $\frac{4}{9} + \frac{5}{6}$

Estimate to check the sum is reasonable.

A Solution

$\frac{4}{9} + \frac{5}{6}$



Estimate first.

$\frac{4}{9}$ is about $\frac{1}{2}$.

$\frac{5}{6}$ is close to 1.

So, $\frac{4}{9} + \frac{5}{6}$ is about $1\frac{1}{2}$.

Use equivalent fractions to write the fractions with a common denominator.

List the multiples of 9: 9, **18**, 27, *36*, 45, …

List the multiples of 6: 6, 12, **18**, 24, 30, *36*, 42, …

18 is a multiple of 9 and 6, so 18 is a common denominator.

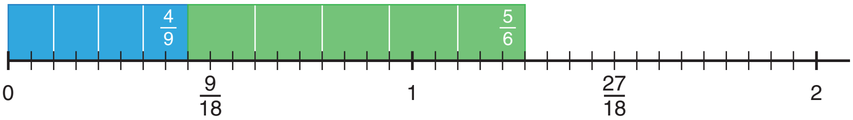
> **36 is also in both lists. So, 36 is another possible common denominator.**

$$\frac{4}{9} \overset{\times 2}{=} \frac{8}{18} \qquad\qquad \frac{5}{6} \overset{\times 3}{=} \frac{15}{18}$$

$$\begin{aligned} \frac{4}{9} + \frac{5}{6} &= \frac{8}{18} + \frac{15}{18} \\ &= \frac{23}{18} \end{aligned}$$

Add the numerators.

We could have found this sum with fraction strips on a number line.



Since 23 > 18, this is an improper fraction.
To write the fraction as a mixed number:

$\frac{23}{18} = \frac{18}{18} + \frac{5}{18}$

$= 1 + \frac{5}{18}$

$= 1\frac{5}{18}$ This is a mixed number.

The estimate was $1\frac{1}{2}$, so the answer is reasonable.

Recall that an improper fraction is a fraction with the numerator greater than the denominator.

Practice

Write all sums in simplest form.
Write improper fractions as mixed numbers.

1. Find a common denominator for each pair of fractions.
a) $\frac{1}{2}$ and $\frac{5}{8}$ b) $\frac{1}{8}$ and $\frac{2}{3}$ c) $\frac{2}{3}$ and $\frac{1}{9}$ d) $\frac{3}{5}$ and $\frac{2}{3}$

2. Copy and complete. Replace each □ with a digit to make each equation true.
a) $\frac{3}{12} = \frac{\square}{4}$ b) $\frac{3}{4} = \frac{6}{\square}$ c) $\frac{3}{6} = \frac{\square}{4}$ d) $\frac{6}{8} = \frac{15}{\square}$

3. Add. Sketch a number line to model each sum.
a) $\frac{4}{9} + \frac{1}{3}$ b) $\frac{1}{2} + \frac{1}{3}$ c) $\frac{3}{8} + \frac{3}{2}$ d) $\frac{3}{4} + \frac{1}{6}$

4. Estimate, then add.
a) $\frac{3}{5} + \frac{4}{8}$ b) $\frac{1}{6} + \frac{5}{8}$ c) $\frac{5}{6} + \frac{7}{9}$
d) $\frac{3}{4} + \frac{4}{7}$ e) $\frac{1}{3} + \frac{2}{5}$ f) $\frac{1}{5} + \frac{5}{6}$

5. One page of a magazine had 2 advertisements.
One was $\frac{1}{8}$ of the page, the other $\frac{1}{16}$.
What fraction of the page was covered?
Show your work.

6. Which sum is greater? Show your thinking.

$\frac{2}{3} + \frac{5}{6}$ or $\frac{3}{4} + \frac{4}{5}$

7. **Assessment Focus** Three people shared a pie.
Which statement is true? Can both statements be true?
Use pictures to show your thinking.

a) Edna ate $\frac{1}{10}$, Farrah ate $\frac{3}{5}$, and Ferris ate $\frac{1}{2}$.

b) Edna ate $\frac{3}{10}$, Farrah ate $\frac{1}{5}$, and Ferris ate $\frac{1}{2}$.

8. Damara and Baldwin had to shovel snow to clear their driveway.
Damara shovelled about $\frac{3}{10}$ of the driveway.
Baldwin shovelled about $\frac{2}{3}$ of the driveway.
What fraction of the driveway was cleared of snow?

9. Each fraction below is written as the sum of two unit fractions.
Which sums are correct? Why do you think so?

a) $\frac{7}{10} = \frac{1}{5} + \frac{1}{2}$ b) $\frac{5}{12} = \frac{1}{3} + \frac{1}{4}$ c) $\frac{5}{6} = \frac{1}{3} + \frac{1}{3}$

d) $\frac{7}{12} = \frac{1}{2} + \frac{1}{6}$ e) $\frac{11}{18} = \frac{1}{2} + \frac{1}{9}$ f) $\frac{2}{15} = \frac{1}{10} + \frac{1}{30}$

A fraction with numerator 1 is a **unit fraction**.

10. **Take It Further** Add.

a) $\frac{3}{8} + \frac{1}{2} + \frac{3}{4}$ b) $\frac{1}{4} + \frac{3}{2} + \frac{2}{5}$ c) $\frac{2}{3} + \frac{5}{6} + \frac{4}{9}$

Reflect

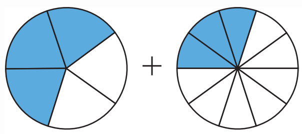
Suppose your friend has forgotten how to add
two fractions with unlike denominators.
What would you do to help?

Mid-Unit Review

LESSON

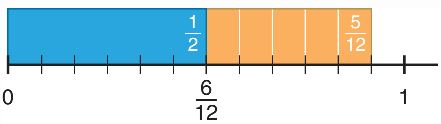
Write all sums in simplest form.
Write improper fractions as mixed numbers.

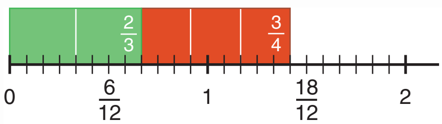
5.1 **1.** Use fraction circles. Model this picture, then find the sum.



2. On Saturday, Howie hiked for $\frac{5}{12}$ h in the morning and $\frac{3}{6}$ h in the afternoon. What fraction of an hour did Howie spend hiking?

5.2 **3.** Write an addition equation for each picture.

a)


b)


4. Add. Sketch fraction strips and a number line to model each addition.

a) $\frac{2}{8} + \frac{3}{8}$ b) $\frac{2}{3} + \frac{1}{6}$

c) $\frac{3}{4} + \frac{2}{6}$ d) $\frac{1}{2} + \frac{2}{5}$

5. Find 3 different ways to add $\frac{2}{3} + \frac{5}{6}$. Draw pictures to help you explain each way.

5.3 **6.** Add. Estimate to check the sum is reasonable.

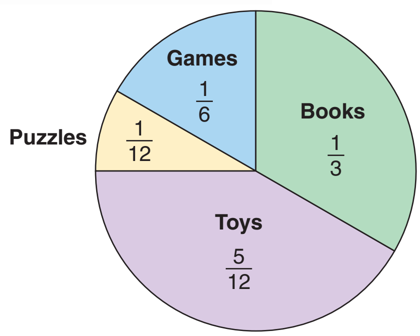
a) $\frac{4}{8} + \frac{5}{8}$ b) $\frac{1}{3} + \frac{3}{5}$

c) $\frac{1}{4} + \frac{1}{8}$ d) $\frac{5}{6} + \frac{7}{12}$

7. Takoda and Wesley are collecting shells on the beach in identical pails. Takoda estimates she has filled $\frac{7}{12}$ of her pail. Wesley estimates he has filled $\frac{4}{10}$ of his pail. Suppose the children combine their shells. Will one pail be full? Explain.

8. Each guest at Tai's birthday party brought one gift.
The circle graph shows the gifts Tai received.

Tai's Birthday Gifts



a) What fraction of the gifts were:
   i) toys or books?
   ii) puzzles or toys?
   iii) games or puzzles?
   iv) books or games?

b) Which 2 types of gifts represent $\frac{1}{4}$ of all the gifts?
Explain how you know.

5.4 Using Models to Subtract Fractions

Focus Use Pattern Blocks, fraction strips, and number lines to subtract fractions.

Explore

You will need congruent squares, grid paper, and coloured pencils.

Use these rules to create a rectangular design.
The design must be symmetrical.

- One-half of the squares must be red.
- One-third of the squares must be blue.
- The remaining squares must be green.

What fraction of the squares are green? How do you know?
How many squares did you use?
Explain why you used that number of squares.
Describe your design.
Record your design on grid paper.

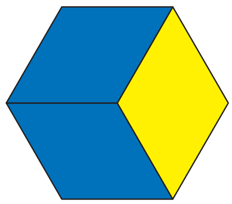
Reflect & Share

Compare your design with that of another pair of classmates.
If the designs are different, explain why your classmates' design obeys the rules.
How could you subtract fractions to find the fraction of the squares that are green?

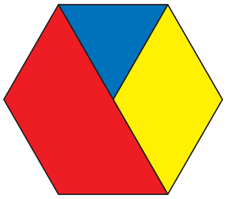
Connect

We can use models to subtract fractions.

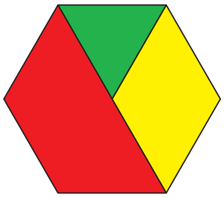
To subtract $\frac{2}{3} - \frac{1}{2}$, we can use Pattern Blocks.
The yellow hexagon represents 1. The blue rhombus represents $\frac{1}{3}$.
The red trapezoid represents $\frac{1}{2}$.
Place 2 blue rhombuses over the hexagon.



To subtract $\frac{1}{2}$, place a red trapezoid over the 2 blue rhombuses.



Find a Pattern Block equal to the difference.
The green triangle represents the difference.



The green triangle is $\frac{1}{6}$ of the hexagon.
So, $\frac{2}{3} - \frac{1}{2} = \frac{1}{6}$

We can also use fraction strips and number lines to subtract.
To subtract fractions with unlike denominators, we use equivalent fractions.

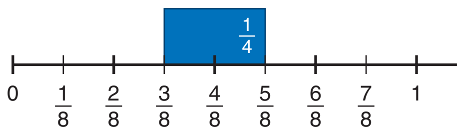
Example

Subtract: $\frac{5}{8} - \frac{1}{4}$

A Solution

$\frac{5}{8} - \frac{1}{4}$
Think addition.
What do we add to $\frac{1}{4}$ to get $\frac{5}{8}$?
Use a number line that shows equivalent fractions for eighths and fourths. That is, use the eighths number line.
Place the $\frac{1}{4}$-strip on the eighths number line with its right end at $\frac{5}{8}$.

Equivalent fractions:
$\frac{1}{4} = \frac{2}{8}$



The left end of the strip is at $\frac{3}{8}$.
So, $\frac{5}{8} - \frac{1}{4} = \frac{3}{8}$

Practice

Use models.

1. Find equivalent fractions with like denominators for each pair of fractions.

a) $\frac{1}{2}$ and $\frac{5}{8}$ b) $\frac{1}{4}$ and $\frac{1}{3}$ c) $\frac{2}{3}$ and $\frac{1}{6}$ d) $\frac{3}{5}$ and $\frac{1}{2}$

2. Is each difference less than $\frac{1}{2}$ or greater than $\frac{1}{2}$?
How can you tell?

a) $\frac{5}{6} - \frac{1}{2}$ b) $\frac{7}{8} - \frac{1}{8}$ c) $\frac{4}{6} - \frac{1}{3}$ d) $1 - \frac{5}{6}$

3. Subtract. Sketch pictures to show each difference.

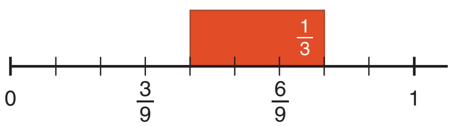
a) $\frac{3}{4} - \frac{2}{4}$ b) $\frac{4}{5} - \frac{1}{5}$ c) $\frac{2}{3} - \frac{1}{3}$ d) $\frac{5}{8} - \frac{3}{8}$

4. a) Write a rule you could use to subtract fractions with like denominators without using number lines or fraction strips.

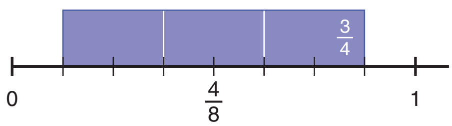
b) Write 3 subtraction questions with like denominators.
Use your rule to subtract the fractions.
Use fraction strips and number lines to check your answers.

5. Write a subtraction equation for each picture.

a)



b)



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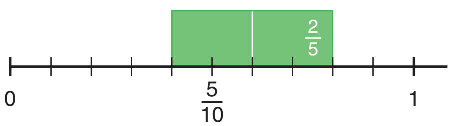
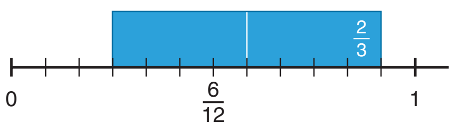


d)



6. Subtract. Sketch pictures to show each difference.

a) $\frac{3}{8} - \frac{1}{4}$ b) $\frac{7}{10} - \frac{1}{2}$ c) $\frac{7}{8} - \frac{1}{2}$ d) $\frac{5}{6} - \frac{1}{4}$

7. Sergio has the lead role in the school play.
He still has to memorize $\frac{1}{2}$ of his lines.
Suppose Sergio memorizes $\frac{1}{3}$ of his lines today.
What fraction of his lines will he have left to memorize?
Show your work.

8. Freida has $\frac{3}{4}$ of a bottle of ginger ale.
She needs $\frac{1}{2}$ of a bottle of ginger ale for her fruit punch.
How much will be left in the bottle after Freida makes the punch?

9. A cookie recipe calls for $\frac{3}{4}$ cup of chocolate chips.
Spencer has $\frac{2}{3}$ cup. Does he have enough?
If your answer is yes, explain why it is enough.
If your answer is no, how much more does Spencer need?

10. Copy and replace each $\square$ with a digit, to make each equation true.
Try to do this more than one way.

a) $\frac{2}{3} - \frac{\square}{\square} = \frac{1}{3}$ b) $\frac{\square}{\square} - \frac{1}{5} = \frac{3}{5}$ c) $\frac{\square}{3} - \frac{2}{\square} = \frac{1}{6}$

11. **Assessment Focus** Kelly had $\frac{3}{4}$ of a tank of gas at the beginning of the week.
At the end of the week, Kelly had $\frac{1}{8}$ of a tank left.

a) Did Kelly use more or less than $\frac{1}{2}$ of a tank? Explain.

b) How much more or less than $\frac{1}{2}$ of a tank did Kelly use?

Show your work.

12. a) Which of these differences is greater than $\frac{1}{2}$?
Why do you think so?

i) $\frac{5}{6} - \frac{2}{3}$ ii) $\frac{5}{6} - \frac{1}{2}$ iii) $\frac{5}{6} - \frac{1}{6}$

b) Explain how you found your answers to part a.
Which other way can you find the fractions with a difference greater than $\frac{1}{2}$? Explain another strategy.

Reflect

When you subtract fractions with unlike denominators, how do you subtract?
Give 2 different examples.
Use diagrams to show your thinking.

5.5 Using Symbols to Subtract Fractions

Focus Use common denominators to subtract fractions.

Addition and subtraction are related operations.
You can use what you know about adding fractions to subtract them.

Explore

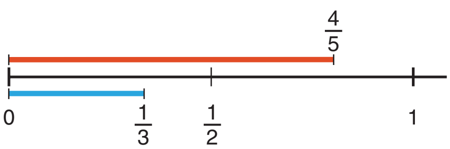
You will need fraction strips and number lines.
Find 2 fractions with a difference of $\frac{1}{2}$.
How many different pairs of fractions can you find?
Record each pair.

Reflect & Share

Discuss with your partner.
How are your strategies for subtracting fractions the same as your strategies for adding fractions? How are they different?
Work together to use common denominators to subtract two fractions.

Connect

To subtract $\frac{4}{5} - \frac{1}{3}$, estimate first.
$\frac{4}{5}$ is close to 1, and $\frac{1}{3}$ is about $\frac{1}{2}$.
So, $\frac{4}{5} - \frac{1}{3}$ is about $1 - \frac{1}{2} = \frac{1}{2}$.



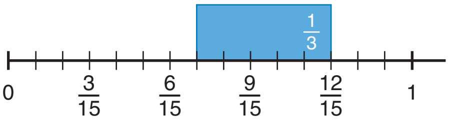
Use equivalent fractions to subtract.
Write $\frac{4}{5}$ and $\frac{1}{3}$ with a common denominator.
List the multiples of 5: 5, 10, **15**, 20, 25, …
List the multiples of 3: 3, 6, 9, 12, **15**, 18, …
15 is a multiple of 5 and 3, so 15 is a common denominator.

$\frac{4}{5} = \frac{12}{15}$ (×3 numerator and denominator) and $\frac{1}{3} = \frac{5}{15}$ (×5 numerator and denominator)

$$\frac{4}{5} - \frac{1}{3} = \frac{12}{15} - \frac{5}{15}$$
$$= \frac{7}{15}$$

Think: 12 fifteenths minus 5 fifteenths is 7 fifteenths.

We could have used a fraction strip on a number line.



Example

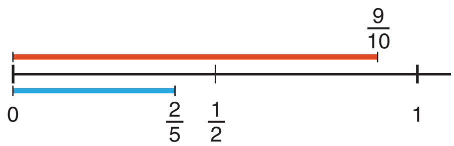
Subtract.

a) $\frac{9}{10} - \frac{2}{5}$ **b)** $\frac{5}{4} - \frac{1}{5}$

Estimate to check the answer is reasonable.

A Solution

a) $\frac{9}{10} - \frac{2}{5}$



Estimate.

$\frac{9}{10}$ is about 1. $\frac{2}{5}$ is close to $\frac{1}{2}$.

So, $\frac{9}{10} - \frac{2}{5}$ is about $1 - \frac{1}{2} = \frac{1}{2}$.

Since 10 is a multiple of 5,

use 10 as a common denominator.

$$\frac{2}{5} \overset{\times 2}{=} \frac{4}{10}$$

$$\frac{9}{10} - \frac{2}{5} = \frac{9}{10} - \frac{4}{10}$$

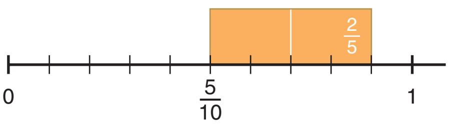
$= \frac{5}{10}$ This is not in simplest form.

$= \frac{5 \div 5}{10 \div 5}$ 5 is a factor of the numerator and denominator.

$= \frac{1}{2}$

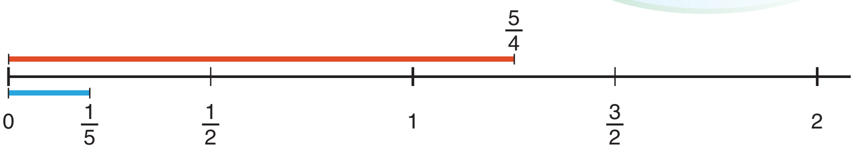
The estimate is $\frac{1}{2}$, so the difference is reasonable.

We could have used a fraction strip on a number line.



Another Strategy

b) $\frac{5}{4} - \frac{1}{5}$



Estimate.

$\frac{5}{4}$ is about 1. $\frac{1}{5}$ is close to 0.

So, $\frac{5}{4} - \frac{1}{5}$ is about $1 - 0 = 1$.

Find a common denominator.

List the multiples of 4: 4, 8, 12, 16, **20**, 24, …

List the multiples of 5: 5, 10, 15, **20**, 25, …

20 is a multiple of 4 and 5, so 20 is a common denominator.

$$\frac{5}{4} \overset{\times 5}{=} \frac{25}{20} \qquad \frac{1}{5} \overset{\times 4}{=} \frac{4}{20}$$

$$\frac{5}{4} - \frac{1}{5} = \frac{25}{20} - \frac{4}{20}$$

$$= \frac{21}{20}$$

This is an improper fraction.

$$\frac{21}{20} = \frac{20}{20} + \frac{1}{20}$$

$$= 1\frac{1}{20}$$

So, $\frac{5}{4} - \frac{1}{5} = 1\frac{1}{20}$

The estimate is 1, so the difference is reasonable.

Practice

Write all differences in simplest form.

1. Subtract.

a) $\frac{4}{5} - \frac{2}{5}$ b) $\frac{2}{3} - \frac{1}{3}$ c) $\frac{7}{9} - \frac{4}{9}$ d) $\frac{5}{7} - \frac{3}{7}$

2. Estimate, then subtract.

a) $\frac{2}{3} - \frac{1}{6}$ b) $\frac{5}{8} - \frac{1}{2}$ c) $\frac{3}{2} - \frac{7}{10}$ d) $\frac{11}{12} - \frac{5}{6}$

3. Subtract.

a) $\frac{3}{4} - \frac{2}{3}$ b) $\frac{4}{5} - \frac{2}{3}$ c) $\frac{7}{4} - \frac{4}{5}$ d) $\frac{3}{5} - \frac{1}{2}$

4. Subtract.

Estimate to check the answer is reasonable.

a) $\frac{4}{6} - \frac{1}{2}$ b) $\frac{5}{3} - \frac{3}{4}$ c) $\frac{7}{5} - \frac{5}{6}$ d) $\frac{5}{6} - \frac{3}{4}$

5. A recipe calls for $\frac{3}{4}$ cup of walnuts and $\frac{2}{3}$ cup of pecans.
Which type of nut is used more in the recipe?
How much more?

6. **Assessment Focus** On Saturday, Terri biked for $\frac{5}{6}$ h.

On Sunday, Terri increased the time she biked by $\frac{7}{12}$ h.

On Saturday, Bastien biked for $\frac{1}{2}$ h.

On Sunday, Bastien increased the time he biked by $\frac{3}{4}$ h.

a) Who biked longer on Sunday?
How can you tell?

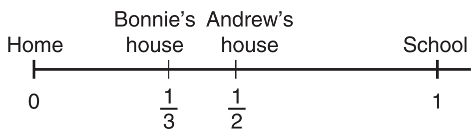
b) For how much longer did this person bike?

c) What did you need to know about fractions to answer these questions?

7. Write as many different subtraction questions as you can where the answer is $\frac{3}{4}$.
Show your work.

8. The difference of 2 fractions is $\frac{1}{2}$.
The lesser fraction is between 0 and $\frac{1}{4}$.
What do you know about the other fraction?

9. **Take It Further** Meagan walks from home to school at a constant speed.
It takes Meagan 3 min to walk the distance between Bonnie's house and Andrew's house.
How long does it take Meagan to get to school?



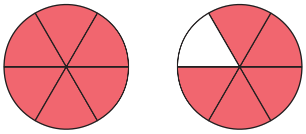
Reflect

Which fractions are easy to subtract?
Which are more difficult?
What makes them more difficult?
Give an example in each case.

5.6 Adding with Mixed Numbers

Focus Use models, pictures, and symbols to add with mixed numbers.

We have used fraction circles to model and add fractions.
We can also use fraction circles to model and add mixed numbers.
These fraction circles model $1\frac{5}{6}$.



1 whole $\frac{5}{6}$

Explore

Use any materials you want.
A recipe calls for $1\frac{1}{3}$ cups of all-purpose flour and $\frac{5}{6}$ cup of whole-wheat flour.
How much flour is needed altogether?
How can you find out?
Show your work.

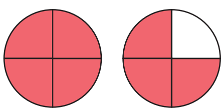
Reflect & Share

Describe your strategy.
Will your strategy work with all mixed numbers?
Test it with $2\frac{1}{3} + \frac{3}{4}$.
Use models or diagrams to justify your strategy.

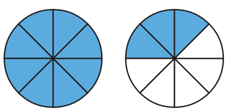
Connect

Use fraction circles to add: $1\frac{3}{4} + 1\frac{3}{8}$

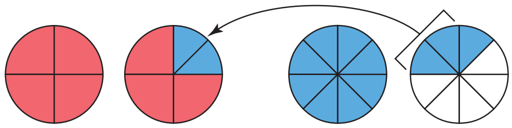
Use fraction circles to model $1\frac{3}{4}$ and $1\frac{3}{8}$.



$1\frac{3}{4}$



$1\frac{3}{8}$



Use eighths to fill the fraction circle for $\frac{3}{4}$.

1 whole and 1 whole and 1 whole and 1 eighth equals 3 wholes and 1 eighth.
So, $1\frac{3}{4} + 1\frac{3}{8} = 3\frac{1}{8}$

To add with mixed numbers, we can:

- Add the fractions and add the whole numbers separately. Or:
- Write each mixed number as an improper fraction, then add.

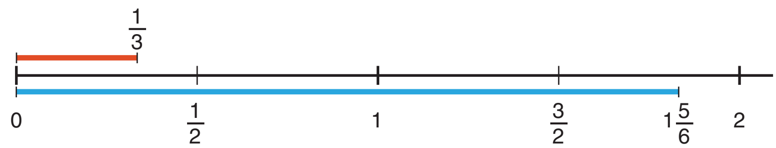
Example

Add: $\frac{1}{3} + 1\frac{5}{6}$

A Solution

$\frac{1}{3} + 1\frac{5}{6}$

Estimate:



$1\frac{5}{6}$ is close to 2.

So, $\frac{1}{3} + 1\frac{5}{6} > 2$, but less than $2\frac{1}{3}$

Add the fractions and
the whole number separately.

$\frac{1}{3} + 1\frac{5}{6} = \frac{1}{3} + \frac{5}{6} + 1$

Add the fractions: $\frac{1}{3} + \frac{5}{6}$

Since 6 is a multiple of 3,

use 6 as a common denominator.

$$\frac{1}{3} \overset{\times 2}{=} \frac{2}{6}$$

$$\frac{1}{3} + \frac{5}{6} = \frac{2}{6} + \frac{5}{6}$$
$$= \frac{7}{6}$$

Since 7 > 6, this is an improper fraction.

To write the improper fraction as a mixed number:

$$\frac{7}{6} = \frac{6}{6} + \frac{1}{6}$$
$$= 1 + \frac{1}{6}$$
$$= 1\frac{1}{6}$$

So, $\frac{1}{3} + \frac{5}{6} + 1 = 1\frac{1}{6} + 1$

$= 2\frac{1}{6}$

Then, $\frac{1}{3} + 1\frac{5}{6} = 2\frac{1}{6}$

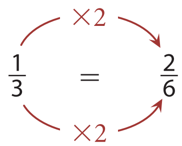
This is close to the estimate of between 2 and $2\frac{1}{3}$, so the sum is reasonable.

Another Solution

Write the mixed number as an improper fraction, then add.

$$1\frac{5}{6} = 1 + \frac{5}{6}$$
$$= \frac{6}{6} + \frac{5}{6}$$
$$= \frac{11}{6}$$

Since 6 is a multiple of 3, use 6 as a common denominator.



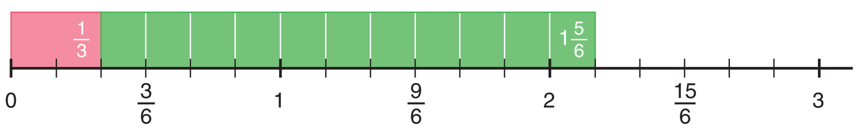
$$\frac{1}{3} + 1\frac{5}{6} = \frac{2}{6} + \frac{11}{6}$$
$$= \frac{13}{6}$$

To write the fraction as a mixed number:

$$\frac{13}{6} = \frac{12}{6} + \frac{1}{6}$$
$$= 2 + \frac{1}{6}$$
$$= 2\frac{1}{6}$$

So, $\frac{1}{3} + 1\frac{5}{6} = 2\frac{1}{6}$

We can model this with a fraction strip on a number line.



Practice

Write all sums in simplest form.

1. Write each mixed number as an improper fraction in simplest form.

a) $1\frac{3}{6}$ b) $4\frac{2}{8}$ c) $1\frac{3}{4}$ d) $3\frac{3}{5}$

2. Write each improper fraction as a mixed number in simplest form.

a) $\frac{17}{5}$ b) $\frac{9}{4}$ c) $\frac{18}{4}$ d) $\frac{28}{6}$

3. Use Pattern Blocks to find each sum.

a) $1\frac{1}{6} + \frac{2}{6}$ b) $1\frac{2}{3} + \frac{2}{3}$ c) $1\frac{4}{6} + 2\frac{1}{2}$ d) $2\frac{1}{3} + 3\frac{5}{6}$

4. Find each sum.

a) $3\frac{2}{3} + 2\frac{1}{3}$ b) $1\frac{1}{8} + 3\frac{5}{8}$ c) $4\frac{2}{9} + 3\frac{5}{9}$ d) $2\frac{3}{5} + 5\frac{4}{5}$

5. Use fraction circles to find each sum.

a) $2\frac{5}{8} + \frac{3}{4}$ b) $2\frac{5}{12} + \frac{2}{3}$ c) $1\frac{3}{8} + 3\frac{3}{4}$ d) $2\frac{2}{5} + 1\frac{7}{10}$

6. We know $\frac{1}{2} + \frac{1}{5} = \frac{7}{10}$.
Use this result to find each sum.
Estimate to check the sum is reasonable.

a) $3\frac{1}{2} + \frac{1}{5}$ b) $\frac{1}{2} + 2\frac{1}{5}$ c) $3\frac{1}{2} + 2\frac{1}{5}$ d) $4\frac{1}{2} + 3\frac{1}{5}$

7. For each pair of numbers, find a common denominator. Then add.

a) $3\frac{1}{3} + \frac{1}{4}$ b) $\frac{1}{2} + 1\frac{9}{10}$ c) $\frac{3}{4} + 2\frac{3}{5}$ d) $\frac{3}{7} + 2\frac{1}{2}$

e) $4\frac{7}{8} + 1\frac{2}{3}$ f) $2\frac{3}{5} + 2\frac{2}{3}$ g) $5\frac{2}{5} + 1\frac{7}{8}$ h) $3\frac{5}{6} + 2\frac{1}{4}$

8. Two students, Galen and Mai, worked on a project.
Galen worked for $3\frac{2}{3}$ h.
Mai worked for $2\frac{4}{5}$ h.
What was the total time spent on the project?

9. **Assessment Focus** Joseph used $1\frac{3}{8}$ cans of paint to paint his room. Juntia used $2\frac{1}{4}$ cans to paint her room.

a) Estimate how many cans of paint were used in all.
b) Calculate how many cans of paint were used.
c) Draw a diagram to model your calculations in part b.

10. A recipe for punch calls for $2\frac{2}{3}$ cups of fruit concentrate and $6\frac{3}{4}$ cups of water.
How many cups of punch will the recipe make?
Show your work.

11. Use the fractions $1\frac{3}{5}$ and $2\frac{1}{10}$.

a) Add the fractions and the whole numbers separately.
b) Write each mixed number as an improper fraction.
c) Add the improper fractions.
d) Which method was easier: adding the mixed numbers or adding the improper fractions? Why do you think so?
When would you use each method?

12. An auto mechanic completed 2 jobs before lunch.
The jobs took $2\frac{2}{3}$ h and $1\frac{3}{4}$ h.
How many hours did it take the mechanic to complete the 2 jobs?

13. **Take It Further** Replace the ☐ with an improper fraction or mixed number to make this equation true.
$3\frac{3}{5} + \square = 5$
Find as many answers as you can.
Draw diagrams to represent your thinking.

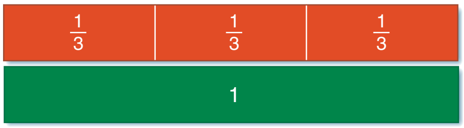
Reflect

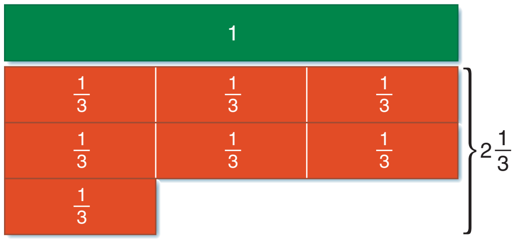
How is adding a mixed number and a fraction like adding two fractions?
How is it different?
Use examples to explain.

5.7 Subtracting with Mixed Numbers

Focus Use models, pictures, and symbols to subtract with mixed numbers.

We can use Cuisenaire rods to model fractions and mixed numbers.
Suppose the dark green rod is 1 whole, then the red rod is $\frac{1}{3}$.
So, seven red rods is $\frac{7}{3}$, or $2\frac{1}{3}$.





Explore

Use any materials you want.
A bicycle shop closed for lunch for $1\frac{2}{3}$ h on Monday and for $\frac{3}{4}$ h on Tuesday.
How much longer was the shop closed for lunch on Monday than on Tuesday?
How can you find out? Show your work.

Reflect & Share

Describe your strategy.
Will your strategy work with all mixed numbers?
Test it with $2\frac{1}{4} - \frac{3}{8}$.
Use models or diagrams to justify your strategy.

Connect

Use Cuisenaire rods to subtract: $1\frac{1}{2} - \frac{3}{4}$

Use Cuisenaire rods to model $1\frac{1}{2}$ and $\frac{3}{4}$.

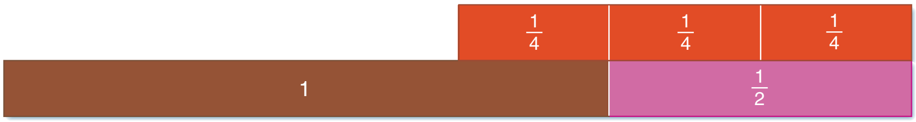
Let the brown rod represent 1 whole.

Then, the purple rod represents $\frac{1}{2}$ and the red rod represents $\frac{1}{4}$.

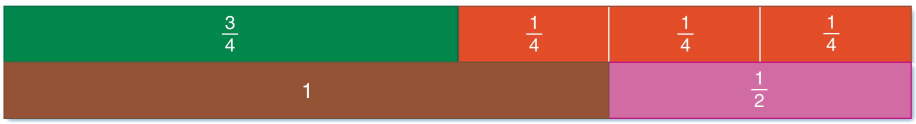
Model $1\frac{1}{2}$ with Cuisenaire rods.

Model $\frac{3}{4}$ with Cuisenaire rods.

Place the rods for $\frac{3}{4}$ above the rods for $1\frac{1}{2}$, so they align at the right.



Find a rod equal to the difference in their lengths.
The difference is equal to the dark green rod.



The dark green rod represents $\frac{3}{4}$ of the brown rod.
So, $1\frac{1}{2} - \frac{3}{4} = \frac{3}{4}$
To subtract with mixed numbers, we can:

- Subtract the fractions and subtract the whole numbers separately. Or:
- Write each mixed number as an improper fraction, then subtract.

Example

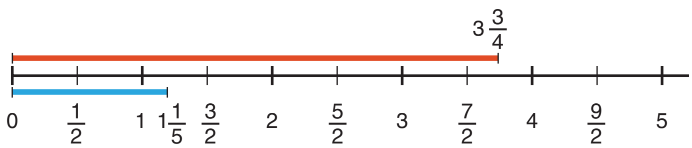
Subtract.

a) $3\frac{3}{4} - 1\frac{1}{5}$ b) $3\frac{1}{5} - \frac{3}{4}$

Estimate to check the answer is reasonable.

A Solution

a) $3\frac{3}{4} - 1\frac{1}{5}$



Estimate.

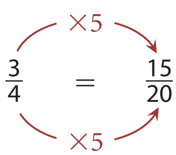
$3\frac{3}{4}$ is about 4. $1\frac{1}{5}$ is about 1.

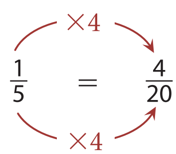
So, $3\frac{3}{4} - 1\frac{1}{5}$ is between 2 and 3.

Subtract the fractions first: $\frac{3}{4} - \frac{1}{5}$

The denominators 4 and 5 have no common factors.

So, a common denominator is: $4 \times 5 = 20$.





$$\frac{3}{4} - \frac{1}{5} = \frac{15}{20} - \frac{4}{20}$$
$$= \frac{11}{20}$$

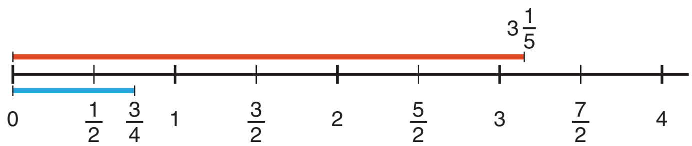
Subtract the whole numbers: $3 - 1 = 2$

Then, $3\frac{3}{4} - 1\frac{1}{5} = 2\frac{11}{20}$

This is close to the estimate of between 2 and 3,

so the answer is reasonable.

b) $3\frac{1}{5} - \frac{3}{4}$



Estimate.

$3\frac{1}{5}$ is about 3.

$\frac{3}{4}$ is close to 1.

So, $3\frac{1}{5} - \frac{3}{4}$ is about $3 - 1 = 2$.

We cannot subtract the fractions because $\frac{1}{5} < \frac{3}{4}$.

So, write $3\frac{1}{5}$ as an improper fraction.

$$3\frac{1}{5} = 3 + \frac{1}{5}$$
$$= \frac{15}{5} + \frac{1}{5}$$
$$= \frac{16}{5}$$

Another Strategy
We could use fraction circles to subtract.

The denominators have no common factors.

So, a common denominator is: $4 \times 5 = 20$

$$\frac{16}{5} \underset{\times 4}{\overset{\times 4}{=}} \frac{64}{20} \qquad \frac{3}{4} \underset{\times 5}{\overset{\times 5}{=}} \frac{15}{20}$$

$$\frac{16}{5} - \frac{3}{4} = \frac{64}{20} - \frac{15}{20}$$
$$= \frac{49}{20}$$
$$= \frac{40}{20} + \frac{9}{20}$$
$$= 2 + \frac{9}{20}$$
$$= 2\frac{9}{20}$$

So, $3\frac{1}{5} - \frac{3}{4} = 2\frac{9}{20}$

This is close to the estimate of 2, so the answer is reasonable.

Before we subtract the fraction parts of two mixed numbers,
we must check the fractions to see which is greater.
When the second fraction is greater than the first fraction,
we cannot subtract directly.

Practice

Write all differences in simplest form.

1. Subtract.

a) $2\frac{3}{5} - 1\frac{2}{5}$ b) $3\frac{7}{8} - 1\frac{5}{8}$ c) $\frac{15}{4} - \frac{3}{4}$ d) $\frac{11}{6} - \frac{1}{6}$

2. Subtract. Use Cuisenaire rods.
Sketch diagrams to record your work.

a) $1\frac{2}{3} - \frac{2}{6}$ b) $3\frac{1}{2} - 1\frac{2}{4}$ c) $3\frac{3}{10} - 2\frac{4}{5}$ d) $2\frac{1}{4} - \frac{1}{2}$

3. We know that $\frac{2}{3} - \frac{1}{2} = \frac{1}{6}$.
Use this result to find each difference.
Estimate to check the answer is reasonable.

a) $2\frac{2}{3} - \frac{1}{2}$ b) $2\frac{2}{3} - 1\frac{1}{2}$ c) $4\frac{2}{3} - 2\frac{1}{2}$ d) $5\frac{2}{3} - 1\frac{1}{2}$

4. Estimate, then subtract.

a) $\frac{7}{2} - \frac{5}{4}$ b) $\frac{13}{6} - \frac{8}{12}$ c) $\frac{5}{4} - \frac{3}{5}$ d) $\frac{9}{5} - \frac{1}{2}$

5. a) Subtract.

i) $3 - \frac{4}{5}$ ii) $4 - \frac{3}{7}$ iii) $5 - \frac{5}{6}$ iv) $6 - \frac{4}{9}$

b) Which methods did you use in part a?
Explain your choice.

6. For the fractions in each pair of numbers, find a common denominator.
Then subtract.

a) $3\frac{3}{4} - 1\frac{1}{5}$ b) $4\frac{9}{10} - 3\frac{1}{2}$ c) $3\frac{3}{4} - 1\frac{1}{3}$ d) $4\frac{5}{7} - 2\frac{2}{3}$

7. For each pair of mixed numbers below:

a) Subtract the fractions and subtract the whole numbers separately.
b) Write the mixed numbers as improper fractions, then subtract.
c) Which method was easier? Why do you think so?

i) $3\frac{3}{5} - 1\frac{3}{10}$ ii) $3\frac{3}{10} - 1\frac{3}{5}$

8. A flask contains $2\frac{1}{2}$ cups of juice.
Ping drinks $\frac{3}{8}$ cup of juice, then Preston drinks $\frac{7}{10}$ cup of juice.
How much juice is in the flask now? Show your work.

9. The running time of a movie is $2\frac{1}{6}$ h.
In the theatre, Jason looks at his watch and sees that $1\frac{1}{4}$ h has passed.
How much longer will the movie run?

10. Subtract.
a) $3\frac{2}{3} - 2\frac{7}{8}$ **b)** $5\frac{1}{2} - 3\frac{7}{9}$ **c)** $4\frac{3}{5} - 1\frac{2}{3}$ **d)** $4\frac{2}{5} - 1\frac{7}{8}$

11. **Assessment Focus** The students in two Grade 7 classes made sandwiches for parents' night.
Mr. Crowe's class used $5\frac{1}{8}$ loaves of bread.
Mme. Boudreau's class used $3\frac{2}{3}$ loaves of bread.
a) Estimate how many more loaves Mr. Crowe's class used.
b) Calculate how many more loaves Mr. Crowe's class used.
c) Draw a diagram to model your calculations in part b.
d) The two classes purchased 10 loaves.
How many loaves were left?

12. **Take It Further** Replace the □ with an improper fraction or mixed number to make this equation true.
$4\frac{1}{8} - \square = 1\frac{1}{2}$
Find as many answers as you can.
Draw diagrams to represent your thinking.

Reflect

You have learned to use improper fractions to subtract mixed numbers.
When is this not the better method? Use an example to explain.

Advertising Sales Representative

Magazines and newspapers make money by selling advertising space.

The advertising sales representative contacts companies whose products might be of interest to readers. She offers to sell them various sizes of advertisement space at different rates. When talking about ads smaller than a full page, the sales rep uses fractions to describe them. It's much simpler to talk about a $\frac{2}{3}$-page ad instead of a 0.666 667 page ad!

The sales rep tries to sell combinations of ads that can fill pages, with no space left over. A sales rep has sold two $\frac{1}{4}$-page ads and one $\frac{1}{6}$-page ad. She wants to know the possible combinations of ad sizes she can sell to fill the rest of the page. What might they be?

Writing a Complete Solution

A question often says "Show your work."
What does this mean?

When you are asked to show your work, you should show your thinking by writing a complete solution.

Work with a partner.
Compare these solutions.

Solution 1

A cookie recipe calls for $\frac{3}{8}$ cup of brown sugar and $\frac{1}{3}$ cup of white sugar.
Which type of sugar is used more in the recipe?
How much more?

Solution

Which is greater, $\frac{3}{8}$ or $\frac{1}{3}$?

Write $\frac{3}{8}$ and $\frac{1}{3}$ with a common denominator.
List the multiples of 8: 8, 16, (24), 32, ...
List the multiples of 3: 3, 6, 9, 12, 15, 18, 21, (24), 27, ...

A common denominator is 24.

$\frac{3 \times 3}{8 \times 3} = \frac{9}{24}$ $\qquad$ $\frac{1 \times 8}{3 \times 8} = \frac{8}{24}$

$\frac{9}{24} > \frac{8}{24}$

So, $\frac{3}{8} > \frac{1}{3}$

Brown sugar is used more.

Subtract to find out how much more.

$\frac{9}{24} - \frac{8}{24} = \frac{1}{24}$

The recipe calls for $\frac{1}{24}$ cup more brown sugar.

Solution 2

$\frac{3}{8} = \frac{9}{24}$ $\qquad$ $\frac{1}{3} = \frac{8}{24}$

Answer: $\frac{9}{24}$, $\frac{1}{24}$

- Which solution is complete?
- Suppose this question is on a test. It is worth 4 marks.
  How many marks would you give each solution above?
  Justify your answers.

Make a list of things that should be included in a complete solution.

Reading and Writing in Math

Tips for writing a complete solution:

- Write down the question.
- Show all steps so that someone else can follow your thinking.
- Include graphs or pictures to help explain your thinking.
- Check that your calculations are accurate.
- Use math symbols correctly.
- Write a concluding sentence that answers the question.

Name: ____________________ Date: __________

1. Which fraction is greater?
How do you know?

$\frac{3}{5}, \frac{2}{3}$

2. Add.

$\frac{3}{4} + \frac{1}{12}$

3. Subtract.

$4\frac{5}{6} - \frac{17}{18}$

4. Marty drank $\frac{4}{5}$ cup of orange juice.

Kobe drank $\frac{3}{4}$ cup of orange juice.

a) Who drank more orange juice?
b) How much more orange juice did he drink?

Unit Review

What Do I Need to Know?

Adding and Subtracting Fractions

- ✓ Use models, such as Pattern Blocks, fraction circles, fraction strips, and number lines.
- ✓ Like denominators: add or subtract the numerators.
  For example, $\frac{5}{6} + \frac{2}{6} = \frac{7}{6}$ $\quad \frac{5}{6} - \frac{2}{6} = \frac{3}{6}$, or $\frac{1}{2}$
- ✓ Unlike denominators: Use a common denominator to write equivalent fractions, then add or subtract the numerators.
  For example:

$$\frac{3}{4} + \frac{3}{5} = \frac{15}{20} + \frac{12}{20} = \frac{27}{20}, \text{ or } 1\frac{7}{20}$$

$$\frac{3}{4} - \frac{3}{5} = \frac{15}{20} - \frac{12}{20} = \frac{3}{20}$$

Adding and Subtracting with Mixed Numbers

- ✓ Use models, such as fraction circles, Pattern Blocks, and Cuisenaire rods.
- ✓ Add or subtract the fractions and the whole numbers separately.
  For example:

$$3\frac{5}{8} + 2\frac{1}{4} = 3 + 2 + \frac{5}{8} + \frac{1}{4} = 5 + \frac{5}{8} + \frac{2}{8} = 5 + \frac{7}{8} = 5\frac{7}{8}$$

$$3\frac{2}{3} - 1\frac{3}{5} = 3 - 1 + \frac{2}{3} - \frac{3}{5} = 2 + \frac{10}{15} - \frac{9}{15} = 2 + \frac{1}{15} = 2\frac{1}{15}$$

Check that $\frac{2}{3} > \frac{3}{5}$.

- ✓ Write each mixed number as an improper fraction, then add or subtract.
  For example:

$$1\frac{5}{6} + 1\frac{2}{5} = \frac{11}{6} + \frac{7}{5} = \frac{55}{30} + \frac{42}{30} = \frac{97}{30}, \text{ or } 3\frac{7}{30}$$

$$2\frac{1}{4} - 1\frac{1}{2} = \frac{9}{4} - \frac{3}{2} = \frac{9}{4} - \frac{6}{4} = \frac{3}{4}$$

Since $\frac{1}{4} < \frac{1}{2}$, use improper fractions.

What Should I Be Able to Do?

LESSON

5.1 **1.** Add.
Use fraction circles.
Draw a picture to show each sum.

a) $\frac{8}{12} + \frac{5}{12}$ b) $\frac{3}{4} + \frac{2}{8}$
c) $\frac{1}{4} + \frac{2}{3}$ d) $\frac{1}{10} + \frac{3}{5}$

5.2 **2.** Add. Use fraction strips on number lines.
Draw a picture to show each sum.

a) $\frac{5}{9} + \frac{2}{3}$ b) $\frac{2}{3} + \frac{5}{6}$
c) $\frac{1}{6} + \frac{7}{12}$ d) $\frac{3}{8} + \frac{6}{8}$

3. Find 2 fractions that add to $\frac{5}{8}$.
Find as many pairs of fractions as you can.

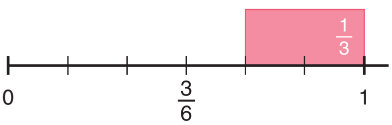
5.3 **4.** Find a common denominator for each set of fractions.
Write equivalent fractions for each pair.

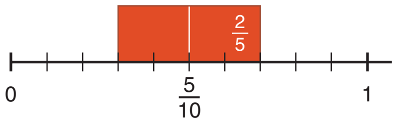
a) $\frac{3}{5}$ and $\frac{3}{4}$ b) $\frac{2}{5}$ and $\frac{3}{15}$
c) $\frac{4}{9}$ and $\frac{1}{2}$ d) $\frac{5}{8}$ and $\frac{1}{6}$

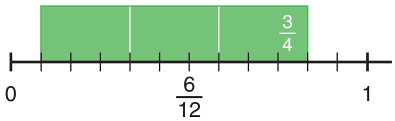
5. Add.

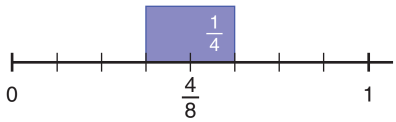
a) $\frac{1}{5} + \frac{3}{5}$ b) $\frac{1}{2} + \frac{3}{7}$
c) $\frac{2}{3} + \frac{3}{10}$ d) $\frac{3}{5} + \frac{1}{4}$

5.4 **6.** Write a subtraction equation for each picture.

a)


b)


c)


d)


7. Subtract. Draw a picture to show each difference.

a) $\frac{4}{5} - \frac{1}{5}$
b) $\frac{5}{6} - \frac{1}{3}$
c) $\frac{11}{12} - \frac{1}{2}$

8. Joyce and Javon each have the same MP3 player.
Joyce has used $\frac{7}{9}$ of her storage capacity.
Javon has used $\frac{5}{6}$ of his storage capacity.

a) Who has used more storage capacity?
b) How much more storage capacity has he or she used?

Show your work.

5.5 **9.** Subtract.

a) $\frac{9}{10} - \frac{2}{5}$ b) $\frac{7}{3} - \frac{5}{6}$
c) $\frac{8}{5} - \frac{1}{4}$ d) $\frac{9}{4} - \frac{2}{3}$

LESSON

10. Write a subtraction question that has each fraction below as the answer.
The two fractions that are subtracted should have unlike denominators.

a) $\frac{1}{2}$ b) $\frac{3}{4}$ c) $\frac{1}{10}$
d) $\frac{1}{6}$ e) $\frac{1}{4}$

11. Anton drank $\frac{3}{4}$ bottle of water.
Brad drank $\frac{7}{8}$ bottle of water.
a) Who drank more water?
b) How much more water did he drink?

12. The gas tank in Eddie's car is $\frac{5}{8}$ full. He uses $\frac{1}{4}$ tank of gas to run his errands. What fraction of a tank of gas is left?

5.6 **13.** Use fraction circles to find each sum.

a) $6\frac{1}{3} + \frac{1}{3}$ b) $1\frac{5}{12} + \frac{1}{6}$
c) $2\frac{3}{10} + 3\frac{1}{5}$ d) $5\frac{1}{4} + 1\frac{2}{5}$

14. Add.

a) $3\frac{5}{6} + \frac{4}{6}$ b) $4\frac{3}{8} + \frac{1}{4}$
c) $7\frac{3}{10} + 2\frac{4}{5}$ d) $2\frac{5}{9} + 5\frac{2}{3}$

15. Danielle mows lawns as a part-time job. On Monday, Danielle spent $1\frac{3}{4}$ h mowing lawns.
On Wednesday, she spent $1\frac{7}{8}$ h mowing lawns.
How much time did she spend mowing lawns over the 2 days?

5.7 **16.** Subtract. Draw a picture to show each difference.

a) $4\frac{1}{2} - \frac{3}{8}$ b) $3\frac{4}{9} - \frac{2}{3}$
c) $5\frac{5}{12} - 3\frac{5}{6}$ d) $4\frac{5}{8} - 2\frac{2}{3}$

17. Amelie wants to bake two kinds of muffins. One recipe calls for $1\frac{3}{4}$ cups of bananas.
The other recipe calls for $1\frac{7}{8}$ cups of cranberries.
a) Which recipe uses more fruit?
b) How much more fruit does the recipe in part a use?

18. Add or subtract as indicated.

a) $2\frac{2}{3} + 1\frac{1}{2}$ b) $3\frac{1}{3} - 1\frac{7}{10}$
c) $2\frac{1}{6} + 4\frac{7}{8}$ d) $3\frac{1}{2} - 2\frac{3}{4}$

19. On a trip from Edmonton to Saskatoon, Carly drove for $2\frac{1}{2}$ h, stopped for gas and lunch, then drove for $2\frac{2}{3}$ h.
The total trip took 6 h. How long did Carly stop for gas and lunch?
Express your answer as a fraction of an hour.

Practice Test

1. Add or subtract.
Draw a picture to show each sum or difference.
Write each sum or difference in simplest form.

a) $\frac{7}{5} + \frac{3}{5}$ **b)** $\frac{13}{10} - \frac{2}{3}$ **c)** $\frac{11}{12} - \frac{8}{12}$ **d)** $\frac{4}{9} + \frac{7}{6}$

2. Find two fractions that have a sum of $\frac{3}{5}$.

a) The fractions have like denominators.
b) The fractions have unlike denominators.

3. Find two fractions that have a difference of $\frac{1}{4}$.

a) The fractions have like denominators.
b) The fractions have unlike denominators.

4. Add or subtract.

a) $6\frac{3}{8} + 2\frac{1}{5}$ **b)** $3\frac{1}{10} - 1\frac{4}{5}$

5. Lana does yard work.
The table shows the approximate time for each job.
For one Saturday, Lana has these jobs:
– mow 3 small lawns
– mow 1 large lawn
– mow lawn/tidy yard in 2 places
– plant annuals in 1 place
Lana needs travel time between jobs,
and a break for lunch.
Do you think she will be able to do all the jobs? Justify your answer.

Job	Time
Mow small lawn	$\frac{1}{2}$ h
Mow large lawn	$\frac{3}{4}$ h
Mow lawn/tidy yard	$1\frac{1}{2}$ h
Plant annuals	$2\frac{1}{2}$ h

6. Write each fraction as the sum of two different unit fractions.

a) $\frac{3}{4}$ **b)** $\frac{5}{8}$

7. A fraction is written on each side of two counters.
All the fractions are different.
The counters are flipped and the fractions are added.
Their possible sums are: $1, 1\frac{1}{4}, \frac{7}{12}, \frac{5}{6}$
Which fractions are written on the counters?
Explain how you found the fractions.

Unit Problem Publishing a Book

The students at Anishwabe School are preparing a special book for the school's 100th anniversary.
They finance the book by selling advertising space to sponsors.
The students sold the following space:

Full page	$\frac{1}{2}$ page	$\frac{1}{3}$ page	$\frac{1}{4}$ page	$\frac{1}{6}$ page	$\frac{1}{8}$ page
1	1	1	3	4	5

All the advertisements are to fit at the back of the book.
Sam asks: "How many pages do we need for the advertisements?"
Ruth asks: "Will the advertisements fill the pages?"
Jiba asks: "Is there more than one way to arrange these advertisements?"
Can you think of other questions students might ask?

1. Find the total advertising space needed.
2. Sketch the pages to find how the advertisements can be placed. Use grid paper if it helps.

3. Compare your group's sketch with those of other groups.
When you made your sketch, what decisions did you make about the shape of each advertisement?
Did other groups make the same decisions?
If your answer is no, explain how another group made its decisions.

4. What are the fewest pages needed to display the advertisements?
Will there be room for any other advertisements?
How can you tell?

5. What else might students need to consider as they prepare the layout for the book?

To encourage students to sell advertisements, the organizing committee offered prizes to the 2 students who sold the most space.

Sandra, Roy, and Edward are the top sellers.
This table shows the advertising space each of these students sold.

	Full page	$\frac{1}{2}$ page	$\frac{1}{3}$ page	$\frac{1}{4}$ page	$\frac{1}{6}$ page	$\frac{1}{8}$ page
Sandra		1			1	2
Roy	1				1	
Edward			1	1	1	1

6. Which two students sold the most space?
Show how you know.

7. How much more space would the third-place student have to sell to receive first prize?
Second prize? Show your work.

Check List

Your work should show:

- ✓ all calculations in detail
- ✓ diagrams of the layout for the advertisements
- ✓ a clear explanation of how you prepared the layout
- ✓ a clear explanation of which students received the prizes, and how much more the third student needed to sell

Reflect on Your Learning

Look back at the goals under *What You'll Learn*.
How well do you think you have met these goals?

UNIT 6

Equations

Many digital music clubs offer albums for subscribers to download.

These tables show the plans for downloading albums for two companies. Each plan includes 5 free album downloads per month.

What patterns do you see in the tables? Write a pattern rule for each pattern. Describe each plan.

Which is the more expensive plan for 8 additional albums? Assume the patterns continue. Will this company always be more expensive? How do you know?

What You'll Learn

- Demonstrate and use the preservation of equality.
- Explain the difference between an expression and an equation.
- Use models, pictures, and symbols to solve equations and verify the solutions.
- Solve equations using algebra.
- Make decisions about which method to use to solve an equation.
- Solve problems using related equations.

Why It's Important

- Using equations is an effective problem-solving tool.
- Using algebra to solve equations plays an important role in many careers. For example, urban planners use equations to investigate population growth.

Company A

Number of Additional Albums Downloaded per Month	Total Cost ($)
0	10
1	18
2	26
3	34
4	42
5	50

Company B

Number of Additional Albums Downloaded per Month	Total Cost ($)
0	20
1	27
2	34
3	41
4	48
5	55

Key Words

- systematic trial
- inspection

6.1 Solving Equations

Focus Solve equations by inspection and by systematic trial.

Look at the algebraic expressions and equations below.
Which ones are equations? Which ones are expressions?
How do you know?

$3n + 12$ $\quad$ $3n = 12$ $\quad$ $5x + 2$ $\quad$ $5x + 2 = 27$

Explore

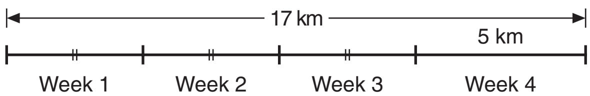
On the way home from school,
10 students got off the bus at the first stop.
There were then 16 students on the bus.
How many students were on the bus
when it left the school?
How many different ways can you solve the problem?

Reflect & Share

Discuss your strategies for finding the answer
with another pair of classmates.
Did you use an equation?
Did you use reasoning?
Did you draw a picture?
Justify your choice.

Connect

Janet walked a total of 17 km in February.
She walked the same number of kilometres
in each of the first 3 weeks.
Then she walked 5 km in the fourth week.
How many kilometres did Janet walk
in each of the first 3 weeks?



Let d represent the distance Janet walked, in kilometres, in each of the first 3 weeks.
So $3 \times d$, or $3d$, represents the total number of kilometres Janet walked in the first 3 weeks.
She walked 5 km in the fourth week, for a total of 17 km.
The equation is: $3d + 5 = 17$

When we use the equation to find the value of d, we *solve the equation*.

Writing and solving equations is a useful strategy for solving problems.

Here are 2 ways to solve this equation.

Method 1: By Systematic Trial

Systematic trial means choosing a value for the variable, then checking by substituting. Use the answer and reasoning to choose the next value to check.

$3d + 5 = 17$
We choose a value for d and substitute.
Try $d = 2$.

$$3d + 5 = 3 \times 2 + 5 \\ = 6 + 5 \\ = 11$$

11 is too small, so choose a greater value for d.

Try $d = 5$.

$$3d + 5 = 3 \times 5 + 5 \\ = 15 + 5 \\ = 20$$

20 is too large, so choose a lesser value for d.

Try $d = 4$.

$$3d + 5 = 3 \times 4 + 5 \\ = 12 + 5 \\ = 17$$

This is correct.
Janet walked 4 km during each of the first 3 weeks of February.

Method 2: By Inspection

By inspection means finding the value for the variable by using these types of number facts: addition, subtraction, multiplication, and division

$3d + 5 = 17$
We first find a number which, when added to 5, gives 17.

$$3d + 5 = 17$$

We know that $12 + 5 = 17$.
So, $3d = 12$
Then we find a number which, when multiplied by 3, has product 12.
We know that $3 \times 4 = 12$; so $d = 4$
Janet walked 4 km during each of the first 3 weeks of February.

We say that the value $d = 4$ makes the equation $3d + 5 = 17$ true.
Any other value of d, such as $d = 6$,
would not make the equation true.
The value $d = 4$ is the only solution to the equation.
That is, there is only one value of d that makes the equation true.

$3 \times 6 + 5$ does not equal 17.

Example

For each situation, write an equation.
Ben has a large collection of baseball caps.

a) Ben takes y caps from a group of 18 caps.
There are 12 caps left.
How many caps did Ben take away?
Solve the equation by inspection.

b) Ben put k caps in each of 6 piles.
There are 108 caps altogether.
How many caps did Ben put in each pile?
Solve the equation by systematic trial.

c) Ben shares n caps equally among 9 piles.
There are 6 caps in each pile.
How many caps did Ben have?
Solve the equation by inspection.

d) Ben combines p groups of 4 caps each into one large group.
He then takes away 7 caps. There are 49 caps left.
How many groups of 4 caps did Ben begin with?
Solve the equation by systematic trial.

A Solution

a) 18 subtract y equals 12.
$18 - y = 12$
Which number subtracted from 18 gives 12?
We know that $18 - 6 = 12$; so $y = 6$.
Ben took away 6 caps.

b) 6 times k equals 108.
$6k = 108$
Try $k = 15$. $\quad 6k = 6(15)$
$\quad = 90$
90 is too small, so choose a greater value for k.

Recall: $6(15) = 6 \times 15$

Try $k = 20$. $6k = 6(20)$
$= 120$

120 is too large, so choose a lesser value for k.

Try $k = 17$. $6k = 6(17)$
$= 102$

102 is too small, but it is close to the value we want.

Try $k = 18$. $6k = 6(18)$
$= 108$

This is correct.

Ben put 18 caps in each pile.

c) n divided by 9 equals 6.

$n \div 9 = 6$, or $\frac{n}{9} = 6$

Which number divided by 9 gives 6?

We know that $54 \div 9 = 6$; so $n = 54$.

Ben had 54 caps.

d) 4 times p subtract 7 equals 49.

$4p - 7 = 49$

Since $4 \times 10 = 40$, we know we need to start with a value for p greater than 10.

Try $p = 12$. $4p - 7 = 4 \times 12 - 7$
$= 48 - 7$
$= 41$, which is too small

41 is 8 less than 49, so we need two more groups of 4.

Try $p = 14$. $4p - 7 = 4 \times 14 - 7$
$= 56 - 7$
$= 49$

This is correct.

Ben began with 14 groups of 4 caps each.

Practice

1. Look at the algebraic expressions and equations below.
Which are expressions? Equations?
How do you know?

a) $4w = 48$ **b)** $g - 11$ **c)** $3d + 5$

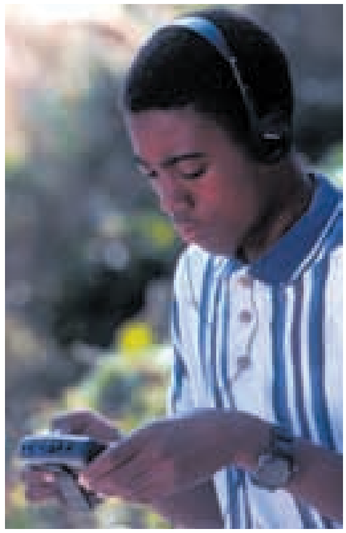
d) $\frac{x}{12} = 8$ **e)** $\frac{j - 5}{10}$ **f)** $6z + 1 = 67$

2. Solve each equation in question 1 by inspection or by systematic trial.
Explain why you chose the method you did.

3. Shenker gives 10 CDs to his brother.
Shenker then has 35 CDs.

a) Write an equation you can solve to find how many CDs Shenker had to begin with.

b) Solve the equation.



4. Write an equation for each sentence.
Solve each equation by inspection.

a) Seven more than a number is 18.

b) Six less than a number is 24.

c) Five times a number is 45.

d) A number divided by six is 7.

e) Three more than four times a number is 19.

5. Write an equation you could use to solve each problem.
Solve each equation by systematic trial.

a) Aiko bought 14 DVDs for $182.
She paid the same amount for each DVD.
How much did each DVD cost?

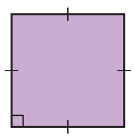
b) Kihew collects beaded leather bracelets. She lost 14 of her bracelets.
Kihew has 53 bracelets left.
How many bracelets did she have to begin with?

c) Manuel gets prize points for reading books.
He needs 100 points to win a set of tangrams.
Manuel has 56 points. When he reads 11 more books, he will have 100 points.
How many points does Manuel get for each book he reads?

6. The perimeter of a square is 48 cm.

a) Write an equation you can solve to find the side length of the square.

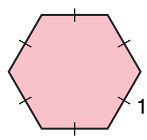
b) Solve the equation.



7. The side length of a regular hexagon is 11 cm.

a) Write an equation you can solve to find the perimeter of the hexagon.

b) Solve the equation.



8. Use questions 6 and 7 as a guide.

a) Write your own problem about side length and perimeter of a figure.

b) Write an equation you can use to solve the problem.

c) Solve the equation.

9. **Assessment Focus** Eli has 130 key chains. He keeps 10 key chains for himself, then shares the rest equally among his friends.
Each friend then has 24 key chains.

a) Write an equation you can solve to find how many friends were given key chains.

b) Solve the equation by inspection, then by systematic trial.
Which method was easier to use? Explain your choice.

10. Find the value of n that makes each equation true.

a) $3n = 27$ b) $2n + 3 = 27$ c) $2n - 3 = 27$ d) $\frac{n}{3} = 27$

11. **Take It Further** Write a problem that can be described by each equation.
Solve each equation. Which equation was the most difficult to solve?
Why do you think so?

a) $2x - 1 = 5$ b) $4y = 24$

c) $\frac{z}{38} = 57$ d) $5x + 5 = 30$

Math Link

Dr. Edward Doolittle, a Mohawk Indian, was the first Indigenous person in Canada to obtain a PhD in Mathematics. Dr. Doolittle has taught at the First Nations University in Saskatchewan, and he is currently an Assistant Professor of Mathematics at the University of Regina. One of Dr. Doolittle's goals is to show his students how much fun mathematics can be. In addition to his academic interests, Dr. Doolittle also writes and performs comedy sketches for radio.

Reflect

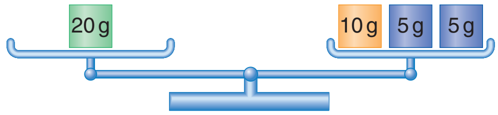
How do you decide whether to solve an equation by inspection or by systematic trial?
How might using a calculator affect your decision?
Give examples to illustrate your thinking.

6.2 Using a Model to Solve Equations

Focus Solve equations by using a balance-scales model and verifying the solution.

Sometimes, systematic trial and inspection are not the best ways to solve an equation.

Balance scales can be used to *model* an equation. When pans are balanced, the mass in one pan is equal to the mass in the other pan.



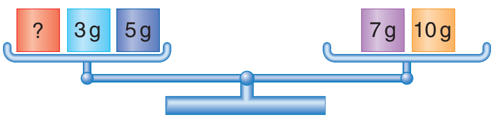
We can write an equation to describe the masses in grams.
$20 = 10 + 5 + 5$

Explore

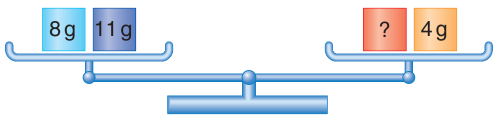
Use balance scales if they are available.
Otherwise, draw diagrams.

Here are some balance scales.
Some masses are known. Other masses are unknown.

Balance A



Balance B



➤ The pans are balanced.
For each balance scales:
- Write an equation to represent the masses.
- Find the value of the unknown mass.

➤ Make up your own balance-scales problem.
Make sure the pans are balanced and one mass is unknown.
Solve your problem.

Reflect & Share

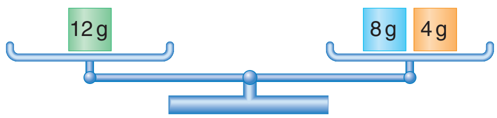
Trade problems with another pair of classmates.
Compare strategies for finding the value of the unknown mass.

Connect

We can use a balance-scales model to solve an equation.

When the two pans of scales are balanced,
we can adjust each pan of the scales in the same way,
and the two pans will still be balanced.

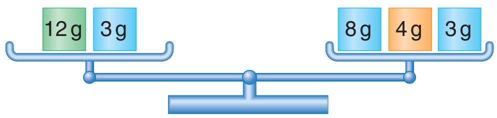
➤ Consider these balance scales:



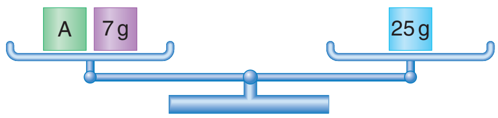
If we add 3 g to each pan,
the masses still balance.

$12\text{ g} + \mathbf{3\text{ g}} = 8\text{ g} + 4\text{ g} + \mathbf{3\text{ g}}$

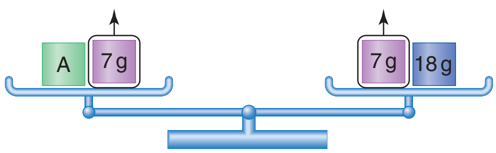
$15\text{ g} = 15\text{ g}$



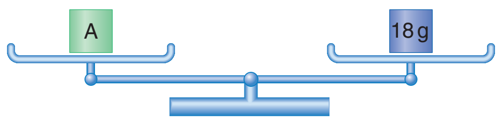
➤ Here is a balance-scales problem.
Mass A is an unknown mass.



If we remove 7 g from the left pan,
then Mass A is alone in that pan.
To keep the pans balanced,
we need to remove 7 g from the right pan too.
We replace 25 g with 7 g and 18 g;
then remove 7 g.



We are left with Mass A in the left pan
balancing 18 g in the right pan.
So, Mass A is 18 g.



We can *verify* the solution to this problem.
Replace Mass A with 18 g.
Then, in the left pan: $18\text{ g} + 7\text{ g} = 25\text{ g}$
And, in the right pan: 25 g
Since the masses are equal, the solution is correct.

To verify means to check the solution is correct.

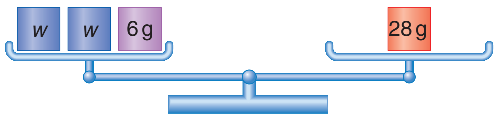
Example

A hockey team gets 2 points for a win,
1 point for a tie, and 0 points for a loss.
The Midland Tornadoes ended the season
with 28 points. They tied 6 games.
How many games did they win?
Write an equation you can use to solve the problem.
Use a model to solve the equation, then verify the solution.

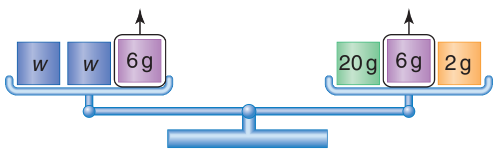
A Solution

Let w represent the number of games won by the Midland Tornadoes.
So, $2w$ represents the number of points earned from wins.
The team has 6 points from ties.
It has 28 points altogether.
So, the equation is: $2w + 6 = 28$

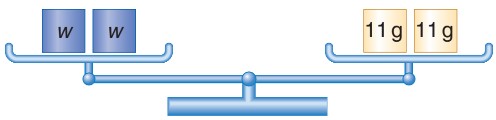
Use balance scales to model the equation.



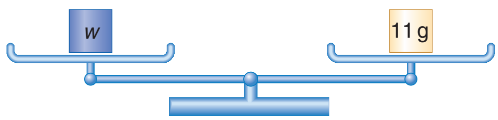
To get w on its own in one pan,
6 g has to be removed from the left pan.
So, select masses in the right pan
so that 6 g can be removed.
One way is to replace 28 g with 20 g + 6 g + 2 g.
Then, remove 6 g from each pan.



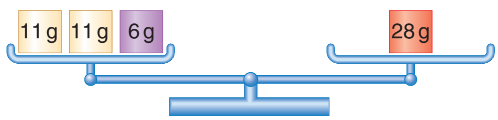
Two identical unknown masses
are left in the left pan.
20 g + 2 g = 22 g are left in the right pan.
Replace 22 g with two 11-g masses.



The two unknown masses balance
with two 11-g masses.
So, each unknown mass is 11 g.



The Midland Tornadoes won 11 games.
Verify the solution by replacing
each unknown mass with 11 g.
$11 + 11 + 6 = 28$, so the solution is correct.



The examples on pages 227 and 228 show these ways in which we preserve balance and equality:

- We can *add* the same mass to each side.
- We can *subtract* the same mass from each side.
- We can *divide* each side into the same number of equal groups.

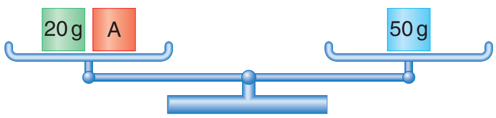
Later, we will show that:

- We can *multiply* each side by the same number by placing equal groups on each side that match the group already there.

Practice

1. Find the value of the unknown mass on each balance scales. Sketch the steps you used.

a)



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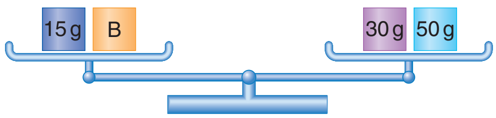

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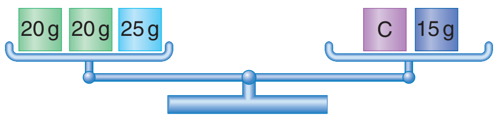

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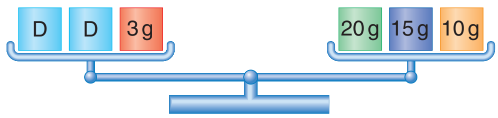


2. **a)** Sketch balance scales to represent each equation.

b) Solve each equation. Verify the solution.

i) $x + 12 = 19$	**ii)** $x + 5 = 19$
iii) $4y = 12$	**iv)** $3m = 21$
v) $3k + 7 = 31$	**vi)** $2p + 12 = 54$

3. **a)** Write an equation for each sentence.

b) Solve each equation. Verify the solution.

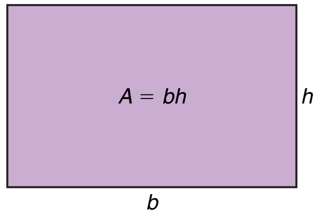
i) Five more than a number is 24.

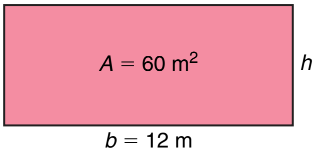
ii) Eight more than a number is 32.

iii) Three times a number is 42.

iv) Five more than two times a number is 37.

4. The area of a rectangle is $A = bh$,
where b is the base of the rectangle and h is its height.
Use this formula for each rectangle below.
Substitute for A and b to get an equation.
Solve the equation for h to find the height.
Show the steps you used to get the answers.



a)


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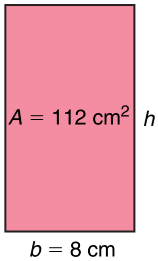

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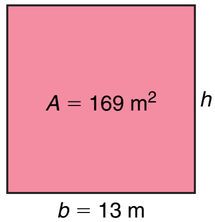


5. **Assessment Focus** Suppose the masses for balance scales are only available in multiples of 5 g.
   a) Sketch balance scales to represent this equation:
   $x + 35 = 60$
   b) Solve the equation. Verify the solution.
   Show your work.

6. **Take It Further**
   a) Write a problem that can be solved using this equation: $x + 4 = 16$
   b) How would your problem change if the equation were $x - 4 = 16$?
   c) Solve the equations in parts a and b.
   Show your steps.

7. **Take It Further** Write an equation you could use to solve this problem.
Replace the □ in the number 5□7 with a digit to make the number divisible by 9.

Reflect

Do you think you can always solve an equation using balance scales?
Justify your answer. Include an example.

6.3 Solving Equations Involving Integers

Focus Use algebra tiles and inspection to solve equations involving integers.

Recall that 1 red unit tile and 1 yellow unit tile combine to model 0.
These two unit tiles form a zero pair.


−1 +1

The yellow variable tile represents a variable, such as *x*.


x

Explore

You will need algebra tiles.
Tyler had some gumdrops and jellybeans.
He traded 5 gumdrops for 5 jellybeans.
Tyler then had 9 gumdrops and 9 jellybeans.
How many gumdrops did he have to begin with?

Let *g* represent the number of gumdrops Tyler began with.
Write an equation you can use to solve for *g*.
Use tiles to represent the equation.
Use the tiles to solve the equation. Sketch the tiles you used.

Reflect & Share

Compare your equation with that of another pair of classmates.
Share your strategies for solving the equation using tiles.
How did you use zero pairs in your solutions?
Work together to find how many jellybeans Tyler began with.
Discuss your strategies for finding out.

Connect

Michaela has a collection of old pennies.
She sells 3 pennies to another collector.
Michaela then has 10 pennies left.
How many pennies did she have before she made the sale?

Let p represent the number of pennies Michaela had before she made the sale.
The equation is: $p - 3 = 10$
One way to solve this equation is to use tiles.

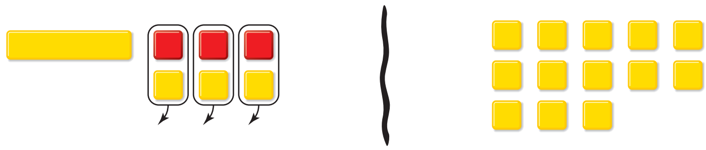
Draw a vertical line in the centre of the page.
It represents the equal sign in the equation.
We arrange tiles on each side of the line to represent an equation.
Recall that subtracting 3 is equivalent to adding -3.
So, we represent subtract 3 with 3 red unit tiles.

On the left side, put algebra tiles to represent $p - 3$.

On the right side, put algebra tiles to represent 10.

To isolate the variable tile, add 3 yellow unit tiles to make zero pairs.
Remove zero pairs.

Add 3 yellow unit tiles to this side, too, to preserve the equality.



The tiles show the solution is $p = 13$.
Michaela had 13 old pennies before she made the sale.

Recall from Unit 1 that we can verify the solution by replacing p with 13 yellow unit tiles.
Then:

becomes

Since there are now 10 yellow unit tiles on each side, the solution is correct.

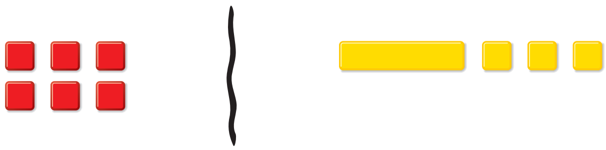
Example

At 10 a.m., it was cold outside.
By 2 p.m., the temperature had risen 3°C to −6°C.
What was the temperature at 10 a.m.?

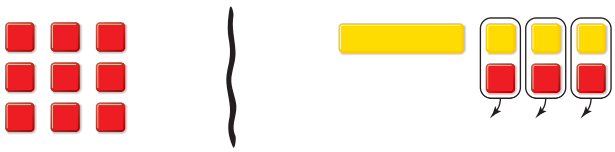
A Solution

Let t represent the temperature, in degrees Celsius, at 10 a.m.
After an increase of 3°C, the temperature was −6°C.
The equation is: $-6 = t + 3$

The variable in an equation can be on the left side or the right side.



Add 3 red unit tiles to each side. Remove zero pairs.



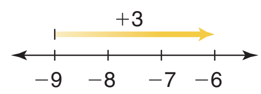
9 red unit tiles equals one variable tile.

We can verify the solution by replacing one yellow variable tile with 9 red unit tiles in the original equation.

The solution is $t = -9$.
At 10 a.m., the temperature was −9°C.

Another Solution

We can also solve equations involving integers by inspection.
To solve $-6 = t + 3$ by inspection:
We find a number which, when 3 is added to it, gives -6.
Think of moving 3 units to the right on a number line.
To arrive at -6, we would have to start at -9.
So $t = -9$.



Practice

1. Use tiles to solve each equation.
Sketch the tiles you used.

a) $x + 4 = 8$ **b)** $3 + x = 10$ **c)** $12 = x + 2$
d) $x - 4 = 8$ **e)** $10 = x - 3$ **f)** $12 = x - 2$

2. Solve by inspection. Show your work.

a) $9 = n - 4$ **b)** $x + 6 = 8$ **c)** $2 = p - 5$
d) $x - 4 = -9$ **e)** $-8 = s + 6$ **f)** $x - 5 = -2$

3. Four less than a number is 13.
Let x represent the number.
Then, an equation is: $x - 4 = 13$
Solve the equation. What is the number?

4. Jody had some friends over to watch movies.
Six of her friends left after the first movie.
Five friends stayed to watch a second movie.
Write an equation you can use to find how many of Jody's friends watched the first movie.
Solve the equation. Verify the solution.

5. Overnight, the temperature dropped 8°C to −3°C.

a) Write an equation you can solve to find the original temperature.
b) Use tiles to solve the equation.
Sketch the tiles you used.

6. **Assessment Focus** Solve each equation using tiles, and by inspection. Verify each solution. Show your work.

a) $x + 6 = 13$ b) $n - 6 = 13$

7. At the Jungle Safari mini-golf course, par on each hole is 5. A score of −1 means a player took 4 strokes to reach the hole. A score of +2 means a player took 7 strokes to reach the hole. Write an equation you can use to solve each problem below. Solve the equation. Show your work.

Par is the number of strokes a good golfer should take to reach the hole.

a) On the seventh hole, Andy scored +2. His overall score was then +4. What was Andy's overall score after six holes?

b) On the thirteenth hole, Bethany scored −2. Her overall score was then +1. What was Bethany's overall score after twelve holes?

c) On the eighteenth hole, Koora reached the hole in one stroke. His overall score was then −2. What was Koora's overall score after seventeen holes?

8. **Take It Further** Consider equations of the form $x + a = b$, where a and b are integers. Make up a problem that can be solved by an equation of this form in which:

a) Both a and b are positive.

b) Both a and b are negative.

c) a is positive and b is negative.

d) a is negative and b is positive.

Solve each equation.
Explain the method you used each time.

Reflect

How did your knowledge of adding and subtracting integers help you in this lesson?

Mid-Unit Review

LESSON

6.1

1. Jaclyn went on a 4-day hiking trip.
   a) For each problem, write an equation you can solve by inspection.
      i) Jaclyn hiked 5 km the first day. After two days she had hiked 12 km. How far did she hike on the second day?
      ii) Jaclyn hiked a total of 12 km on the third and fourth days. She hiked the same distance each day.
      How far did she hike on each of these two days?
   b) For each problem, write an equation you can solve by systematic trial.
      i) Jaclyn counted squirrels. During the first three days, she counted a total of 67 squirrels. After four days, her total was 92 squirrels.
      How many squirrels did she see on the fourth day?
      ii) Jaclyn drank the same volume of water on each of the first three days. On the fourth day, she drank 8 cups of water. She drank 29 cups of water over the four days.
      How many cups of water did she drink on each of the first three days?

6.2

2. a) Write an equation for each sentence.
   b) Sketch balance scales to represent each equation.
   c) Solve each equation.
      Verify the solution.
      i) Nine more than a number is 17.
      ii) Three times a number is 21.
      iii) Seven more than two times a number is 19.

3. Andre's age is 14 more than twice Bill's age. Andre is 40 years old. How old is Bill? Write an equation you can use to solve this problem. Solve the equation using a balance-scales model.

6.3

4. a) Write an equation you can use to solve each problem.
   b) Use tiles to solve each equation. Sketch the tiles you used. Verify each solution.
   c) Solve each equation by inspection.
      i) Eight years ago, Susanna was 7 years old.
      How old is she now?
      ii) The temperature dropped 6°C to −4°C. What was the original temperature?
      iii) Hannah borrowed money. She paid back $7. Hannah still owes $5. How much money did she borrow?

6.4 Solving Equations Using Algebra

Focus Solve a problem by solving an equation algebraically.

Explore

Solve this problem.
My mother's age is 4 more than 2 times my brother's age.
My mother is 46 years old. How old is my brother?

Reflect & Share

Discuss the strategies you used for finding the brother's age with those of another pair of classmates.
Did you draw a picture? Did you use tiles? Did you use an equation?
If you did not use an equation, how could you represent this problem with an equation?

Connect

Solving an equation *using algebra* is often the quickest way to find a solution, especially if the equation involves large numbers.

Recall that when we solve an equation, we find the value of the variable that makes the equation true.
That is, we find the value of the variable which, when substituted into the equation, makes the left side of the equation equal to the right side.

When we solve an equation using algebra, remember the balance-scales model.
To preserve the equality, always perform the same operation on both sides of the equation.

Whatever you do to one side of an equation, you do to the other side too.

Example

Three more than two times a number is 27. What is the number?

a) Write an equation to represent this problem.
b) Solve the equation. Show the steps.
c) Verify the solution.

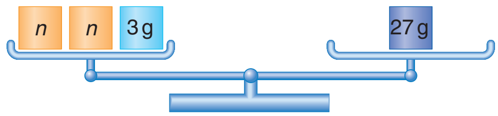
A Solution

a) Let n represent the number.

Then two times the number is: $2n$

And, three more than two times the number is: $2n + 3$

The equation is: $2n + 3 = 27$

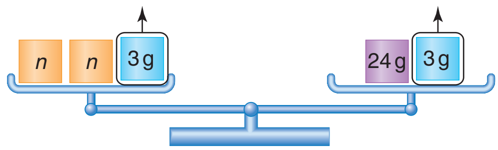


b) $2n + 3 = 27$

To isolate $2n$, subtract 3 from each side.

$2n + 3 - 3 = 27 - 3$

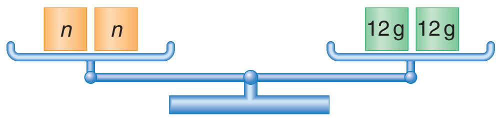
$2n = 24$



Divide each side by 2.

$\frac{2n}{2} = \frac{24}{2}$

$n = 12$



c) To verify the solution, substitute $n = 12$ into $2n + 3 = 27$.

Left side $= 2n + 3$ Right side $= 27$

$= 2(12) + 3$

$= 24 + 3$

$= 27$

Since the left side equals the right side, $n = 12$ is correct.

The number is 12.

We could also verify by following the instructions in the *Example*.
Start with 12.
Multiply by 2: 24
Add 3: 27

Practice

Solve each equation using algebra.

1. Solve each equation. Verify the solution.

a) $x - 27 = 35$ **b)** $11x = 132$ **c)** $4x + 7 = 75$

2. Write, then solve, an equation to find each number. Verify the solution.

a) Nineteen more than a number is 42.

b) Ten more than three times a number is 25.

c) Fifteen more than four times a number is 63.

3. Five years after Jari's age now doubles, he will be 27. How old is Jari now?

a) Write an equation you can use to solve the problem.

b) Solve the equation. Show the steps. How old is Jari?

c) Verify the solution.

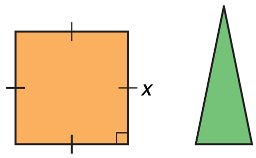
4. Jenny baby-sat on Saturday for $6/h. She was given a $3 bonus.
How many hours did Jenny baby-sit if she was paid $33?

a) Write an equation you can use to solve the problem.
b) Solve the equation. How many hours did Jenny baby-sit?
c) Verify the solution.

5. In x weeks and 4 days, the movie *Math-Man IV* will be released.
The movie will be released in 25 days. Find the value of x.

a) Write an equation you can use to solve the problem.
b) Solve the equation. Verify the solution.

6. Look at the square and triangle on the right.
The sum of their perimeters is 56 cm.
The perimeter of the triangle is 24 cm.
What is the side length of the square?



a) Write an equation you can use to find the side length of the square.
b) Solve the equation. Verify the solution.

7. **Assessment Focus** Sunita has $72 in her savings account.
Each week she saves $24. When will Sunita have a total savings of $288?

a) Write an equation you can use to solve the problem.
b) Solve the equation. Show the steps.
When will Sunita have $288 in her savings account?
c) How can you check the answer?

8. **Take It Further** Use the information on the sign to the right.

a) Write a problem that can be solved using an equation.
b) Write the equation, then solve the problem.
c) Show how you could solve the problem without writing an equation.

9. **Take It Further** The nth term of a number pattern is $9n + 1$.
What is the term number for each term value?

a) 154 b) 118 c) 244

Reflect

What advice would you give someone who is having difficulty solving equations using algebra?

6.5 Using Different Methods to Solve Equations

Focus Decide which method to use to solve an equation.

Recall the methods you have used to solve an equation:

- using algebra tiles
- by inspection
- by systematic trial
- using a balance-scales model
- using algebra

Explore

Lila, Meeka, and Noel are playing darts.
Each player throws 3 darts at the board.
A player's score is the sum of the numbers in the areas the darts land.
This picture shows a score of:
$8 + 20 + 2 = 30$

Write an equation for each problem.
Solve the equation using a method of your choice.

➤ Lila's first two darts scored a total of 12 points.
Lila scored 20 points in the round.
How many points did she score with her third dart?

➤ All three of Meeka's darts landed in the same area.
She scored 63 points. In which area did all her darts land?

➤ Noel's first two darts landed in the same area.
Her third dart was a bull's-eye, scoring 50 points.
She scored a total of 72 points.
In which area did her first two darts land?

Reflect & Share

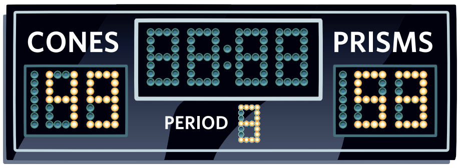
Compare your equations with those of another pair of classmates.
Explain why you chose the method you did to solve them.
Use a different method to solve one of the equations.
Did this method work better for you? Why do you think so?

Connect

We can use any method to solve an equation, as long as the steps we take make sense, and the correct solution is found.

You can always check if the solution is correct by substituting the solution into the original equation.

Example



In a basketball game between the Central City Cones and the Park Town Prisms, the lead changed sides many times.
Write, then solve, an equation to solve each problem.

a) Early in the game, the Cones had one-half as many points as the Prisms.
The Cones had 8 points. How many points did the Prisms have?

b) Near the end of the first half, the Cones were 12 points ahead of the Prisms.
The Prisms had 39 points. How many points did the Cones have?

c) The Prisms scored 32 points in the fourth quarter.
Twenty of these points were scored by foul shots and field goals.
The rest of the points were scored by 3-point shots.
How many 3-point shots did the Prisms make in the fourth quarter?

A Solution

a) Let p represent the number of points the Prisms had.
The Cones had one-half as many: $\frac{p}{2}$
The Cones had 8 points.
The equation is: $\frac{p}{2} = 8$
Solve using algebra.
Multiply each side by 2.

$$\frac{p}{2} \times 2 = 8 \times 2$$
$$\frac{2p}{2} = 16$$
$$p = 16$$

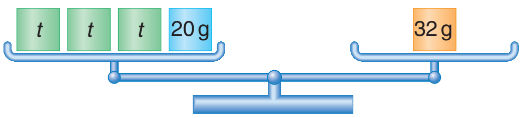
The Prisms had 16 points.

Another Strategy
We could use inspection to solve this equation.

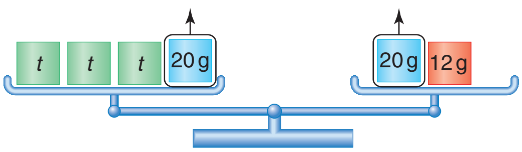
b) Let d represent the number of points the Cones had.
The Prisms had 12 fewer points: $d - 12$
An equation is: $d - 12 = 39$
Solve using systematic trial. We choose a value for d and substitute.
Try $d = 50$. $\quad d - 12 = 50 - 12$
$\qquad\qquad\qquad\qquad = 38$
38 is close. Choose a greater value for d.
Try $d = 51$. $\quad d - 12 = 51 - 12$
$\qquad\qquad\qquad\qquad = 39$
This is correct. The Cones had 51 points.

Another Strategy
We could use algebra to solve this equation.

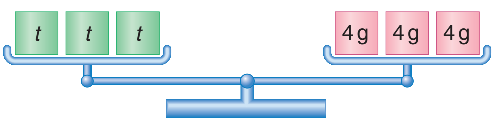
c) Let t represent the number of 3-point shots the Prisms made in the fourth quarter.
So, $3t$ represents the number of points scored by 3-point shots.
The equation is: $3t + 20 = 32$
Use a balance-scales model.



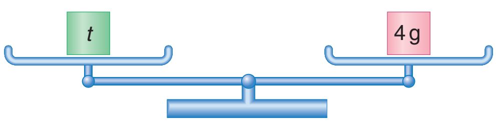
To isolate t, 20 g has to be removed from the left pan.
So, replace 32 g in the right pan with masses of 20 g and 12 g,
since 20 g + 12 g = 32 g. Then, remove 20 g from each pan.



Three identical unknown masses remain in the left pan.
12 g remain in the right pan. Replace 12 g with three 4-g masses.



The three unknown masses balance with three 4-g masses.
So, each unknown mass is 4 g.



The Prisms made four 3-point shots in the fourth quarter.

In *Example,* part c, we can solve the equation using algebra to check.

$$3t + 20 = 32$$

$$3t + 20 - 20 = 32 - 20$$ To isolate $3t$, subtract 20 from each side.

$$3t = 12$$

$$\frac{3t}{3} = \frac{12}{3}$$ To isolate t, divide each side by 3.

$$t = 4$$

The algebraic solution and the balance-scales solution are the same.
It was much quicker to solve the equation using algebra.

Practice

Use algebra, systematic trial, inspection, algebra tiles, or a balance-scales model to solve each equation.

1. Use algebra to solve each equation. Verify each solution.

a) $\frac{x}{2} = 4$ **b)** $\frac{x}{3} = 7$ **c)** $\frac{x}{4} = 16$ **d)** $\frac{x}{5} = 10$

2. Which method would you choose to solve each equation?
Explain your choice.
Solve each equation using the method of your choice.

a) $x + 5 = 12$ **b)** $x - 5 = 12$ **c)** $\frac{x}{6} = 9$ **d)** $x + 4 = -9$
e) $4x = 36$ **f)** $16x = 112$ **g)** $4x + 2 = 30$ **h)** $8x + 17 = 105$

3. George and Mary collect friendship beads.
George gave Mary 7 beads.
Mary then had 21 beads.
How many friendship beads did Mary have to start with?

a) Write, then solve, an equation you can use to solve this problem.
b) Verify the solution.

4. Jerome baked some cookies.
He shared them among his eight friends.
Each friend had 4 cookies.
Write, then solve, an equation to find how many cookies Jerome baked.

5. Which method do you prefer to use to solve an equation?
Explain. Give an example.

6. **Assessment Focus** Carla has 20 songs downloaded to her MP3 player.
Each month she downloads 8 additional songs.
After how many months will Carla have a total of 92 songs?
  a) Use an equation to solve the problem.
  b) Which method did you choose to solve the equation?
     Explain why you chose this method.

7. Write, then solve, an equation to answer each question.
Verify the solution.
Sheng sorted 37 cans.
  a) He divided the cans into 4 equal groups.
     He had 5 cans left over.
     How many cans were in each group?
  b) He divided the cans into 9 equal groups.
     He had 10 cans left over.
     How many cans were in each group?

8. Write, then solve, an equation to answer each question.
Verify the solution.
At Pascal's Pet Store, a 5-kg bag of dog food costs $10.
The 10-kg bag costs $15.
  a) Pascal sold $85 worth of dog food.
     He sold four 5-kg bags. How many 10-kg bags did he sell?
  b) Pascal sold $140 worth of dog food.
     He sold six 10-kg bags. How many 5-kg bags did he sell?

9. **Take It Further** Refer to the dart problem in *Explore*, page 240.
  a) Write two more problems using the given information.
     For each problem, write an equation you can use
     to solve your problem. Solve the equation.
     Use a different method for each equation.
  b) How could a player score 35 points with 3 darts?
     Find as many different ways as you can.

Reflect

Talk to a partner.
Tell how you choose the method you use to solve an equation.

Equation Baseball

Each game card is marked with a circled number to indicate how many bases you move for a correct answer. Each time a player crosses home plate on the way around the board, one run is awarded.

YOU WILL NEED

A set of Equation Baseball cards; one game board; a die; different coloured game pieces; algebra tiles; paper; pencils

NUMBER OF PLAYERS

4

GOAL OF THE GAME

To get the most runs

HOW TO PLAY THE GAME:

1. Shuffle the equation cards.
   Place them face down in the middle of the game board.
   Each player places a game piece on home plate.

2. Each player rolls the die. The player with the greatest number goes first. Play moves in a clockwise direction.

3. The first player turns over the top card for everyone to see.
   She solves the equation using a method of her choice.
   The other players check the answer.
   If the answer is correct, she moves the number of bases indicated by the circled number on the card and places the card in the discard pile. If the answer is incorrect, the card is placed in the discard pile.

4. The next player has a turn.

5. The player with the most runs when all cards have been used wins.

Decoding Word Problems

A word problem is a math question that has a story. Word problems often put math into real-world contexts.

The ability to read and understand word problems helps you connect math to the real world and solve more complicated problems.

Work with a partner. Compare these two questions.

Question 1	Question 2
Taylor has three more apples than Judy. Judy has six apples. How many apples does Taylor have?	$6 + 3$

List three reasons why the first question is more difficult than the second question.

A word problem can be challenging because it may not be obvious which math operations are needed ($+$, $-$, $\times$, $\div$) to solve it.

Reading and Writing in Math

Work with a partner.
Solve each word problem below.

1. Julio has 36 photos of his favourite singing star.
He wants to arrange the photos in groups that have equal numbers of rows and columns.
How many different arrangements can Julio make?
Show your work.

2. A rectangular garden is 100 m long and 44 m wide.
A fence encloses the garden.
The fence posts are 2 m apart.
Predict how many posts are needed.

3. A digital clock shows this time.
Seven minutes past 7 is a palindromic time.
List all the palindromic times between noon and midnight.

What's the Question?

The *key words* in a word problem are the words that tell you what to do.

Here are some common key words:

solve	explain
describe	predict
estimate	simplify
show your work	graph
find	list
compare	

Work with a partner. Discuss each question:
- What does each key word ask you to do?
- Which key words require an exact answer?
- Which key words tell you to show your thinking?

Unit Review

What Do I Need to Know?

✓ We can solve equations:

- by inspection
- by systematic trial
- using a balance-scales model
- using algebra tiles
- using algebra

✓ To keep the balance of an equation, what you do to one side you must also do to the other side. This is called preserving equality.

What Should I Be Able to Do?

LESSON

6.1

1. Jan collects foreign stamps.
Her friend gives her 8 stamps.
Jan then has 21 stamps.
How many stamps did Jan have to start with? Let *x* represent the number of stamps.
Then, an equation is: $8 + x = 21$
Solve the equation.
Answer the question.

2. Write an equation for each sentence.
Solve each equation by inspection.

a) Five more than a number is 22.
b) Seven less than a number is 31.
c) Six times a number is 54.
d) A number divided by eight is 9.
e) Nine more than three times a number is 24.

3. Write an equation you can use to solve each problem. Solve each equation by systematic trial.

a) Ned spent $36 on a new shuttlecock racquet.
He then had $45 left.
How much money did Ned have before he bought the racquet?

b) Laurie sold 13 books for $208.
All books had the same price.
What was the price of each book?

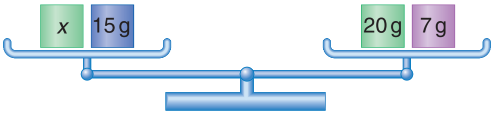
c) Maurice sorts some dominoes.
He divides them into 15 groups, with 17 dominoes in each group.
How many dominoes does Maurice sort?

LESSON

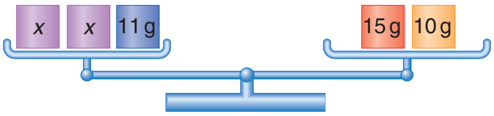
6.2

4. Write the equation that is represented by each balance scales. Solve the equation. Sketch the steps.

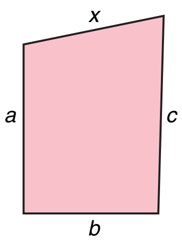
a)



b)



5. Look at the polygon below.



Recall that perimeter is the distance around.

Find the value of *x* when:

a) the perimeter of the polygon is 21 cm and $a = 5$ cm, $b = 3$ cm, and $c = 7$ cm

b) the perimeter of the polygon is 60 cm and $a = 15$ cm, $b = 11$ cm, and $c = 18$ cm

6. Jerry makes some photocopies. He pays 25¢ for a copy-card, plus 8¢ for each copy he makes. Jerry paid a total of 81¢. How many photocopies did Jerry make?

a) Write, then solve, an equation you can use to solve this problem. Show the steps.

b) Verify the solution.

6.3

7. Solve each equation using tiles, and by inspection. Verify each solution.

a) $x + 6 = 9$ **b)** $n + 9 = 6$

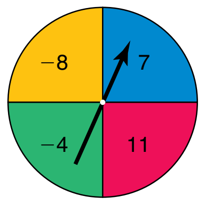
c) $w - 6 = 9$ **d)** $x - 9 = 6$

8. Adriano thinks of two numbers. When he adds 5 to the first number, the sum is -7. When he subtracts 5 from the second number, the difference is $+7$. What are the two numbers?

a) Write 2 equations you can use to solve this problem.

b) Use algebra tiles to solve the equations. Sketch the tiles you used.

9. Max spins the pointer on this spinner. He adds the number the pointer lands on to his previous total each time.



a) Write an equation you can use to solve each problem below.

b) Solve the equation using algebra tiles.

c) Verify each solution.

i) Max gets -8 on his first spin. After his next spin, his total is $+3$. Which number did he get on his second spin?

ii) After 3 spins, Max has a total of -1. Which number did he get on his third spin?

LESSON

6.4

10. Sara collects 56 leaves for a science project. She collects the same number of each of 7 different types of leaves. How many of each type did Sara collect?

a) Write an equation you can use to solve the problem.

b) Solve the equation. Verify the solution.

11. Serena walks 400 m from home to school. Serena is 140 m from school. How far is Serena from home?

a) Write an equation you can use to solve the problem.

b) Solve the equation using algebra. Verify the solution.

12. The Grade 7 classes sold pins to raise money for charity. They raised $228. Each pin sold for $4. How many pins did they sell?

a) Write an equation you can use to solve the problem.

b) Solve the equation using algebra. Verify the solution.

13. Write a problem that can be described by each equation below. Solve each equation using algebra. Explain the meaning of each answer.

a) $x + 15 = 34$ b) $7x = 49$

c) $\frac{x}{5} = 9$ d) $4x + 5 = 37$

6.5

14. Solve each equation using a method of your choice. Explain why you chose each method.

a) $x + 12 = 24$ b) $x + 7 = -3$

c) $x - 18 = -15$ d) $4x = 28$

e) $\frac{x}{11} = 9$ f) $5x + 8 = 73$

15. Jaya has 25 hockey cards. She has one more than 3 times the number of cards her brother has. Write, then solve, an equation to find how many cards he has.

16. The school's sports teams held a banquet. The teams were charged $125 for the rental of the hall, plus $12 for each meal served. The total bill was $545. How many people attended the banquet?

a) Write an equation you could use to solve the problem.

b) Solve your equation.

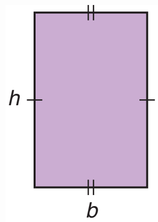
c) Verify the solution.

Practice Test

1. Solve each equation using a method of your choice.
Explain your steps clearly.
a) $x - 9 = -7$ **b)** $12p = 168$ **c)** $\frac{c}{7} = 9$ **d)** $7q + 11 = 102$

2. The area of a rectangle is $A = bh$,
where b is the base of the rectangle and h is its height.
The perimeter of a rectangle is $P = 2b + 2h$.

- Write an equation you can use to solve each problem below.
- Solve the equation. Verify the solution.



a) What is the height of the rectangle when $b = 4$ cm and $A = 44$ cm^2?
b) What is the base of the rectangle when $h = 16$ cm and $P = 50$ cm?

3. The formula $s = \frac{d}{t}$ relates average speed, s, distance, d, and time, t.
Brad took part in a mini-marathon race.
a) Brad jogged at an average speed of 5 km/h for 2 h.
How far did he jog?
b) Brad then rode his bike at an average speed of 16 km/h for 3 h.
How far did he ride his bike?
c) What distance was the race?
Show how you used the formula to solve this problem.

4. Wapeka saves pennies. She has 12¢ in her jar at the start.
Wapeka starts on January 1st. She saves 5¢ every day.

- Write an equation you can use to solve each problem.
- Solve the equation. Verify the solution.

a) By which day had Wapeka saved a total of 47¢?
b) By which day had Wapeka saved a total of $1.07?

5. Anoki is holding a skating party.
The rental of the ice is $75, plus $3 per skater.
a) Write an expression for the cost in dollars for 25 skaters.
b) Suppose Anoki has a budget of $204. Write an equation
you can solve to find how many people can skate.
Solve the equation.

Unit Problem Choosing a Digital Music Club

Suppose your older sister has bought an MP3 player.
She wants to download songs to play on it.
She asks for your help to find the best digital music club.

Part 1

1. Here are three plans from digital music clubs.
Each plan includes 10 free downloaded songs per month.

Songs4U: $20 per month, plus $3 per additional song
YourHits: $30 per month, plus $2 per additional song
Tops: $40 per month, plus $1 per additional song

Copy and complete this table.

Number of Additional Songs per Month	3	6	9	12	15
Cost of Songs4U ($)					
Cost of YourHits ($)					
Cost of Tops ($)					

What patterns do you see in the table?

2. Which club would you recommend if your sister plans to download 3 additional songs per month? 9 additional songs? 15 additional songs? Explain your choice each time.

Part 2

3. a) For each plan, write an expression for the monthly cost of *n* additional songs.

b) Use each expression to find the total monthly cost for 10 additional songs for each club.

c) Suppose your sister can afford to spend $80 a month on downloading songs. Write an equation you can solve to find how many additional songs she can afford with each plan.

d) Solve each equation. Explain what each solution means.

Part 3

Write a paragraph to explain what decisions you made about choosing the best digital music club.

Check List

Your work should show:

- ✓ the completed table
- ✓ the expressions and equations you wrote, and how you used them to solve the problems
- ✓ detailed, accurate calculations
- ✓ clear explanations of your solutions

Reflect on Your Learning

You have learned different methods to solve an equation.
Which method do you prefer? Why?
Which method do you find most difficult?
What is it that makes this method so difficult?

Units 1–6 Cumulative Review

UNIT

1

1. Use the divisibility rules to find the factors of each number.
   a) 120 b) 84 c) 216

2. Grace collects autographs of sports celebrities. She collected 7 autographs at the BC Open Golf Tournament. She then had 19 autographs. How many autographs did she have before the BC Open?
   a) Write an equation you can solve to find how many autographs Grace had before the BC Open.
   b) Solve the equation.
   c) Verify the solution.

2

3. Write the integer suggested by each situation below. Draw yellow or red tiles to model each integer.
   a) You walk down 8 stairs.
   b) You withdraw $10 from the bank.
   c) The temperature rises 9°C.

4. Use tiles to subtract.
   a) $(-9) - (-3)$ b) $(+9) - (-3)$
   c) $(+9) - (+3)$ d) $(-9) - (+3)$

3

5. Find a number between each pair of numbers.
   a) 1.6, 1.7 b) $\frac{6}{11}, \frac{7}{11}$
   c) $2\frac{1}{7}, \frac{16}{7}$ d) $2.7, 2\frac{4}{5}$

6. Find the area of a rectangular vegetable plot with base 10.8 m and height 5.2 m.

7. The regular price of a scooter is $89.99. The scooter is on sale for 20% off.
   a) What is the sale price of the scooter?
   b) There is 14% sales tax. What would a person pay for the scooter?

4

8. a) How many radii does a circle have?
   b) How many diameters does a circle have?

9. A DVD has diameter 12 cm.
   a) Calculate the circumference of the DVD. Give your answer to two decimal places.
   b) Estimate to check if your answer is reasonable.

10. Which has the greatest area? The least area?
   a) a rectangle with base 10 cm and height 5 cm
   b) a parallelogram with base 7 cm and height 8 cm
   c) a square with side length 7 cm

11. Zacharie has 50 m of plastic edging. He uses all the edging to enclose a circular garden. Find:
   a) the circumference of the garden
   b) the radius of the garden
   c) the area of the garden

UNIT

12. Adele recorded the hair colours of all Grades 7 and 8 students in her school.

Hair Colour	Students
Black	60
Brown	20
Blonde	30
Red	10

a) Find the total number of students.
b) Write the number of students with each hair colour as a fraction of the total number of students.
c) Write each fraction as a percent.
d) Draw a circle graph to represent the data.

5 **13.** A cookie recipe calls for $\frac{3}{8}$ cup of brown sugar and $\frac{1}{3}$ cup of white sugar. How much sugar is needed altogether? Show your work.

14. Estimate, then add or subtract.
a) $\frac{3}{5} + \frac{1}{6}$ b) $\frac{5}{6} - \frac{5}{12}$
c) $\frac{2}{3} - \frac{1}{8}$ d) $\frac{1}{4} + \frac{2}{9}$

15. The Boudreau family started a trip with the gas gauge reading $\frac{3}{4}$ full. At the end of the trip, the gauge read $\frac{1}{8}$ full. What fraction of a tank of gas was used?

16. Add or subtract.
a) $5\frac{1}{6} + 3\frac{3}{4}$ b) $1\frac{3}{10} - \frac{2}{3}$
c) $1\frac{3}{5} + 3\frac{2}{3}$ d) $2\frac{5}{6} - 1\frac{5}{8}$

6 **17.** a) Sketch balance scales to represent each equation.
b) Solve each equation. Verify the solution.
i) $s + 9 = 14$ ii) $s + 5 = 14$
iii) $3s = 27$ iv) $3s + 5 = 23$

18. Use tiles to solve each equation. Sketch the tiles you used.
a) $x + 5 = 11$ b) $13 = x - 4$

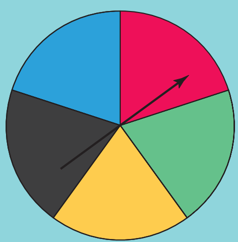
19. Juan works as a counsellor at a summer camp. He is paid $7/h. He was given a $5 bonus for organizing the scavenger hunt. How many hours did Juan work if he was paid $250?
a) Write an equation you can use to solve the problem.
b) Solve the equation. How many hours did Juan work?

20. In a game of cards, black cards are worth +1 point each, and red cards are worth –1 point each. Four players are each dealt 13 cards per round, their scores are recorded, then the cards are shuffled. Write an equation you can use to solve each problem below. Solve the equation.
a) In Round Two, Shin was dealt 8 black cards and 5 red cards. His overall score was then +10. What was Shin's score after Round One?
b) In Round Two, Lucia was dealt 6 black cards and 7 red cards. Her overall score was then −4. What was Lucia's score after Round One?

UNIT 7

Data Analysis

Many games involve probability.
One game uses this spinner or a die labelled 1 to 6.



You can choose to spin the pointer or roll the die.
You win if the pointer lands on red.
You win if you roll a 6.
Are you more likely to win if you spin the pointer or roll the die?
Why do you think so?

What You'll Learn

- Find the mean, mode, median, and range of a set of data.
- Determine the effect of an outlier on the mean, median, and mode.
- Determine the most appropriate average to report findings.
- Express probabilities as ratios, fractions, and percents.
- Identify the sample space for an experiment involving two independent events.
- Compare theoretical and experimental probability.

Why It's Important

- You see data and their interpretations in the media. You need to understand how to interpret these data.
- You need to be able to make sense of comments in the media relating to probability.

Key Words

- mean
- mode
- measure of central tendency
- average
- range
- median
- outlier
- chance
- impossible event
- certain event
- independent events
- tree diagram
- sample space

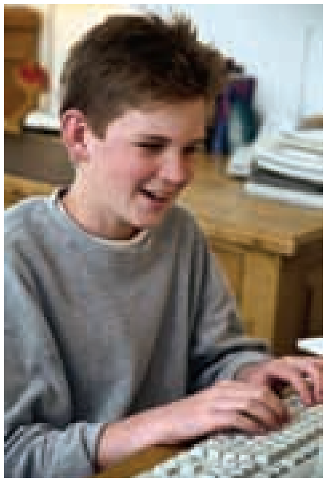
7.1 Mean and Mode

Focus Calculate the mean and mode for a set of data.

Questionnaires, experiments, databases, and the Internet are used to collect data. These collected data can be displayed in tables and graphs, which can be used to make predictions. In this lesson, you will learn ways to describe all the numbers in a data set.

Explore

You will need counters.
Three friends compared the time, in hours, they spent on the computer in one particular week.
Ali spent 5 h,
Bryn spent 9 h,
and Lynne spent 10 h.



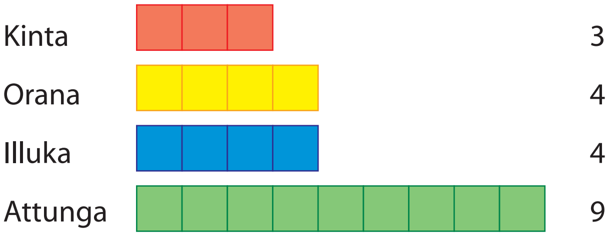
Use counters to represent the time each person spent on the computer.
Find one number that best represents this time.

Reflect & Share

Share your findings with another pair of classmates.
How did you use counters to help you decide on the number?
Explain to your classmates why your number best represents the data.

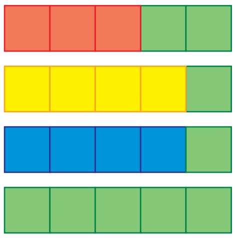
Connect

Allira surveyed 4 friends on the number of first cousins each has.
To find a number that best represents the number of cousins, Allira used linking cubes.



The **mean** is a number that can represent the centre of a set of numbers.

➤ One way to find the mean is to rearrange the cubes to make rows of equal length.



There are 5 cubes in each row.
The mean number of first cousins is 5.

When you make equal rows or columns, the total number of cubes does not change.

➤ You can use the total number of cubes to calculate the mean.
The number of cubes in each row is 3, 4, 4, and 9.
Add these numbers: $3 + 4 + 4 + 9 = 20$
Then divide by the number of rows, 4: $20 \div 4 = 5$

The mean is 5.

The **mode** is the number that occurs most often.

➤ To find the mode, determine which number occurs most often.
In Allira's data, the number 4 occurs twice.
The mode is 4 cousins.
Two people have 4 cousins.

In a set of data, there may be no mode or there may be more than one mode.

Each of the mean and the mode is a **measure of central tendency**.
We say the word **average** to describe a measure of central tendency.
An average is a number that represents all numbers in a set.

Example

Here are Ira's practice times, in seconds, for the 100-m backstroke:
121, 117, 123, 115, 117, 119, 117, 120, 122
Find the mean and mode of these data.

A Solution

To find the mean practice time, add the practice times:
$121 + 117 + 123 + 115 + 117 + 119 + 117 + 120 + 122 = 1071$
Divide by the number of data, 9: $1071 \div 9 = 119$
The mean practice time is 119 s.

The mode is the practice time that occurs most often.
117 occurs three times, so the mode practice time is 117 s.

Practice

1. Use linking cubes to find the mean of each set of data.
a) 3, 4, 4, 5 **b)** 1, 7, 3, 3, 1 **c)** 2, 2, 6, 1, 3, 4

2. Calculate the mean of each set of data.
a) 2, 4, 7, 4, 8, 9, 12, 4, 7, 3 **b)** 24, 34, 44, 31, 39, 32

3. Find the mode of each set of data in question 2.

4. Here are the weekly allowances for 10 Grade 7 students:
$9, $11, $13, $15, $20, $10, $12, $15, $10, $15
a) What is the mean allowance?
b) What is the mode allowance?
c) Suppose two allowances of $19 and $25 are added to the list. What is the new mean? What happens to the mode?

5. Here are the ages of video renters at *Movies A Must* during one particular hour: 10, 26, 18, 34, 64, 18, 21, 32, 21, 54, 36, 16, 30, 18, 25, 69, 39, 24, 13, 22
a) What is the mean age? The mode age?
b) During another hour, the mode age of twelve video renters is 36. What might the ages of the renters be? Explain your answer.

6. Jordin Tootoo is the first Inuk athlete to play in the National Hockey League. On October 9, 2003, he played his first game for the Nashville Predators. This table shows Jordin's statistics when he played junior hockey for the Brandon Wheat Kings.

Jordin Tootoo's Scoring Records 1999-2003

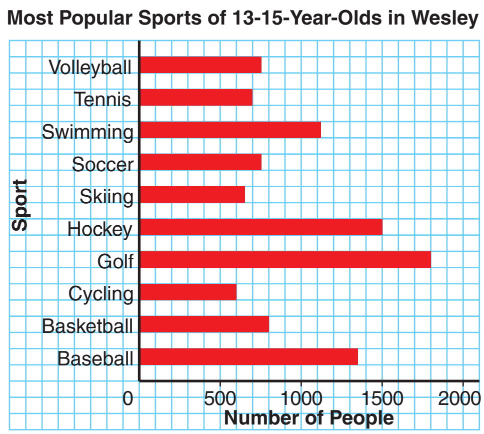
Year	Games Played	Goals	Assists	Points
1999-2000	45	6	10	16
2000-2001	60	20	28	48
2001-2002	64	32	39	71
2002-2003	51	35	39	74

Find the mean and mode for each set of data.

a) Games Played
b) Goals
c) Assists
d) Points

7. **Assessment Focus** The graph shows the most popular sports of 13–15-year-olds in Wesley.

a) Which sports are equally popular?
b) How could you use the bar graph to find the mode? Explain and show your work.
c) Calculate the mean. Use estimated values from the graph.



8. **Take It Further** A data set has 6 numbers. Four of the numbers are: 6, 3, 7, 9
Find the other two numbers in each case.

a) The mean is 6.
b) The mode is 3 and the mean is 6.

Find as many different answers as you can.

Reflect

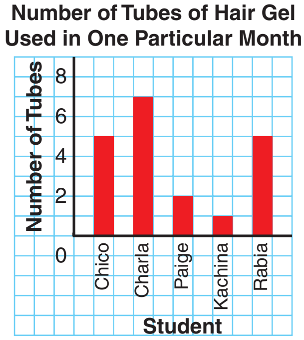
What is the difference between mean and mode? Create a set of data to explain.

7.2 Median and Range

Focus Find the median and the range of a set of data.

The graph shows the number of tubes of hair gel used by each of 5 students in one particular month.

How many tubes of gel did each student use?
What is the mean number of tubes used?
The mode number?
How did you find the mean and the mode?



Explore

Your teacher will give you a bag of Cuisenaire rods.
You will need a ruler.

➤ Without looking, each person takes 3 rods from the bag.
Work together to arrange the 9 rods from shortest to longest.
Find the middle rod.
How many rods are to its right? To its left?
In what way is the middle rod typical of the rods your group picked?
What do you notice about the rods to the left and right of the middle rod?

➤ Each of you takes 1 more rod from the bag.
Place them among the ordered rods in the appropriate places.
Is there a middle rod now? Explain.
Sketch the rods.
Below each rod in your sketch, write its length.
How could you use the lengths to find a "middle" length?
How is the middle length typical of the rods in your sketch?

Reflect & Share

Is it possible to have two different sets of rods with the same middle length?
Share your results with other groups to find out.

Connect

The **median** of a data set is the middle number when the data are arranged in order.

➤ There are 11 Grade 7 students in Ms. Shim's combined Grades 6 and 7 class.
To find the median mark on the last science test, she listed their marks from greatest to least:

95, 92, 87, 85, 80, 78, 76, 73, 70, 66, 54

The middle number is 78.
There are 5 marks greater than 78, and 5 marks less than 78.
The median mark is 78.

When there is an odd number of data, the median is the middle number.

➤ Another Grade 7 student transfers to Ms. Shim's class.
He writes the same test and receives a mark of 72.

To find the new median, the teacher includes his mark in the ordered list:

95, 92, 87, 85, 80, 78, 76, 73, 72, 70, 66, 54

There are two middle numbers, 78 and 76.
There are 5 marks greater than 78, and 5 marks less than 76.
The median is the mean of the 2 middle numbers:
$(78 + 76) \div 2 = 77$
The median mark is now 77.

When there is an even number of data, the median is the mean of the two middle numbers.

The median is also a measure of central tendency, or an average.

➤ Now that the marks are arranged in order, we can easily find the range.
The **range** of a data set tells how spread out the data are.
It is the difference between the greatest and least numbers.
To find the range of the marks on the science test, subtract the least mark from the greatest mark:
$95 - 54 = 41$
The range of the marks is 41.

When there is an even number of data, the median might *not* be one of the numbers in the data set.

Example

The hourly wages, in dollars, of 10 workers are: 8, 8, 8, 8, 9, 9, 9, 11, 12, 20

Find:

a) the mean **b)** the mode **c)** the median **d)** the range

How does each average relate to the data?

A Solution

a) Mean wage:

Add: $8 + 8 + 8 + 8 + 9 + 9 + 9 + 11 + 12 + 20 = 102$

Divide by the number of workers, 10: $102 \div 10 = 10.2$

The mean wage is $10.20.

Three workers have a wage greater than the mean and 7 workers have a wage less than the mean.

b) Mode wage:

8, **8**, **8**, **8**, 9, 9, 9, 11, 12, 20

The mode wage is $8. It occurs 4 times.

This is the least wage; that is, 6 workers have a wage greater than the mode.

c) Median wage:

List the 10 wages in order from least to greatest:

8, 8, 8, 8, **9**, **9**, 9, 11, 12, 20

The median wage is the mean of the 5th and 6th wages.

Both the 5th and 6th wages are 9.

The median wage is $9.

There are 3 wages above the median and 4 wages below the median.

d) Range:

8, 8, 8, 8, 9, 9, 9, 11, 12, **20**

Subtract the least wage from the greatest wage: $20 - 8 = 12$

The range of the wages is $12.

Practice

1. Find the median and the range of each set of data.

a) 85, 80, 100, 90, 85, 95, 90

b) 12 kg, 61 kg, 85 kg, 52 kg, 19 kg, 15 kg, 21 kg, 30 kg

2. The Grade 7 students in two combined Grades 6 and 7 classes wrote the same quiz, marked out of 15.
Here are the results:
Class A: 8, 9, 9, 12, 12, 13, 13, 14, 15, 15
Class B: 10, 10, 11, 11, 12, 12, 13, 13, 14, 14
a) Find the median mark for each class.
b) Find the range of each set of marks.
c) Which class do you think is doing better? Explain.

3. **a)** Find the mean, median, and mode for each data set.
i) 4, 5, 7, 8, 11 **ii)** 50, 55, 65, 70, 70, 50
iii) 7, 63, 71, 68, 71 **iv)** 6, 13, 13, 13, 20
b) Which data sets have:
- the same values for the mean and median? What do you notice about the numbers in each set?
- the same values for the mean, median, and mode? What do you notice about the numbers in each set?
- different values for the mean, median, and mode? What do you notice about the numbers in each set?

4. **Assessment Focus** Write two different data sets with 6 numbers, so that:
a) The mode is 100. The median and the mean are equal.
b) The mode is 100. The mean is less than the median.
Show your work.

5. **a)** The median height of ten 12-year-old girls is 158 cm. What might the heights be? How do you know?
b) The mode height of ten 12-year-old boys is 163 cm. What might the heights be? How do you know?

6. Jamal was training for a 400-m race. His times, in seconds, for the first five races were: 120, 118, 138, 124, 118
a) Find the median and mode times.
b) Jamal wants his median time after 6 races to be 121 s. What time must he get in his 6th race? Show your work.
c) Suppose Jamal fell during one race and recorded a time of 210 s. Which of the mean, median, and mode would be most affected? Explain.

7. In 2005, the Edmonton Miners hosted The Minto Cup Junior A Lacrosse Championship. Here are the 2005 statistics, as of June 30, 2005, for 10 players on the team.

Player	Games	Goals	Assists	Points	Penalty Minutes
Jeremy Boyd	13	2	8	10	54
Dan Claffey	11	3	11	14	33
Dalen Crouse	11	10	10	20	6
Andrew Dixon	15	4	5	10	47
Dan Hartzell	11	5	21	26	8
Cole Howell	12	21	13	34	0
Aiden Inglis	12	3	4	7	23
Ryan Polny	17	7	14	21	2
Chris Schmidt	5	8	4	12	2
Neil Tichkowsky	17	34	19	53	8

a) Calculate the mean, the median, and the mode of each set of data.

b) Make up a question about the mean, the median, or the mode that can be answered using these data. Answer your question.

8. **Take It Further** This is how Edward calculated the mean of these data.
48, 49, 50, 50, 51, 53, 57, 58
Estimated mean is 51.

Score	48	49	50	50	51	53	57	58
Deviation	−3	−2	−1	−1	0	+2	+6	+7

$$\text{Mean} = 51 + \frac{(-3) + (-2) + (-1) + (-1) + 0 + 2 + 6 + 7}{8}$$
$$= 52$$

Check that Edward's answer is correct. How does his method work?

Reflect

A median is the strip of land or concrete barrier separating lanes of highway traffic travelling in opposite directions.
How is this meaning similar to its meaning in math?

7.3 The Effects of Outliers on Average

Focus Understand how mean, median, and mode are affected by outliers.

Explore

Students in a Grade 7 class measured their pulse rates.
Here are their results in beats per minute:
97, 69, 83, 66, 78, 8, 55, 82, 47, 52, 67, 76, 84,
64, 72, 80, 72, 70, 69, 80, 66, 60, 72, 88, 88

- Calculate the mean, median, and mode for these data.
- Are there any numbers that are significantly different from the rest?
  If so, remove them.
  Calculate the mean, median, and mode again.
  Explain how the three averages are affected.

Reflect & Share

Compare your results with those of another pair of classmates.
How did you decide which numbers were significantly different?
Why do you think they are so different?

Connect

A number in a set of data that is significantly different from the other numbers is called an **outlier**.

An outlier is much greater than or much less than most of the numbers in the data set.

Outliers sometimes occur as a result of error in measurement or recording.
In these cases, outliers should be ignored.

Sometimes an outlier is an important piece of information that should not be ignored. For example, if one student does much better or much worse than the rest of the class on a test.

Outliers may not always be obvious.
Identifying outliers is then a matter of choice.

Example

Here are the marks out of 100 on an English test for students in a Grade 7 class:

21, 23, 24, 24, 27, 29, 29, 29, 32, 37, 37, 38, 39,
40, 50, 50, 51, 54, 56, 57, 58, 59, 61, 71, 80, 99

a) How many students were in the class? How do you know?

b) What is the outlier? Explain your choice.

c) Calculate the mean, median, and mode.

d) Calculate the mean, median, and mode without the outlier.
What do you notice?

e) Should the outlier be used when reporting the average test mark? Explain.

A Solution

a) Count the number of marks to find the number of students in the class.
There are 26 students.

b) There is only one number, 99, that is significantly different.
The outlier is 99.
The difference between the outlier and the nearest mark is $99 - 80 = 19$.
This difference is much greater than that between other pairs of adjacent marks.

c) There are 26 marks. To find the mean mark, add the marks:
$21 + 23 + 24 + 24 + 27 + 29 + 29 + 29 + 32 + 37 + 37 + 38 + 39 +$
$40 + 50 + 50 + 51 + 54 + 56 + 57 + 58 + 59 + 61 + 71 + 80 + 99 = 1175$
Divide the total by the number of marks, 26: $1175 \div 26 \doteq 45.2$
The answer is written to the nearest tenth.
The mean mark is about 45.2.
The median mark is the mean of the 13th and 14th marks.
The 13th mark is 39. The 14th mark is 40.
So, the median is: $\frac{39 + 40}{2} = \frac{79}{2} = 39.5$
The mode is the mark that occurs most often. This is 29.

d) Without the outlier, there are 25 marks and the sum of the marks is: $1175 - 99 = 1076$
The mean is: $1076 \div 25 = 43.04$
The median is the 13th mark: 39
The mode is 29.
When the outlier was removed, the mean and median decreased.
The mode remained the same.

e) The outlier should be used when reporting the average test mark.
To understand how the class is performing, all test marks should be included.

Practice

1. This set of data represents the waiting time, in minutes, at a fast-food restaurant:
5, 5, 5, 6, 5, 7, 0, 5, 1, 7, 7, 5, 6, 5, 5, 5, 8, 5, 0, 5, 4, 5, 2, 7, 9

a) Calculate the mean, median, and mode.
b) Identify the outliers. Explain your choice.
c) Calculate the mean, median, and mode without the outliers. How is each average affected when the outliers are not included?

Remember to arrange the data in order before finding the median.

2. Bryan recorded the time he spent on the school bus each day for one month. Here are the times, in minutes:
15, 21, 15, 15, 18, 19, 14, 20, 95, 18, 21, 14, 15, 20, 16, 14, 22, 21, 15, 19

a) Calculate the mean, median, and mode times.
b) Identify the outlier. How can you explain this time?
c) Calculate the mean, median, and mode times without the outlier. How is each average affected when the outlier is not included?
d) A classmate asks Bryan, "What is the average time you spend on the bus each day?" How should Bryan answer? Give reasons.

3. A clothing store carries pant sizes 28 to 46.
A sales clerk records the sizes sold during her 6-h shift:
28, 36, 32, 32, 34, 4, 46, 44, 42, 38, 36, 36, 40, 32, 36

a) Calculate the mean, median, and mode sizes.
b) Is there an outlier? If so, why do you think it is an outlier?
c) Calculate the mean, median, and mode sizes without the outlier. How is each average affected when the outlier is not included?
d) Should the outlier be used when the sales clerk reports the average pant size sold during her shift? Explain your thinking.

4. Here are the science test marks out of 100 for the Grade 7 students in a combined-grades class:
0, 66, 65, 72, 78, 93, 82, 68, 64, 90, 65, 68

a) Calculate the mean, median, and mode marks.
b) Identify the outlier. How might you explain this mark?
c) Calculate the mean, median, and mode marks without the outlier. How is each average affected when the outlier is not included?
d) Should the outlier be used when reporting the average test mark? Explain.

5. a) Give an example of a situation in which outliers would not be used in reporting the averages. Explain why they would not be included.
   b) Give an example of a situation in which outliers would be used in reporting the averages. Explain why they would be included.

6. **Assessment Focus** A Grade 7 class wanted to find out if a TV advertisement was true. The ad claimed that *Full of Raisins* cereal guaranteed an average of 23 raisins per cup of cereal. Each pair of students tested one box of cereal. Each box contained 20 cups of cereal. The number of raisins in each cup was counted.

   a) Assume the advertisement is true. How many raisins should there be in 1 box of cereal?
   b) Here are the results for the numbers of raisins in 15 boxes of cereal:
      473, 485, 441, 437, 489, 471, 400, 453, 465, 413, 499, 428, 419, 477, 467
      i) Calculate the mean, median, and mode numbers of raisins.
      ii) Identify the outliers. Explain your choice.
      iii) Calculate the mean, median, and mode without the outliers. How do the outliers affect the mean?
      iv) Should the outliers be used when reporting the average number of raisins? Explain.
      v) Was the advertisement true? Justify your answer.

7. **Take It Further** Here is a set of data: 2, 3, 5, 5, 7, 8
   An outlier has been removed.
   a) Calculate the mean, median, and mode without the outlier.
   b) The outlier is returned to the set.
      The averages become:
      Mean: 7 Median: 5 Mode: 5
      What is the outlier? Show your work.

Reflect

Your friend is having difficulty recognizing outliers in a data set.
What advice would you give your friend?

7.4 Applications of Averages

Focus Understand which average best describes a set of data.

Explore

Record on the board how many siblings you have.
Use the class data.
Find the mean, the median, and the mode.
Find the range.

Reflect & Share

With a classmate, discuss which measure best describes the average number of siblings.

Connect

A clothing store sold jeans in these sizes in one day:

28 30 28 26 30 32 28 32 26 28 34 38 36 30 34 32 30

To calculate the mean jeans size sold, add the sizes, then divide by the number of jeans sold.

$$\text{Mean} = \frac{28 + 30 + 28 + 26 + 30 + 32 + 28 + 32 + 26 + 28 + 34 + 38 + 36 + 30 + 34 + 32 + 30}{17}$$

$$= \frac{522}{17}$$

$$\doteq 30.7$$

The mean size is approximately 30.7.

To calculate the median, order the jeans sold from least size to greatest size. There are 17 numbers.
The middle number is the median. The middle number is the 9th.
26, 26, 28, 28, 28, 28, 30, 30, **30**, 30, 32, 32, 32, 34, 34, 36, 38
The median size is 30.

The mode is the number that occurs most often.
But, there are two numbers that occur most often.
So, there are two modes.
They are 28 and 30.
So, the mode sizes are 28 and 30.

When there is an odd number of data, to find the middle number: Add 1 to the number of data, then divide by 2. This gives the position of the middle number. For example: $\frac{17 + 1}{2} = \frac{18}{2} = 9$; the middle number is the 9th.

In this situation, the mean, 30.7, is of little use.
The mean does not represent a size.

The median, 30, shows about one-half of the customers bought jeans of size 30 or smaller, and about one-half of the customers bought jeans of size 30 or larger.

The modes, 28 and 30, tell which sizes are purchased more often.
The mode is most useful to the storeowner.
He may use the mode to order extra stock of the most popular sizes.

Example

A bookstore has 15 books in its young adult section.
There are 5 different prices.
This table shows the number of books at each price.

a) Find the mean, median, and mode prices.
b) Which measure best represents the average price of a young adult book?
c) What is the range of the prices?

Young Adult Books

Price ($)	Number of Books
8.99	3
9.99	5
13.99	5
32.99	1
37.99	1

A Solution

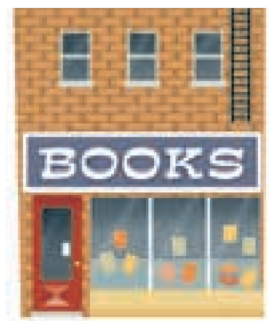
Make a list of the prices, in dollars:
8.99, 8.99, 8.99, 9.99, 9.99, 9.99, 9.99, 9.99, 13.99, 13.99, 13.99, 13.99, 13.99, 32.99, 37.99

a) Mean price:

- Multiply each price by the total number of books at that price, then add the prices.
  $(8.99 \times 3) + (9.99 \times 5) + (13.99 \times 5) + 32.99 + 37.99 = 217.85$
- Divide the total price by the total number of books: 15
  $\frac{217.85}{15} \doteq 14.52$, to two decimal places

The mean price is approximately $14.52.

Median price:
There are 15 books.
The list shows the books in order from least price to greatest price.
The median price is the 8th price. The 8th price is $9.99.
The median price is $9.99.



Mode price:
There are two mode prices. They are $9.99 and $13.99.

b) The mean price is not charged for any of the books.
Only two books cost more than the mean of $14.52.

There are two mode prices.
One mode, $9.99, is the same as the median price.

One-half the books cost the median price or less.
One-half cost more. So, the median price, $9.99, best represents the average price of a young adult book at the store.

c) For the range, subtract the lowest price from the highest price:
$37.99 - 8.99 = 29.00$
The range of prices is $29.00.

The mean is usually the best average when no numbers in the data set are significantly different from the other numbers.

The median is usually the best average when there are numbers in the data set that are significantly different.

The mode is usually the best average when the data represent measures, such as shoe sizes or clothing sizes.
A store needs to restock the sizes that sell most often.

Practice

1. The daily high temperatures for one week at Clearwater Harbour were: 27°C, 31°C, 23°C, 25°C, 28°C, 23°C, 28°C

a) Find the mean, median, and mode for these data.

b) Which average do you think best describes the daily high temperature at Clearwater Harbour that week? Explain.

c) The weather channel reported the average temperature for Clearwater Harbour that week was 23°C. Is this correct? Explain.

2. Caitlin received these test marks in each subject.

a) Find the mean, median, and mode mark for each subject.

b) Explain what information each average gives.

c) Which subject do you think Caitlin is best at? Worst at? Explain your reasoning.

Caitlin's Marks

Math	85	69	92	55	68	75	78
Music	72	81	50	69	81	96	92
French	68	74	82	80	76	67	74

3. The table shows the tips earned by five waiters and waitresses during two weeks in December.

a) Calculate the mean, median, and mode tips for each week.

b) Calculate the mean, median, and mode tips for the two-week period.

c) Compare your answers in parts a and b. Which are the same? Which are different? Explain why.

d) Explain which average best represents the tips earned during the two weeks.

Weekly Tips Earned ($)

Waiter	Week 1	Week 2
James	1150	600
Kyrra	700	725
Tamara	800	775
Jacob	875	860
George	600	1165

4. A small engineering company has an owner and 5 employees. This table shows their salaries.

a) Calculate the mean, median, and mode annual salaries.

b) What is the range of the annual salaries?

c) Which measure would you use to describe the average annual salary in each case? Explain.

i) You want to attract a new employee.

ii) You want to suggest the company does not pay its employees well.

Company Salaries

Position	Annual Salary ($)
Owner	130 000
Manager	90 000
2 Engineers	50 000
Receptionist	28 000
Secretary	28 000

5. Is each conclusion correct? Explain your reasoning.

a) The mean cost of a medium pizza is $10.
So, the prices of three medium pizzas could be $9, $10, and $11.

b) The number of raisins in each of 30 cookies was counted.
The mean number of raisins was 15.
So, in 10 cookies, there would be a total of 150 raisins.

6. **Assessment Focus** In each case, which average do you think is most useful: the mean, median, or mode? Justify your answer.

a) A storeowner wants to know which sweater sizes she should order. Last week she sold 5 small, 15 medium, 6 large, and 2 X-large sweaters.

b) Five of Robbie's friends said their weekly allowances are: $10, $13, $15, $11, and $10.
Robbie wants to convince his parents to increase his allowance.

c) Tina wants to know if her math mark was in the top half or bottom half of the class.

7. A quality control inspector randomly selects boxes of crackers from the production line.
She measures their masses.
On one day she selects 15 boxes, and records these data:

- 6 boxes: 405 g each
- 2 boxes: 395 g each
- 4 boxes: 390 g each
- 2 boxes: 385 g each
- 1 box: 380 g

a) Calculate the mean, median, and mode masses.
b) What is the range of the masses?
c) For the shipment of crackers to be acceptable, the average mass must be at least 398 g. Which average would you use to describe this shipment to make it acceptable? Explain.

8. **Take It Further** Andrew has these marks:
English 82%, French 75%, Art 78%, Science 80%
a) What mark will Andrew need in math if he wants his mean mark in these 5 subjects to be each percent?
i) 80% **ii)** 81% **iii)** 82%
b) Is it possible for Andrew to get a mean mark of 84% or higher? Justify your answer.

9. **Take It Further** Celia received a mean mark of 80% in her first three exams.
She then had 94% on her next exam.
Celia stated that her overall mean mark was 87% because the mean of 80 and 94 is 87.
Is Celia's reasoning correct? Explain.

Reflect

Use your answers from *Practice*. Describe a situation for each case.
a) The mean is the best average.
b) The median is the best average.
c) The mode is the best average.
Justify your choices.

Using Spreadsheets to Investigate Averages

Focus Investigate averages using a spreadsheet.

You can use spreadsheet software to find the mean, median, and mode of a set of data.
A spreadsheet program allows us to calculate the averages for large sets of data values quickly and efficiently.
You can also use the software to see how these averages are affected by outliers.

Here are the heights, in centimetres, of all Grade 7 students who were on the school track team: 164, 131, 172, 120, 175, 168, 146, 176, 175, 173, 155, 170, 172, 160, 168, 178, 174, 184, 189

In some spreadsheet software, the mean is referred to as the average.

Use spreadsheet software.

- ➤ Input the data into a column of the spreadsheet.
- ➤ Use the statistical functions of your software to find the mean, median, and mode. Use the Help menu if you have any difficulties.
- ➤ Investigate the effect of an outlier on the mean, median, and mode.
  Delete 120.
  What happens to the mean? Median? Mode? Explain.

- ➤ Suppose one member of the track team with height 155 cm is replaced by a student with height 186 cm.
  How does this substitution affect the mean, median, and mode? Explain.

Notice that when you add or remove data values, the averages change to reflect the adjustments.

Check

1. Enter these data into a spreadsheet.
They are the donations, in dollars, that were made to a Toy Wish Fund.
5, 2, 3, 9, 10, 5, 2, 8, 7, 15, 14, 17, 28, 30, 16, 19, 4, 7, 9, 11, 25, 30,
32, 15, 27, 18, 9, 10, 16, 22, 34, 19, 25, 18, 20, 17, 9, 10, 15, 35
   a) Find the mean, median, and mode.
   b) Add some outliers to your spreadsheet.
   State the values you added.
   How do the new mean, median, and mode compare to their original values? Explain.

2. Enter these data into a spreadsheet.
They are purchases, in dollars, made by customers at a grocery store.
55.40, 48.26, 28.31, 14.12, 88.90, 34.45, 51.02, 71.87, 105.12, 10.19,
74.44, 29.05, 43.56, 90.66, 23.00, 60.52, 43.17, 28.49, 67.03, 16.18,
76.05, 45.68, 22.76, 36.73, 39.92, 112.48, 81.21, 56.73, 47.19, 34.45
   a) Find the mean, median, and mode.
   b) Add some outliers to your spreadsheet.
   State the values you added.
   How do the new mean, median, and mode compare to their original values? Explain.

3. Enter these data into a spreadsheet.
They are the number of ice-cream bars sold at the community centre each day in the month of July.
101, 112, 127, 96, 132, 125, 116, 97, 124, 136, 123, 113, 78, 102, 118, 130,
87, 108, 114, 99, 126, 86, 94, 117, 121, 107, 122, 119, 111, 105, 93
Find the mean, median, and mode.
What happens when you try to find the mode? Explain.

4. Repeat question 1 parts a and b.
This time, enter data you find in the newspaper or on the Internet.

Reflect

List some advantages of using a spreadsheet to find the mean, median, and mode of a set of data. What disadvantages can you think of?

Mid-Unit Review

LESSON

7.1 7.2 **1.** Here are the heights, in centimetres, of the students in a Grade 7 class:
162, 154, 166, 159, 170, 168, 158, 162, 172, 166, 157, 170, 171, 165, 162, 170, 153, 167, 164, 169, 167, 173, 170

a) Find the mean, median, and mode heights.

b) What is the range of the heights?

2. The mean of five numbers is 20.
The median is 23.
What might the numbers be?
Find 2 different sets of data.

7.3 **3.** The cost of hotel rooms at *Stay in Comfort* range from $49 to $229 per night. Here are the rates charged, in dollars, for one particular night:
70, 75, 85, 65, 75, 90, 70, 75, 60, 80, 95, 85, 75, 20, 65, 229

a) Calculate the mean, median, and mode costs.

b) Identify the outliers.
How can you explain these costs?

c) Calculate the mean, median, and mode costs without the outliers. How is each average affected when the outliers are not included?

d) Should the outliers be used when reporting the average cost of a hotel room? Explain.

7.4 **4.** A quality control inspector measures the masses of boxes of raisins. He wants to know if the average mass of a box of raisins is 100 g. The inspector randomly chooses boxes of raisins.
The masses, in grams, are:
99.1, 101.7, 99.8, 98.9, 100.8, 100.3, 98.3, 100.0, 97.8, 97.6, 98.5, 101.7, 100.2, 100.2, 99.4, 100.3, 98.8, 102.0, 100.3, 98.0, 99.4, 99.0, 98.1, 101.8, 99.8, 101.3, 100.5, 100.7, 98.7, 100.3, 99.3, 102.5

a) Calculate the mean, median, and mode masses.

b) For the shipment to be approved, the average mass of a box of raisins must be at least 100 g. Which average could someone use to describe this shipment to get it approved? Explain.

5. Is each conclusion true or false? Explain.

a) The mode number of books read last month by students in James' class is 5. Therefore, most of the students read 5 books.

b) A random sample of 100 people had a mean income of $35 000. Therefore, a random sample of 200 people would have a mean income of $70 000.

7.5 Different Ways to Express Probability

Focus Express probabilities as ratios, fractions, and percents.

When the outcomes of an experiment are equally likely, the probability of an event occurring is:

$$\frac{\text{Number of outcomes favourable to that event}}{\text{Number of possible outcomes}}$$

Explore

At the pet store, Mei buys 100 biscuits for her dog, Ping-Ping.
She buys 75 beef-flavoured biscuits, 15 cheese-flavoured, and 10 chicken-flavoured.
The clerk puts them all in one bag.
When she gets home, Mei shakes the bag and pulls out one biscuit.

- What is the probability that Mei pulls out a cheese-flavoured biscuit from the bag?
- How many different ways could you write this probability?
- What is the probability of pulling out a beef-flavoured biscuit? A chicken-flavoured biscuit? Write each probability 3 different ways.
- What is the probability of pulling out a vegetable-flavoured biscuit?
- What is the probability of pulling out a flavoured biscuit?

Reflect & Share

Compare your results with those of another pair of classmates.
How many different ways did you write a probability?
Are all of the ways equivalent?
How do you know?
What is the probability of an event that always occurs?
An event that never occurs?

Connect

A probability can be written as a ratio, a fraction, and a percent.

Sam buys a box of different flavours of food for his cat.
In a box, there are 14 packets of fish flavour, 2 of chicken flavour, and 4 of beef flavour.
Sam takes a packet out of the box without looking.
What is the probability that he picks a packet of chicken-flavoured food?

There are 20 packets in a box of cat food.

- Using words:
  Only 2 of the 20 packets are chicken.
  So, picking chicken is unlikely.
- Using a fraction:
  Two of the 20 packets are chicken.
  The probability of picking chicken is $\frac{2}{20}$, or $\frac{1}{10}$.
- Using a ratio:
  The probability of picking chicken is $\frac{1}{10}$.
  We can write this as the part-to-whole ratio 1:10.
- Using a percent:
  To express $\frac{2}{20}$ as a percent, find an equivalent fraction with denominator 100.

  $$\frac{2 \times 5}{20 \times 5} = \frac{10}{100}, \text{ or } 10\%$$

  The chance of picking chicken is 10%.

Recall that you can also use a calculator to help you write a fraction as a percent.

When we express a probability as a percent, we often use the word *chance* to describe it.

When *all* the outcomes are favourable to an event, then the fraction:

$$\frac{\text{Number of outcomes favourable to that event}}{\text{Number of possible outcomes}}$$

has numerator equal to denominator, and the probability is 1, or 100%.
For example, the probability of picking a packet of cat food is: $\frac{20}{20} = 1$

When *no* outcomes are favourable to an event, then the fraction:

$$\frac{\text{Number of outcomes favourable to that event}}{\text{Number of possible outcomes}}$$

has numerator equal to 0, and the probability is 0, or 0%.
For example, the probability of picking a packet of pork-flavoured cat food is: $\frac{0}{20} = 0$

The probability that an **impossible event** will occur is 0, or 0%.
The probability that a **certain event** will occur is 1, or 100%.
All other probabilities lie between 0 and 1.

Example

Twenty-five cans of soup were immersed in water.
Their labels came off so the cans now look identical.
There are: 2 cans of chicken soup; 4 cans of celery soup; 5 cans of vegetable soup; 6 cans of mushroom soup; and 8 cans of tomato soup.
One can is picked, then opened.

a) What is the probability of each event?
Write each probability as a ratio, fraction, and percent.
  i) The can contains celery soup.
  ii) The can contains fish.
  iii) The can contains celery soup or chicken soup.
  iv) The can contains soup.

b) State which event in part a is:
  i) certain **ii)** impossible

A Solution

a) There are 25 cans, so there are 25 possible outcomes.
  i) Four cans contain celery soup.
  The probability of opening a can of celery soup is:
  4:25, or $\frac{4}{25} = \frac{16}{100}$, or 16%
  ii) None of the cans contain fish.
  The probability of opening a can of fish is: 0, or 0%
  iii) Four cans contain celery soup and two contain chicken soup.
  This is 6 cans in all.
  The probability of opening a can of celery soup or chicken soup is:
  6:25, or $\frac{6}{25} = \frac{24}{100}$, or 24%
  iv) Since all the cans contain soup, the probability of opening a can of soup is:
  20:20, or $\frac{20}{20}$, or 100%

b) **i)** The event that is certain to occur is opening a can that contains soup.
  This event has the greatest probability, 1.
  ii) The event that is impossible is opening a can that contains fish.
  This event has the least probability, 0.

Practice

Use a calculator when you need to.

1. Write the probability of each event as many different ways as you can.

a) Roll a 3 or 5 on a die labelled 1 to 6.

b) January immediately follows June.

c) Pick an orange out of a basket that contains 2 oranges, 6 apples, and 8 peaches.

d) The sun will set tomorrow.

2. A bag contains these granola bars: 12 apple, 14 peanut butter, 18 raisin, and 10 oatmeal. You pick one bar at random.
Find the probability of picking:

a) a peanut butter granola bar

b) an apple granola bar

3. Two hundred fifty tickets for a draw were sold.
One ticket, drawn at random, wins the prize.

a) Joe purchased 1 ticket.
What is the probability Joe will win?

b) Maria purchased 10 tickets.
What is the probability Maria will win?

c) Ivan purchased 25 tickets.
What is the probability Ivan will *not* win?

Express each probability three ways.

4. Thanh has 20 felt pens in a pencil case.
He has 6 blue pens, 5 red pens, 2 yellow pens, 3 green pens, 2 brown pens, 1 purple pen, and 1 orange pen.
Thanh reaches into the case without looking and pulls out one pen.
Write a ratio, fraction, and percent to describe the probability that Thanh picks:

a) either a yellow or a green pen

b) either a blue or a red pen

c) a coloured pen

d) a grey pen

e) a purple pen

5. The names of 8 students are in a hat.
You pick one name without looking.
Find each probability.
Express each probability as many ways as you can.

a) Laura will be picked.
b) Jorge will *not* be picked.
c) A three-letter name will be picked.
d) A five-letter name will be picked.
e) A name with 4 or more vowels will be picked.
f) A boy's or a girl's name will be picked.

6. Think of an experiment for which an event occurs with each probability.
Explain your choice.

a) 100% b) $\frac{1}{2}$ c) 1:6 d) 0

7. **Assessment Focus** Construct a spinner with red, yellow, blue, and green sectors, so the following probabilities are true.

- The probability of landing on red is $\frac{1}{5}$.
- The probability of landing on yellow is 50%.
- The probability of landing on blue is 1:10.
- The probability of landing on green is $\frac{2}{10}$.

Explain how you drew your spinner.

8. **Take It Further** A box contains 3 red, 2 green, and 4 white candies.
Carmen picked one candy, found it was white, and ate it.
She picked a second candy at random, found it was red, and ate it.
Carmen picked a third candy at random.

a) Which colour is the third candy most likely to be? Explain.
b) Write the probability that the third candy will be the colour named in part a. Use a ratio, fraction, and percent to write the probability.
c) What is the probability that the candy will *not* be the colour named in part a?

Reflect

The weather forecast shows a 90% chance of rain tomorrow.
How would this affect your plans for a class picnic? Why?

7.6 Tree Diagrams

Focus Investigate outcomes of probability experiments.

Recall that an outcome is the possible result of an experiment or action.
When you roll a die, the outcomes are equally likely.
When you toss a coin, the outcomes are equally likely.
Some experiments have two or more actions.

Explore

You will need a die labelled 1 to 6 and a coin.

➤ List the possible outcomes of rolling the die and tossing the coin.
How many possible outcomes are there?
How many outcomes include rolling a 4? Tossing a head?

➤ What is the theoretical probability of the event "a head on the coin and a 2 on the die?"

➤ Conduct the experiment.
One of you tosses the coin and one rolls the die.
Record the results.
Calculate the experimental probability of the event "a head on the coin and a 2 on the die" after each number of trials.
- 10 trials
- 20 trials
- 50 trials
- 100 trials

➤ How do the experimental and theoretical probabilities compare?

Reflect & Share

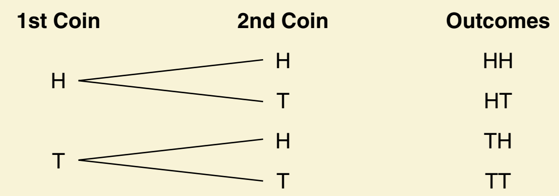
Compare the strategy you used to find the outcomes with that of another pair of classmates. Was one strategy more efficient than another? Explain.
Compare your probabilities. Combine your results to get 200 trials.
What is the experimental probability of the event
"a head on the coin and a 2 on the die?"
How do the experimental and theoretical probabilities compare?

Connect

Two events are **independent events** if the result of the one event does not depend on the result of the other event.
Tossing two coins is an example of two independent events.
The outcome of the first toss does not affect the outcome of the second toss.
The outcome of the second toss does not depend on the outcome of the first toss.
We can use a **tree diagram** to show the possible outcomes for an experiment that has two independent events.

When 2 coins are tossed, the outcomes for each coin are heads (H) or tails (T).
List the outcomes of the first coin toss.
This is the first branch of the tree diagram.
For each outcome, list the outcomes of the second coin toss.
This is the second branch of the tree diagram.
Then list the outcomes for the coins tossed together.

You could also use a table to list the outcomes.

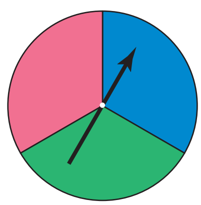


		First Coin	
		H	**T**
Second Coin	**H**	HH	HT
	T	TH	TT

There are 4 possible outcomes: HH, HT, TH, TT

Example

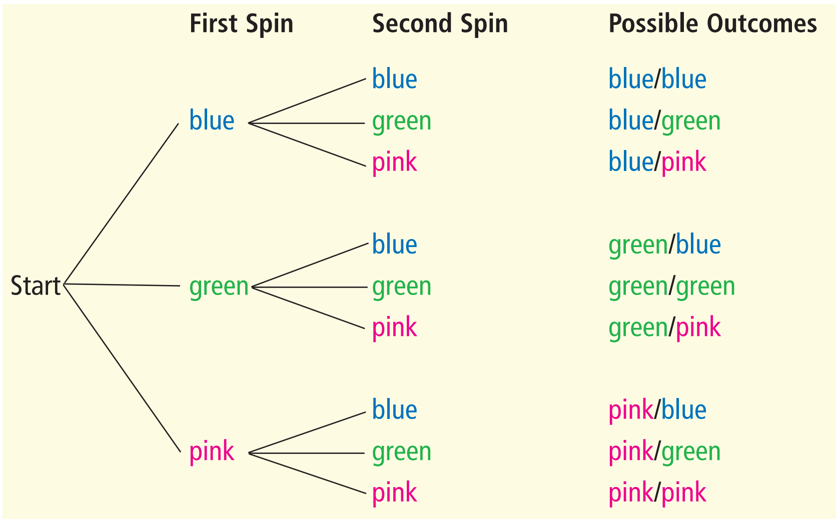
On this spinner, the pointer is spun once.
The colour is recorded.
The pointer is spun a second time.
The colour is recorded.



a) Draw a tree diagram to list the possible outcomes.
b) Find the probability of getting the same colours.
c) Find the probability of getting different colours.
d) Carina and Paolo carry out the experiment 100 times.
There were 41 same colours and 59 different colours.
How do the experimental probabilities compare to the theoretical probabilities? Explain.

A Solution

a) The first branch of the tree diagram lists the equally likely outcomes of the first spin: blue, green, pink
The second branch lists the equally likely outcomes of the second spin: blue, green, pink
For each outcome from the first spin, there are 3 possible outcomes for the second spin.
Follow the paths from left to right. List all the possible outcomes.



A list of all outcomes is the *sample space*.

b) From the tree diagram, there are 9 possible outcomes.
3 outcomes have the same colours: blue/blue, green/green, pink/pink
The probability of the same colour is:
$\frac{3}{9} = \frac{1}{3} \doteq 0.33$, or about 33%

c) 6 outcomes have different colours: blue/green, blue/pink, green/blue, green/pink, pink/blue, pink/green
The probability of different colours is:
$\frac{6}{9} = \frac{2}{3} \doteq 0.67$, or about 67%

d) The experimental probability of the same colour is:
$\frac{41}{100} = 0.41$, or 41%
The experimental probability of different colours is:
$\frac{59}{100} = 0.59$, or 59%
These probabilities are different from the theoretical probabilities.
The greater the number of times the experiment is carried out, the closer the theoretical and experimental probabilities may be.

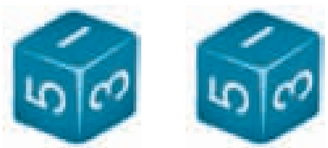
Practice

1. List the sample space for each pair of independent events.
Why are the events independent?

a) Rolling a die labelled 3 to 8 and tossing a coin

b) Rolling a tetrahedron labelled 1 to 4 and spinning a pointer on this spinner

c) Rolling a pair of dice labelled 1 to 6



2. Use the outcomes from question 1a. Aseea wins if an odd number or a head shows. Roberto wins if a number less than 5 shows. Who is more likely to win? Explain.

3. Use the outcomes from question 1b.
Name an outcome that occurs about one-half of the time.

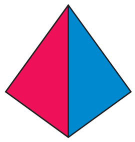
4. Use the outcomes from question 1c.
How often are both numbers rolled greater than 4?

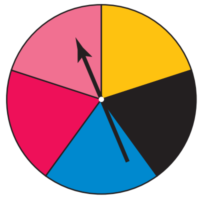
5. Hyo Jin is buying a new mountain bike.
She can choose from 5 paint colours—black, blue, red, silver, or gold—and 2 seat colours—grey or black.

a) Use a table to display all the possible combinations of paint and seat colours.

b) Suppose Hyo Jin were to choose colours by pointing at lists without looking. What is the probability she would end up with a silver or black bike with a grey seat?

6. Assessment Focus Tara designs the game *Mean Green Machine.*
A regular tetrahedron has its 4 faces coloured red, pink, blue, and yellow.
A spinner has the colours shown.
When the tetrahedron is rolled, the colour on its face down is recorded.
A player can choose to:





- roll the tetrahedron and spin the pointer, or
- roll the tetrahedron twice, or
- spin the pointer twice

To win, a player must make green by getting blue and yellow.
With which strategy is the player most likely to win?
Justify your answer. Play the game to check.
Show your work.

7. An experiment is: rolling a regular octahedron, labelled 1 to 8, and drawing a counter from a bag that contains 4 counters: green, red, yellow, blue
The number on the octahedron and the colour of the counter are recorded.

a) Use a regular octahedron, and a bag that contains the counters listed above. Carry out the experiment 10 times. Record your results.
b) Combine your results with those of 9 classmates.
c) What is the experimental probability of each event?
  i) green and a 4
  ii) green or red and a 7
  iii) green or yellow and an odd number
d) Draw a tree diagram to list the possible outcomes.
e) What is the theoretical probability of each event in part c?
f) Compare the theoretical and experimental probabilities of the events in part c.
What do you think might happen if you carried out this experiment 1000 times? Explain.

Reflect

Which method do you prefer to find the sample space?
Why?

Math Link

Your World
The Canadian Cancer Society runs a lottery every year to raise money for cancer research in Canada. One year, the chances of winning were given as the ratio 1:12. This could also be represented by the fraction $\frac{1}{12}$.

All the Sticks

Game

This game is based on a game originally played by the Blackfoot Nation.
The original materials were 4 animal bones and sticks.

YOU WILL NEED

4 popsicle sticks (or tongue depressors); markers; 12 counters

NUMBER OF PLAYERS

2

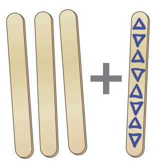
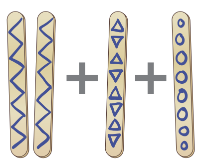
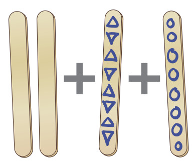
GOAL OF THE GAME

To get all 12 counters

HOW TO PLAY THE GAME:

1. Decorate:
   - 2 popsicle sticks with a zigzag pattern on one side
   - 1 popsicle stick with a circle pattern on one side
   - 1 popsicle stick with a pattern of triangles on one side

   Leave the other side of each popsicle stick blank.
2. Decide who will go first.
3. Place the counters in a pile on the floor.
4. Hold the 4 popsicle sticks in one hand, then drop them to the floor.
   Points are awarded according to the patterns that land face up.
   Find your pattern in the chart to determine your score.
   Take that number of counters.
   For example, if you score 4 points, take 4 counters.
   Take the counters from the pile until it has gone, then take counters from each other.

Pattern				Any other combination
Points	6	4	2	0

5. Take turns.
   The first player to have all 12 counters wins.

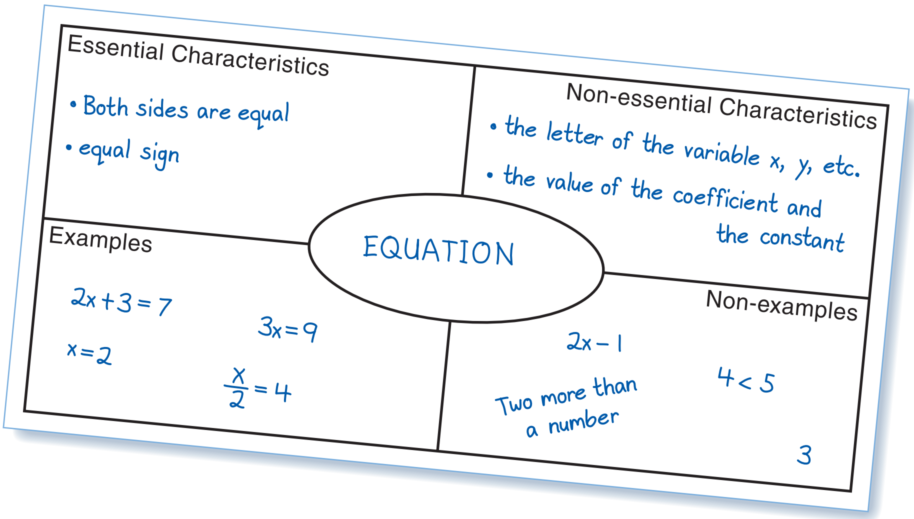
**What is the theoretical probability of scoring 6 points?
How many points are you most likely to score in one turn?
How did you find out?**

Using a Frayer Model

Every new math topic has new words and ideas.

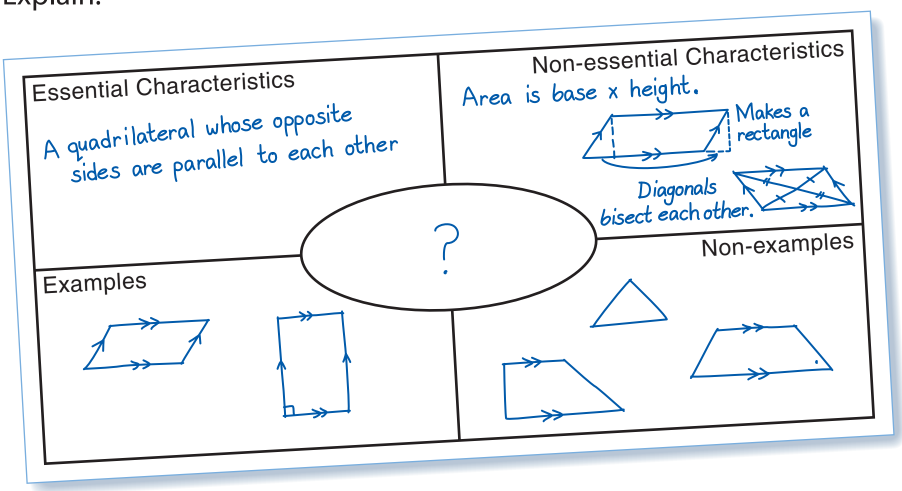
You can use a **Frayer Model** to help you remember new words and to better understand new ideas. A Frayer Model helps you make connections to what you already know.

Here is an example of a Frayer Model.



This model lists the essential and non-essential characteristics of the word.

What word do you think belongs in the centre of this Frayer Model? Explain.



Reading and Writing in Math

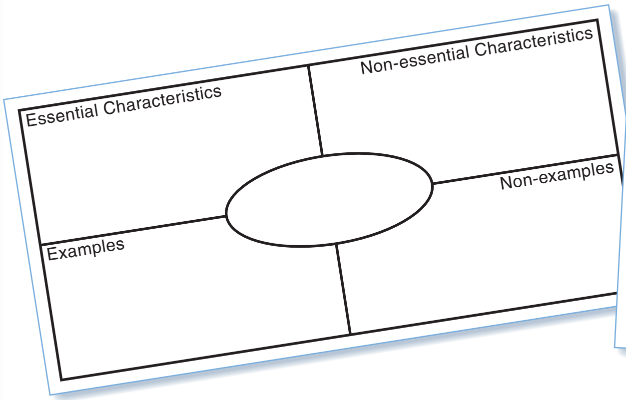
✓ Check

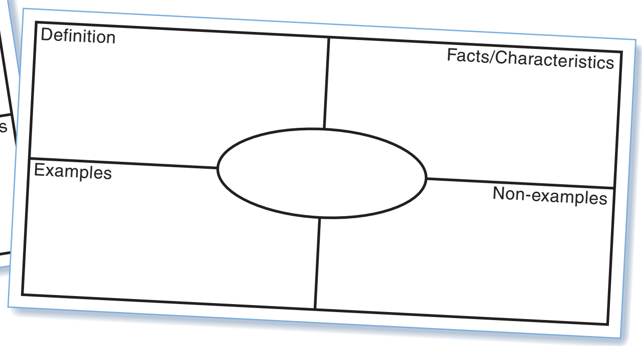
Work with a partner.

1. With your partner, make a Frayer Model for 3 of these words.

- Mean
- Median
- Mode
- Range
- Outliers
- Independent events
- Probability
- Tree diagram

Choose the Frayer Model type that best suits each word.





This model gives the definition of the word, then lists its characteristics.

2. Work on your own.
Choose a word you did not use in question 1.
Each of you should choose a different word.
Make a Frayer Model.

3. Share your Frayer Model with your partner.
- Suggest ways your partner could improve his Frayer Model.
- Make changes to reflect your partner's suggestions.

4. Discuss with your partner.
- Did making Frayer Models help you understand the new words and ideas? Explain.
- Do you think your Frayer Models will be a useful review tool? Explain.

Unit Review

What Do I Need to Know?

- Each of the *mean*, *median*, and *mode* is a measure of central tendency, or an average.

- In a set of data:
  The *mean* is the sum of the numbers divided by the number of numbers in the set.
  The *median* is the middle number when the data are arranged in order. When there is an odd number of data in the set, the median is the middle number. When there is an even number of data, the median is the mean of the two middle numbers.
  The *mode* is the number that occurs most often.
  A set of data can have no mode, one mode, or more than one mode.
  The *range* is the difference between the greatest and least numbers.
  An *outlier* is a number that is much greater than or much less than most of the numbers in the data set.

- Probability Range:
  The probability of an event that is impossible is 0, or 0%.
  The probability of an event that is certain is 1, or 100%.
  All probabilities lie within this range.

- Two events are independent if the result of one event does not depend on the result of the other event.

What Should I Be Able to Do?

LESSON

7.1
7.2

1. To celebrate his birthday, Justin and his friends played miniature golf. Here are their scores:
29, 33, 37, 24, 41, 38, 48, 26, 36, 33,
40, 29, 36, 22, 31, 38, 42, 35, 33
It was a par 36 course.
This means that a good golfer takes 36 strokes to complete the course.

a) How many scores were under par? At par? Over par?
b) Find the range of the scores.
c) Calculate the mean, median, and mode scores.

LESSON

7.2 **2.** The median shoe size of eight 12-year-old boys is $6\frac{1}{2}$. What might the shoe sizes be? Explain your answer.

7.3 **3.** Josephine recorded the hours she worked each week at her part-time job, for 10 weeks.
Here are the hours:
15, 12, 16, 10, 15, 15, 3, 18, 12, 10
a) Calculate the mean, median, and mode hours.
b) Identify the outlier. How might you explain this value?
c) Calculate the mean, median, and mode hours without the outlier. How is each measure affected when the outlier is not included?
d) Should the outlier be used in reporting the average number of hours Josephine worked? Explain.

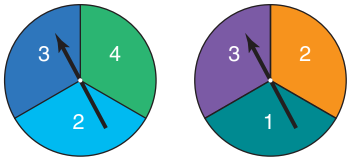
7.4 **4.** The times, in minutes, that 14 students spent doing math homework over the weekend are:
27, 36, 48, 35, 8, 40, 41,
39, 74, 47, 44, 125, 37, 47
a) Calculate the mean, median, and mode for the data.
b) What are the outliers?
Justify your choice.
Calculate the mean without the outliers.
What do you notice? Explain.
c) Which average best describes the data? Explain.
d) Should the outliers be used in reporting the average? Explain.

5. Annette's practice times for a downhill ski run, in seconds, are:
122, 137, 118, 119, 124, 118, 120, 118
a) Find the mean, median, and mode times.
b) Which measure best represents the times? Explain.
c) What is the range?
d) What time must Annette get in her next run so the median is 120 s? Explain.
e) What time must Annette get in her next run so the mean is 121 s? Is this possible? Explain.

6. Which average best describes a typical item in each data set? Explain.
a) size of sandals sold in a shoe store
b) test scores of a Grade 7 class
c) salaries in a company with 1000 employees
d) salaries in a company with 6 employees

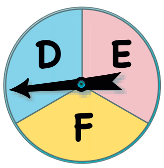
7.5 **7.** Twenty cards are numbered from 1 to 20. The cards are shuffled. A card is drawn. Find the probability that the card has:
a) an odd number
b) a multiple of 4
c) a prime number
d) a number greater than 30
e) a number divisible by 1
Express each probability as many ways as you can.

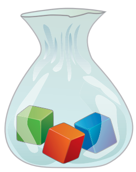
8. In a board game, players take turns to spin pointers on these spinners. The numbers the pointers land on are multiplied.
The player moves that number of squares on the board.



a) List the possible products.
b) What is the probability of each product in part a?
c) Which products are equally likely? Explain.
d) What is the probability of a product that is less than 10? Explain.

9. A spinner has 3 equal sectors labelled D, E, and F. A bag contains 3 congruent cubes: 1 green, 1 red, and 1 blue





The pointer is spun and a cube is picked at random.
a) Use a tree diagram to list the possible outcomes.
b) What is the probability of:
   i) spinning E?
   ii) picking a green cube?
   iii) spinning E and picking a green cube?
   iv) spinning D and picking a red cube?

10. Conduct the experiment in question 9.
a) Record the results for 10 trials.
   i) State the experimental probability for each event in question 9, part b.
   ii) How do the experimental and theoretical probabilities compare?
b) Combine your results with those of 9 other students. You now have the results of 100 trials. Repeat part a.
c) What happens to the experimental and theoretical probabilities of an event when the experiment is repeated hundreds of times?

11. The student council runs a coin toss game during School Spirit week.
All the profits go to charity.
Each player pays 50¢ to play.
A player tosses two coins and wins a prize if the coins match.
Each prize is $2.00. Can the student council expect to make a profit? Justify your answer.

Practice Test

1. The data show the time, in seconds, for swimmers to swim a 400-m freestyle race.
208, 176, 265, 222, 333, 237, 225, 269, 303, 295, 238, 175, 257, 208, 271, 210

a) What is the mean time?
b) What is the mode time?
c) What is the range of the times?
d) What is the median time?

2. A sports store carries women's ice skates, sizes 4 to 10.
In a particular 4-h period, these skate sizes are sold:
4, 9, 8.5, 7.5, 6, 7, 6.5, 7, 7.5, 9, 8, 8, 18, 6.5, 8.5, 7, 5, 7, 9.5, 7

a) Calculate the mean, median, and mode sizes.
Which average best represents the data? Explain.
b) Identify the outlier. How can you explain this size?
c) Calculate the mean, median, and mode sizes without the outlier.
How is each average affected when the outlier is not included?
d) Should the outlier be used when the sales clerk reports the average skate size sold? Explain.

3. Match each probability to an event listed below.

a) 0 **b)** $\frac{1}{2}$ **c)** 100% **d)** 1:4

i) You roll an even number on a die labelled 1 to 6.
ii) You pick an orange out of a bag of apples.
iii) You draw a red counter from a bag that contains 3 yellow counters and 1 red counter.
iv) You roll a number less than 7 on a die labelled 1 to 6.

4. Two dice are rolled. Each die is labelled 1 to 6.
The lesser number is subtracted from the greater number, to get the difference. For example: $5 - 3 = 2$

a) List the possible differences.
b) Express the probability of each difference as a ratio, fraction, and percent.
i) exactly 1 **ii)** greater than 3 **iii)** an odd number
c) Carry out this experiment 10 times. Record your results.
d) What is the experimental probability of each event in part b?
e) Compare the theoretical and experimental probabilities.
What do you notice?

Unit Problem Board Games

Many board games involve rolling pairs of dice, labelled 1 to 6.
Suppose you are on a particular square of a game board.
You roll the dice. You add the numbers on the uppermost faces.
How likely are you to roll 7?
Investigate to find out.

Part 1

Work with a partner.
Copy and complete this table.
Show the sums when two dice are rolled.

When one die shows 1 and the other die shows 4, then the sum is 5.

Sum of Numbers on Two Dice

Number on Die	1	2	3	4	5	6
1	2	3	4	5	6	7
2				6		
3						
4	5					
5						
6						

How many different outcomes are there?
In how many ways can the sum be 6? 9? 2? 12?
Why do you think a table was used instead of a tree diagram?
Find the theoretical probability for each outcome.

Part 2

Work in groups of four.

1. Choose one of the following options:
   - Each of you rolls a pair of dice 25 times.
     Combine the results of your group.
   - Use a website your teacher gives you. Use the website to simulate rolling a pair of dice 100 times.

   Record the sum each time.

   Calculate the experimental probability of each outcome after 100 rolls.

2. Repeat *Step 1* three more times, to get results for 200 rolls, 300 rolls, and 400 rolls.
Calculate the experimental probability of each outcome after each number of rolls.
Summarize your results for 400 rolls in a table.

Sum	Number of Times Sum Occurred	Experimental Probability	Theoretical Probability
2			
3			
4			
5			

Find the mode of the sum rolled.
You can use a software program to help.
What does this tell you?

Check List

Your work should show:

- ✓ all records and calculations in detail
- ✓ clear explanations of your results, with the correct use of language of data analysis
- ✓ tables to show outcomes and probabilities
- ✓ an explanation of who is more likely to win

Part 3

What happens to the experimental probabilities as the number of rolls increases?

How does the experimental probability of each outcome compare with the theoretical probability? Explain.

Part 4

To win a board game, you must land on "home."
Suppose you are 7 squares from "home."
Your opponent is 4 squares from "home."
Who is more likely to win on the next roll? Explain.
Why do you think 7 is a lucky number?

Reflect on Your Learning

Why do you think you are learning about data analysis?

UNIT 8

Geometry

Many artists use geometric concepts in their work.

Think about what you have learned in geometry. How do these examples of First Nations art and architecture show geometry ideas?

What You'll Learn

- Identify and construct parallel and perpendicular line segments.
- Construct perpendicular bisectors and angle bisectors, and verify the constructions.
- Identify and plot points in the four quadrants of a grid.
- Graph and describe transformations of a shape on a grid.

Why It's Important

- A knowledge of the geometry of lines and angles is required in art and sports, and in careers such as carpentry, plumbing, welding, engineering, interior design, and architecture.

Key Words

- parallel lines
- perpendicular lines
- line segment
- bisect
- bisector
- perpendicular bisector
- angle bisector
- coordinate grid
- Cartesian plane
- *x*-axis
- *y*-axis
- origin
- quadrants

8.1 Parallel Lines

Focus Use different methods to construct and identify parallel line segments.

Identify parallel line segments in these photos. How could you check they are parallel?

Explore

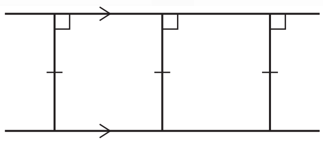
You may need a ruler, plastic triangle, tracing paper, protractor, and Mira.
Use any methods or tools. Draw a line segment on plain paper.
Draw a line segment parallel to the line segment.
Find as many ways to do this as you can using different tools.

Reflect & Share

Compare your methods with those of another pair of classmates.
How do you know the line segment you drew is parallel to the line segment?
Which method is most accurate? Explain your choice.

Connect

Parallel lines are lines on the same flat surface that never meet.
They are always the same distance apart.



Here are 3 strategies to draw a line segment parallel to a given line segment.

➤ Use a ruler.
Place one edge of a ruler along the line segment.
Draw a line segment along the other edge of the ruler.

➤ Use a ruler and protractor.
Choose a point on the line segment.
Place the centre of the protractor on the point.

Align the base line of the protractor with the line segment.
Mark a point at 90°.
Repeat this step once more.
Join the 2 points to draw a line segment parallel to the line segment.

➤ Use a ruler and compass as shown below.

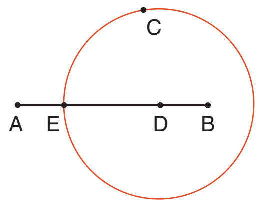
Example

Use a ruler and compass to draw a line segment parallel to line segment AB that passes through point C.

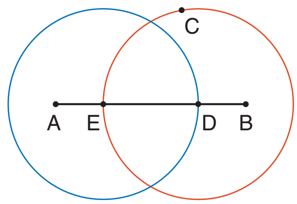
A *line segment* is the part of a line between two points on the line.

A Solution

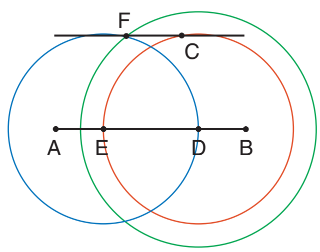
- Mark any point D on AB.



- Place the compass point on D.
  Set the compass so the pencil point is on C.
  Draw a circle.
  Label point E where the circle intersects AB.



- Do not change the distance between the compass and pencil points.
  Place the compass point on E.
  Draw a circle through D.
- Place the compass point on E.
  Set the compass so the pencil point is on C.



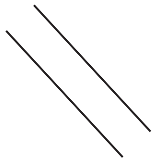
- Place the compass point on D.
  Draw a circle to intersect the circle through D.
  Label the point of intersection F.
- Draw a line through points C and F.
  Line segment CF is parallel to AB.

The 2 line segments are parallel because they are always the same distance apart.

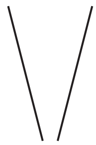
Practice

1. Which lines are parallel? How do you know?

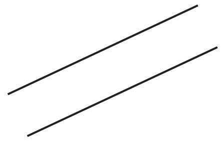
a)



b)



c)



2. a) Draw line segment CD of length 5 cm.
Use a ruler to draw a line segment parallel to CD.

b) Choose 3 different points on CD.
Measure the shortest distance from each point to the line segment you drew.
What do you notice?

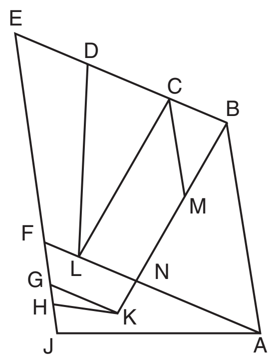
3. Draw line segment EF of length 8 cm.

a) Use a ruler and protractor to draw a line segment parallel to EF.

b) Use a ruler and compass to draw a line segment parallel to EF.

4. Suppose there are 2 line segments that look parallel.
How could you tell if they are parallel?

5. Make a list of where you see parallel line segments in your community or around the house.
Sketch diagrams to illustrate your list.



6. **Assessment Focus** Your teacher will give you a large copy of this diagram.
Find as many pairs of parallel line segments as you can.
How do you know they are parallel?

7. **Take It Further** Draw line segment CD.
Use what you know about drawing parallel line segments to construct parallelogram CDEF.
Explain how you can check you have drawn a parallelogram.

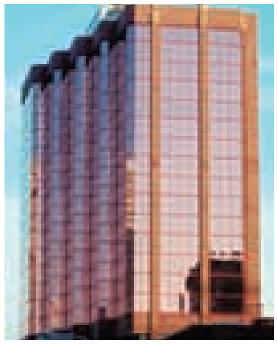
Reflect

Describe 3 different methods you can use to draw a line segment parallel to a given line segment. Which method do you prefer? Which method is most accurate? Explain your choice.

8.2 Perpendicular Lines

Focus Use different methods to construct and identify perpendicular line segments.

Identify perpendicular line segments in these photos. How could you check they are perpendicular?



Explore

You may need a ruler, plastic triangle, protractor, and Mira.
Use any methods or tools. Draw a line segment on plain paper.
Draw a line segment perpendicular to the line segment.
Find as many ways to do this as you can using different tools.

Reflect & Share

Compare your methods with those of another pair of classmates. How do you know the line segment you drew is perpendicular to the line segment?
Which method is most accurate? Explain your choice.

Recall that 2 lines intersect if they meet or cross.

Connect

Two line segments are **perpendicular** if they intersect at right angles.
Here are 5 strategies to draw a line segment perpendicular to a given line segment.

➤ Use a plastic right triangle.
Place the base of the triangle along the line segment.
Draw a line segment along the side that is the height of the triangle.

➤ Use paper folding.
Fold the paper so that the line segment coincides with itself. Open the paper.
The fold line is perpendicular to the line segment.

➤ Use a ruler and protractor.
Choose a point on the line segment.
Place the centre of the protractor on the point.
Align the base line of the protractor with the line segment. Mark a point at 90°.
Join the 2 points to draw a line segment perpendicular to the line segment.

➤ Use a Mira. Place the Mira so that the reflection of the line segment coincides with itself when you look in the Mira.
Draw a line segment along the edge of the Mira.

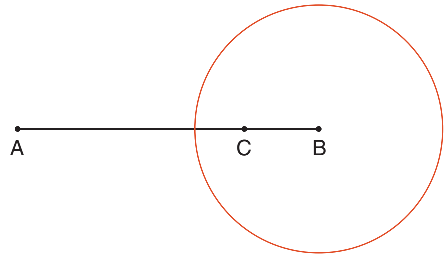
➤ Use a ruler and compass as shown below.

Example

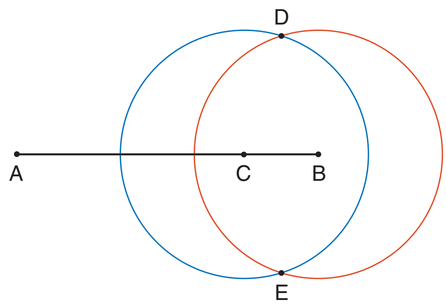
Use a ruler and compass to draw a line segment perpendicular to line segment AB.

A Solution

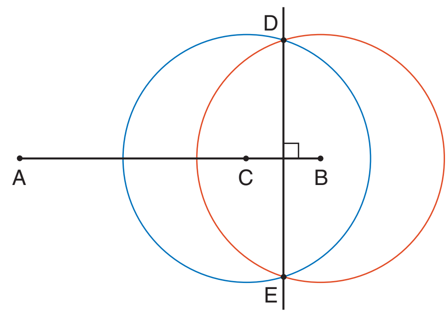
- Mark a point C on AB.



- Set the compass so the distance between the compass and pencil points is greater than one-half the length of CB.
  Place the compass point on B.
  Draw a circle that intersects AB.



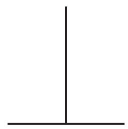
- Do not change the distance between the compass and pencil points.
  Place the compass point on C.
  Draw a circle to intersect the first circle you drew.
  Label the points D and E where the circles intersect.
- Draw a line through points D and E.
  DE is perpendicular to AB.

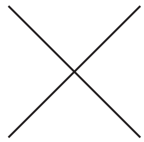


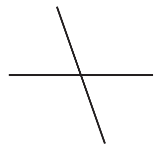
To check, measure the angles to make sure each is 90°.

Practice

1. Which lines are perpendicular? How do you know?

a) 

b) 

c) 

2. a) Draw line segment AB of length 6 cm.
Use a Mira to draw a line segment perpendicular to AB.

b) Draw line segment CD of length 8 cm. Mark a point on the segment.
Use paper folding to construct a line segment perpendicular to CD that passes through the point.
How do you know that each line segment you drew is perpendicular to the line segment?

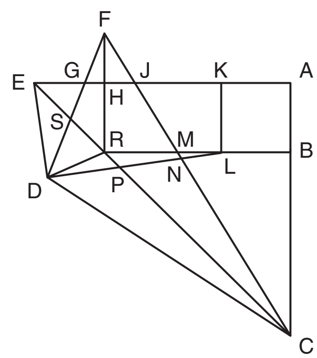
3. Draw line segment EF of length 10 cm.

a) Use a ruler and protractor to draw a line segment perpendicular to EF.

b) Use a ruler and compass to draw a line segment perpendicular to EF.

c) Check that the line segments you drew are perpendicular to EF.

4. Make a list of where you see perpendicular line segments in the world around you. Sketch diagrams to illustrate your list.



5. **Assessment Focus** Your teacher will give you a large copy of this diagram.
Find as many pairs of perpendicular line segments as you can.
How do you know they are perpendicular?

6. **Take It Further** Draw line segment JK of length 10 cm.
Use what you know about drawing perpendicular and parallel line segments to construct a rectangle JKMN, where KM is 4 cm.
Explain how you can check you have drawn a rectangle.

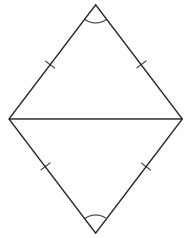
Reflect

Describe 4 different methods you can use to draw a line perpendicular to a given line segment.
Which method do you prefer?
Which method is most accurate? Explain your choice.

8.3 Constructing Perpendicular Bisectors

Focus Use a variety of methods to construct perpendicular bisectors of line segments.

Recall that a rhombus has all sides equal and opposite angles equal.



Each diagonal divides the rhombus
into 2 congruent isosceles triangles.
How do you know the triangles are isosceles?
How do you know the triangles are congruent?

You will investigate ways to cut line segments into 2 equal parts.

Explore

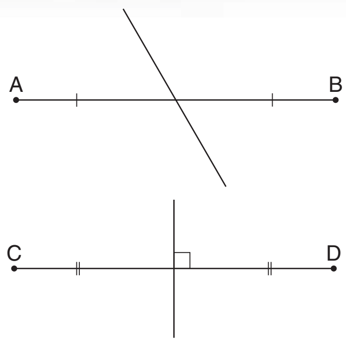
You may need rulers, protractors, tracing paper, plain paper, and Miras.
Use any methods or tools.
Draw a line segment on plain paper.
Draw a line segment perpendicular to the line segment that cuts the line segment in half.

Reflect & Share

Compare your results and methods with those of another pair of classmates.
How could you use your method to cut your classmate's line segment in half?

Connect

When you draw a line to divide a line segment into two equal parts, you **bisect** the segment.
The line you drew is a **bisector** of the segment.



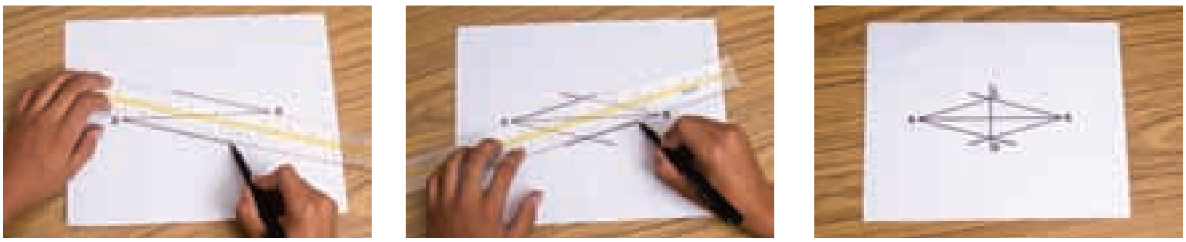
When the bisector is drawn at right angles to the segment, the line is the **perpendicular bisector** of the segment.

Here are 3 strategies to draw the perpendicular bisector of a given line segment.

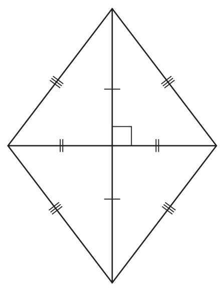
➤ Use paper folding.
Fold the paper so that point A
lies on point B.
Crease along the fold. Open the paper.
The fold line is the perpendicular
bisector of AB.

➤ Use a Mira. Place the Mira so that the reflection
of point A lies on point B.
Draw a line segment along the edge of the Mira.

➤ Use a ruler. Place the ruler so that A is on one side
of the ruler and B is on the other.
Draw line segments along both edges of the ruler.
Repeat this step once more so that A and B are now
on opposite sides of the ruler.
Draw line segments along both edges of the ruler.
Label the points C and D where the line segments you drew intersect.
Join CD. CD is the perpendicular bisector of AB.



Recall that each diagonal of a rhombus is a line of symmetry.
The diagonals intersect at right angles.
The diagonals bisect each other.
So, each diagonal is the perpendicular bisector of
the other diagonal.



We can use these properties of a rhombus to construct
the perpendicular bisector of a line segment.
Think of the line segment as a diagonal of a rhombus.
As we construct the rhombus, we also construct the
perpendicular bisector of the segment.

Example

Use a ruler and compass to draw the perpendicular bisector of any line segment AB.

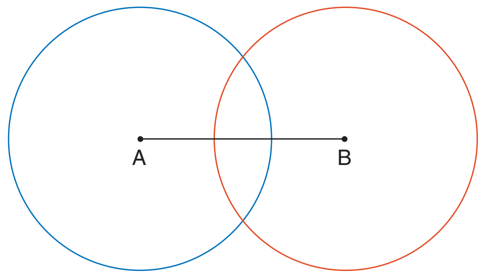
A Solution

Use a ruler and compass.

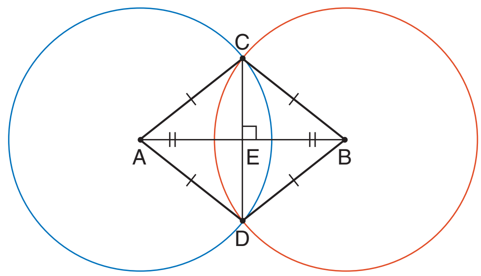
- Draw any line segment AB.

- Set the compass so the distance between the compass and pencil points is greater than one-half the length of AB.

- Place the compass point on A.
  Draw a circle.
  Do not change the distance between the compass and pencil points.
  Place the compass point on B.
  Draw a circle to intersect the first circle you drew.



- Label the points C and D where the circles intersect.
  Join the points to form rhombus ACBD.
  Draw the diagonal CD.
  The diagonals intersect at E.
  CD is the perpendicular bisector of AB.
  That is, AE = EB and ∠AEC = ∠CEB = 90°



To check that the perpendicular bisector has been drawn correctly, measure the two parts of the segment to check they are equal, and measure the angles to check each is 90°.

Note that any point on the perpendicular bisector of a line segment is the same distance from the endpoints of the segment.
For example, AC = BC and AD = BD

Practice

Show all construction lines.

1. a) Draw line segment CD of length 8 cm.
Use paper folding to draw its perpendicular bisector.

b) Choose three different points on the bisector.
Measure the distance to each point from C and from D.
What do you notice?

2. a) Draw line segment EF of length 6 cm.
Use a Mira to draw its perpendicular bisector.
b) How do you know that you have drawn the perpendicular bisector of EF?

3. Draw line segment GH of length 4 cm.
Use a ruler to draw its perpendicular bisector.

4. a) Draw line segment AB of length 5 cm.
Use a ruler and compass to draw its perpendicular bisector.
b) Choose three different points on the bisector.
Measure the distance to each point from A and from B.
What do you notice? Explain.

5. Find out what happens if you try to draw the perpendicular bisector of a line segment when the distance between the compass and pencil points is:
a) equal to one-half the length of the segment
b) less than one-half the length of the segment

6. **Assessment Focus** Draw line segment RS of length 7 cm.
Use what you know about perpendicular bisectors to construct rhombus RTSU.
How can you check that you have drawn a rhombus?

7. Look around you. Give examples of perpendicular bisectors.

8. **Take It Further** Draw a large △PQR.
Construct the perpendicular bisector of each side.
Label point C where the bisectors meet.
Draw the circle with centre C and radius CP.

"Circum" is Latin for "around." So, the *circumcircle* is the circle that goes around a triangle.

9. **Take It Further**
a) How could you use the construction in question 8 to draw a circle through any 3 points that do not lie on a line?
b) Mark 3 points as far apart as possible. Draw a circle through the points. Describe your construction.

The point at which the perpendicular bisectors of the sides of a triangle intersect is called the *circumcentre*.

Reflect

How many bisectors can a line segment have?
How many perpendicular bisectors can a line segment have?
Draw a diagram to illustrate each answer.

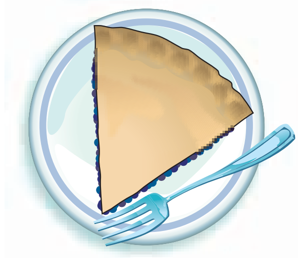
8.4 Constructing Angle Bisectors

Focus Use a variety of methods to construct bisectors of angles.

You will investigate ways to divide an angle into 2 equal parts.

Explore

Your teacher will give you a large copy of this picture.
You may need rulers, protractors, tracing paper, plain paper, and Miras.
Use any methods or tools.
George wants to share this slice of pie equally with a friend.
Show how he could divide the slice of pie into 2 equal parts.



Reflect & Share

Compare your results and methods with those of another pair of classmates.
How could you use your classmates' methods to divide the slice of pie in half?

Connect

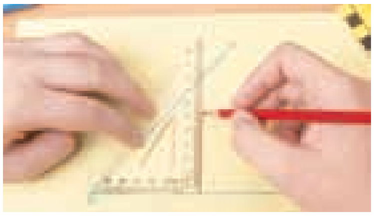
When you divide an angle into two equal parts, you *bisect* the angle.

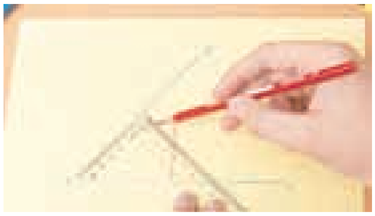
Here are 3 strategies to draw the bisector of a given angle.

➤ Use paper folding.
Fold the paper so that XY lies along ZY.
Crease along the fold line.
Open the paper.
The fold line is the bisector of ∠XYZ.

➤ Use a Mira. Place the Mira so that the reflection of one arm of the angle lies along the other arm.
Draw a line segment along the edge of the Mira.
This line segment is the bisector of the angle.

➤ Use a plastic right triangle.
Place the triangle with one angle at B and one edge along BC.
Draw a line segment.
Place the triangle with the same angle at B and the same edge along AB.
Draw a line segment.
Label M where the line segments you drew intersect. BM is the bisector of ∠ABC.





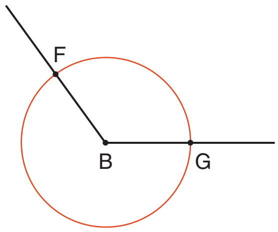
We can use the properties of a rhombus to construct the bisector of an angle.
Think of the angle as one angle of a rhombus.

Example

Draw obtuse ∠B of measure 126°.
Use a ruler and a compass to bisect the angle.
Measure the angles to check.

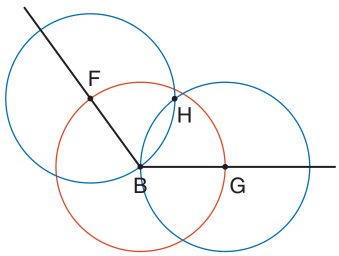
A Solution

Use a ruler and protractor to draw ∠B = 126°.
Use ∠B as one angle of a rhombus.
With compass point on B, draw a circle that intersects one arm at F and the other arm at G.

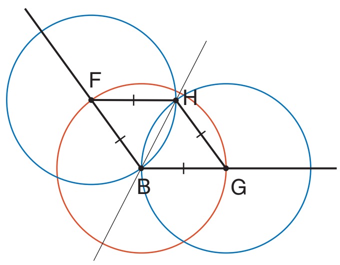


FB and BG are 2 sides of the rhombus; FB = BG

Do not change the distance between the compass and pencil points.
Place the compass point on F.
Draw a circle.
Place the compass point on G.
Draw a circle to intersect the second circle you drew.
Label the point H where the circles intersect.



Join FH and HG to form rhombus BFHG.
Draw a line through BH.
This line is the **angle bisector** of ∠FBG.
That is, ∠FBH = ∠HBG



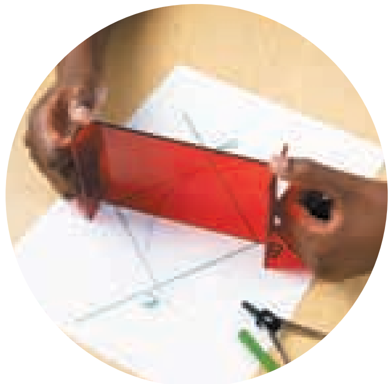
FH and HG are the other 2 sides of the rhombus.

BH is a diagonal of the rhombus.

Use a protractor to check. Measure each angle.
$\angle FBG = 126°$
$\angle FBH = 63°$ and $\angle GBH = 63°$
$\angle FBH + \angle GBH = 63° + 63°$
$= 126°$
$= \angle FBG$

To check that the bisector of an angle has been drawn correctly, we can:

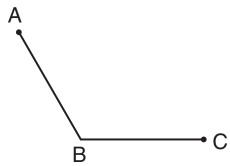
- Measure the two angles formed by the bisector. They should be equal.
- Fold the angle so the bisector is the fold line. The two arms should coincide.
- Place a Mira along the angle bisector. The reflection image of one arm of the angle should coincide with the other arm, and vice versa.



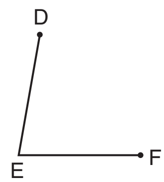
Practice

Show any construction lines.

1. Your teacher will give you a copy of this obtuse angle.
Use a Mira to bisect the angle.
Measure the two parts of the angle.
Are they equal?



2. Your teacher will give you a copy of this acute angle.
Use a plastic right triangle to bisect the angle.
Measure the two parts of the angle.
Are they equal?



3. Use a ruler and compass.
a) Draw acute $\angle PQR = 50°$. Bisect the angle.
b) Draw obtuse $\angle GEF = 130°$. Bisect the angle.

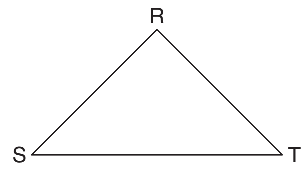
4. Draw a reflex angle of measure 270°.
a) How many different methods can you find to bisect this angle?
b) Describe each method.
Check that the bisector you draw using each method is correct.

A *reflex angle* is an angle between 180° and 360°.

5. You have used Miras, triangles, and paper folding to bisect an angle.
What is the advantage of using a ruler and compass?

6. a) Draw line segment HJ of length 8 cm.
Draw the perpendicular bisector of HJ.
b) Bisect each right angle in part a.
c) How many angle bisectors did you need to draw in part b?
Explain why you needed this many bisectors.

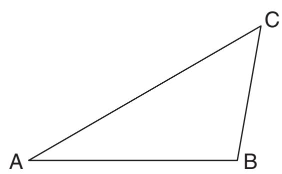
7. **Assessment Focus** Your teacher will give you a large copy of this isosceles triangle. Use a ruler and compass.



a) Bisect ∠R.
b) Show that the bisector in part a is the perpendicular bisector of ST.
c) Is the result in part b true for:
i) a different isosceles triangle?
ii) an equilateral triangle?
iii) a scalene triangle?
How could you find out? Show your work.

8. Describe examples of angle bisectors that you see in the environment.

9. **Take It Further** Your teacher will give you a copy of this triangle.
Cut it out.
Fold the triangle so BC and BA coincide. Open the triangle.
Fold it so AB and AC coincide. Open the triangle.
Fold it so AC and BC coincide. Open the triangle.



a) Measure the angles each crease makes at each vertex.
What do you notice?
b) Label point K where the creases meet.
Draw a circle in the triangle that touches each side of △ABC.
What do you notice?
c) What have you constructed by folding?

Reflect

How many bisectors can an angle have?
Draw a diagram to illustrate your answer.

Mid-Unit Review

LESSON

8.1

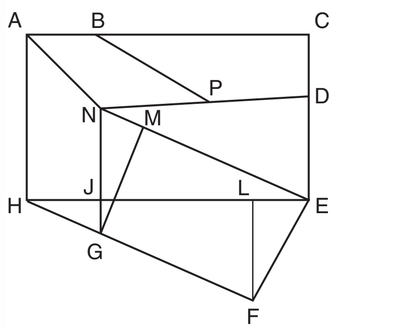
1. a) Draw a line segment CD of length 9 cm. Draw a point F not on the line segment.
Use a ruler and compass to construct a line segment parallel to CD that passes through F.

b) Draw line segment CD again.
Use a different method to construct a line segment parallel to CD that passes through F.

c) Which method is more accurate?
Explain you choice.

8.2

2. Your teacher will give you a large copy of this picture.



a) Identify as many parallel line segments as you can.
How do you know they are parallel?

b) Find as many perpendicular line segments as you can.
How do you know they are perpendicular?

8.3

3. a) Draw line segment AB of length 10 cm. Use a ruler and compass to draw the perpendicular bisector of AB.

b) Draw line segment AB again.
Use a different method to draw the perpendicular bisector.

c) How can you check that you have drawn each bisector correctly?

4. a) Draw line segment AB of length 6 cm. AB is the base of a triangle.

b) Construct the perpendicular bisector of AB. Label point C where the perpendicular bisector intersects AB.
Mark a point D on the perpendicular bisector.
Join AD and DB.

c) What kind of triangle have you drawn? How do you know?
What does CD represent?

8.4

5. a) Draw obtuse $\angle PQR = 140°$.
Use a ruler and compass to bisect $\angle PQR$.

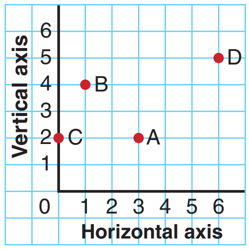
b) Draw acute $\angle CDE = 50°$.
Use a different method to bisect $\angle CDE$.

c) How can you check that you have drawn each bisector correctly?

8.5 Graphing on a Coordinate Grid

Focus Identify and plot points in four quadrants of a coordinate grid.

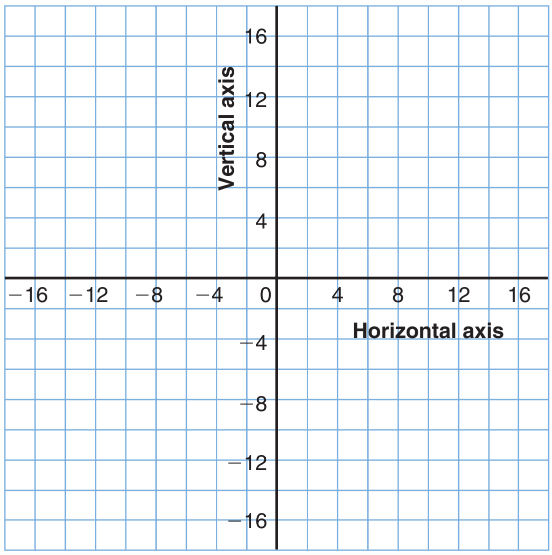
You have plotted points with whole-number coordinates on a grid.
Point A has coordinates (3, 2).
What are the coordinates of point B? Point C? Point D?



A vertical number line and a horizontal number line intersect at right angles at 0.
This produces a grid on which you can plot points with integer coordinates.

Explore

You will need grid paper and a ruler. Copy this grid.



- Plot these points: A(14, 0), B(6, 2), C(8, 8), D(2, 6), E(0, 14)
  Join the points in order.
  Draw a line segment from each point to the origin.
- Reflect the shape in the vertical axis.
  Draw its image.
  Write the coordinates of each vertex of the image.
- Reflect the original shape and the image in the horizontal axis.
  Draw the new image.
  Write the coordinates of each vertex of the new image.

Your design should be symmetrical about the horizontal and vertical axes.
Describe the design. What shapes do you see?

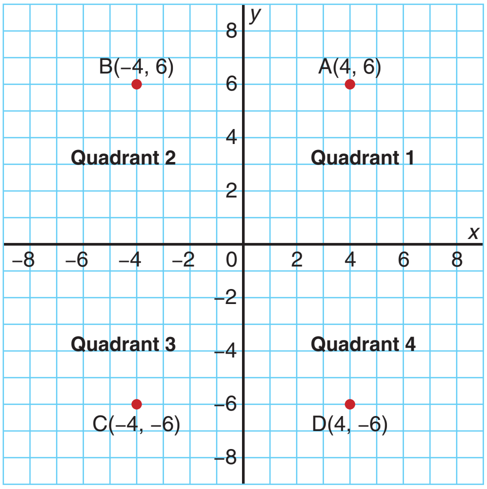
Reflect & Share

Compare your design and its coordinates with those of another pair of classmates.
Describe any patterns you see in the coordinates of corresponding points.

Connect

A vertical number line and a horizontal number line that intersect at right angles at 0 form a **coordinate grid**.
The horizontal axis is the ***x*-axis**.
The vertical axis is the ***y*-axis**.
The axes meet at the **origin**, (0, 0).
The axes divide the plane into four **quadrants**.
They are numbered counterclockwise.

This coordinate grid is also called a *Cartesian plane*.



We do not need arrows on the axes.

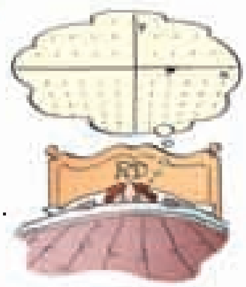
A pair of coordinates is called an *ordered pair*.

In Quadrant 1, to plot point A, start at 4 on the *x*-axis and move up 6 units.
Point A has coordinates (4, 6).
In Quadrant 2, to plot point B, start at −4 on the *x*-axis and move up 6 units.
Point B has coordinates (−4, 6).
In Quadrant 3, to plot point C, start at −4 on the *x*-axis and move down 6 units.
Point C has coordinates (−4, −6).
In Quadrant 4, to plot point D, start at 4 on the *x*-axis and move down 6 units.
Point D has coordinates (4, −6).

We do not have to include a + sign for a positive coordinate.

Math Link

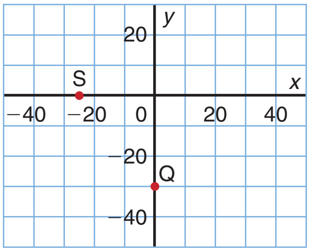
History
René Descartes lived in the 17th century.
He developed the coordinate grid.
It is named the Cartesian plane in his honour.
There is a story that René was lying in bed and watching a fly on the ceiling.
He invented coordinates as a way to describe the fly's position.



Example

a) Write the coordinates of each point.

i) Q **ii)** S



Notice that each grid square represents 10 units.

b) Plot each point on a grid.

i) F(0, −15) **ii)** G(−40, 0)

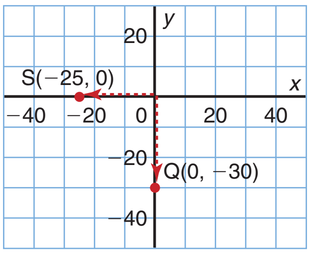
A Solution

a) Start at the origin each time.

i) To get to Q, move 0 units right and 30 units down.
So, the coordinates of Q are (0, −30).

ii) To get to S, move 25 units left and 0 units down.
So, the coordinates of S are (−25, 0).

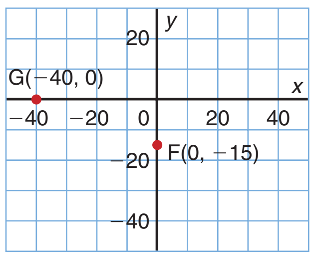
Remember, first move left or right, then up or down.



Point S is halfway between –20 and –30 on the *x*-axis.

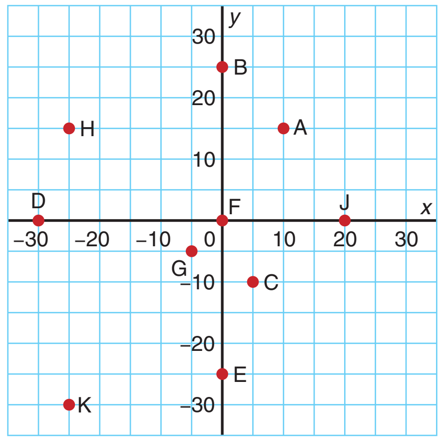
b) **i)** F(0, −15)
Since there is no movement left or right, point F lies on the *y*-axis.
Start at the origin.
Move 15 units down the *y*-axis. Mark point F.
It is halfway between −10 and −20.

ii) G(−40, 0)
Start at −40 on the *x*-axis.
Since there is no movement up or down, point G lies on the *x*-axis. Mark point G.



Practice

1. What is the scale on each axis?
Write the coordinates of each point from A to K.



2. Use the coordinate grid to the right.
Which points have:

a) *x*-coordinate 0?
b) *y*-coordinate 0?
c) the same *x*-coordinate?
d) the same *y*-coordinate?
e) equal *x*- and *y*-coordinates?
f) *y*-coordinate 2?

3. Draw a coordinate grid. Look at the ordered pairs below.
Label the axes. How did you choose the scale?
Plot each point.

a) A(30, −30) **b)** B(25, 0) **c)** C(−10, 35)
d) D(−15, 40) **e)** E(15, 5) **f)** F(0, −20)
g) O(0, 0) **h)** H(−20, −5) **i)** I(−40, 0)

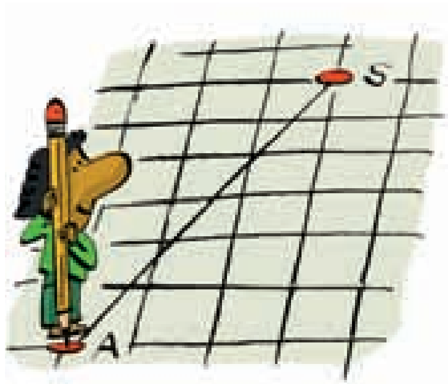
Which point is the origin?

4. How could you use the grid in question 3 to plot these points?

a) K(3, 5) **b)** P(−10, 2) **c)** R(−7, −8)

5. Which quadrant has all negative coordinates? All positive coordinates?
Both positive and negative coordinates?

6. a) Plot these points: A(0, 5), B(−1, 4), C(−1, 3), D(−2, 3), E(−3, 2), F(−2, 1), G(−1, 1), H(−1, 0), J(0, −1), K(1, 0), L(1, 1), M(2, 1), N(3, 2), P(2, 3), R(1, 3), S(1, 4)
b) Join the points in order. Then join S to A.
c) Describe the shape you have drawn.



7. Draw a design on a coordinate grid.
Each vertex should be at a point where grid lines meet.
List the points used to make the design, in order.
Trade lists with a classmate.
Use the list to draw your classmate's design.

8. Use a 1-cm grid.

a) Plot the points A(−3, 2) and B(5, 2).
Join the points to form line segment AB.
What is the horizontal distance between A and B?
How did you find this distance?

b) Plot the points C(3, −4) and D(3, 7).
Join the points to form line segment CD.
What is the vertical distance between C and D?
How did you find this distance?

9. Use question 8 as a guide.
Plot 2 points that lie on a horizontal or vertical line.
Trade points with a classmate.
Find the horizontal or vertical distance between your classmate's points.

10. **Assessment Focus** Use a coordinate grid.
How many different parallelograms can you draw that have area 12 square units?
For each parallelogram you draw, label its vertices.

11. a) Plot these points: K(−15, 20), L(5, 20), M(5, −10)

b) Find the coordinates of point N that forms rectangle KLMN.

12. a) Plot these points on a grid: A(16, −14), B(−6, 12), and C(−18, −14).
Join the points.
What scale did you use? Explain your choice.

b) Find the area of $\triangle$ABC.

13. **Take It Further** The points A(−4, 4) and B(2, 4) are two vertices of a square.
Plot these points on a coordinate grid.
What are the coordinates of the other two vertices?
Find as many different answers as you can.

Reflect

How did your knowledge of integers help you plot points on a Cartesian plane?

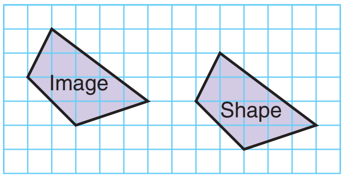
8.6 Graphing Translations and Reflections

Focus Graph translation and reflection images on a coordinate grid.

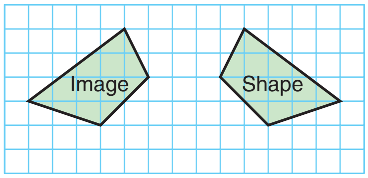
Recall that a translation slides a shape in a straight line. When the shape is on a square grid, the translation is described by movements right or left, and up or down.

A translation and a reflection are transformations.

Which translation moved this shape to its image?



A shape can also be reflected, or flipped, in a mirror line. Where is the mirror line that relates this shape and its image?



Explore

You will need 0.5-cm grid paper and a ruler.
Draw axes on the grid paper to get 4 quadrants.
Use the whole page. Label the axes.
Draw and label a quadrilateral.
Each vertex should be where the grid lines meet.

➤ Translate the quadrilateral. Draw and label the translation image. What do you notice about the quadrilateral and its image?

➤ Choose an axis. Reflect the quadrilateral in this axis. Draw and label the reflection image. What do you notice about the quadrilateral and its image?

➤ Trade your work with that of a classmate. Identify your classmate's translation. In which axis did your classmate reflect?

Reflect & Share

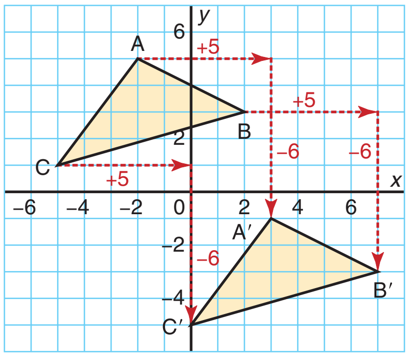
Did you correctly identify each transformation? Explain.
If not, work with your classmate to find the correct transformations.

Connect

➤ To translate △ABC 5 units right and 6 units down:
Begin at vertex A(−2, 5).
Move 5 units right and 6 units down to point A′(3, −1).
From vertex B(2, 3), move 5 units right and 6 units down to point B′(7, −3).
From vertex C(−5, 1), move 5 units right and 6 units down to point C′(0, −5).

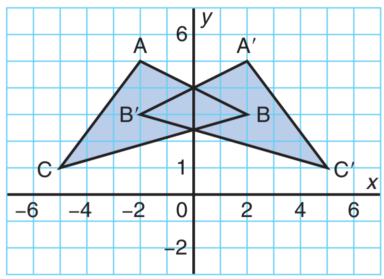
We read A′ as "A prime."

Each vertex of the image is labelled with a prime symbol.



Then, △A′B′C′ is the image of △ABC after a translation 5 units right and 6 units down.
△ABC and △A′B′C′ are congruent.

➤ To reflect △ABC in the *y*-axis:
Reflect each vertex in turn.
The reflection image of A(−2, 5) is A′(2, 5).
The reflection image of B(2, 3) is B′(−2, 3).
The reflection image of C(−5, 1) is C′(5, 1).



Then, △A′B′C′ is the image of △ABC after a reflection in the *y*-axis.
△ABC and △A′B′C′ are congruent.
The triangles have different orientations: we read △ABC clockwise; we read △A′B′C′ counterclockwise.

We could use a Mira to check the reflection.

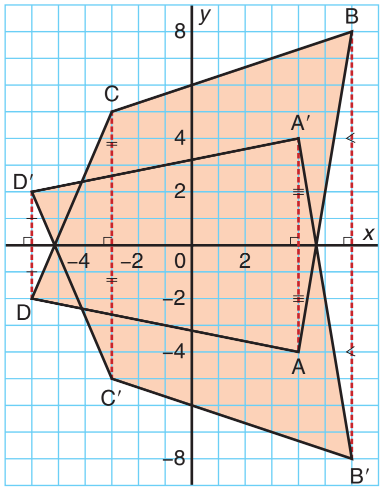
Example

a) Plot these points: A(4, −4), B(6, 8), C(−3, 5), D(−6, −2)
Join the points to draw quadrilateral ABCD.
Reflect the quadrilateral in the *x*-axis.
Draw and label the reflection image A′B′C′D′.

b) What do you notice about the line segment joining each point to its reflection image?

A Solution

a) To reflect quadrilateral ABCD in the *x*-axis:
Reflect each vertex in turn.
The reflection image of A(4, −4) is A′(4, 4).
The reflection image of B(6, 8) is B′(6, −8).
The reflection image of C(−3, 5) is C′(−3, −5).
The reflection image of D(−6, −2) is D′(−6, 2).



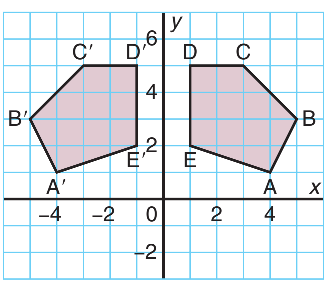
b) The line segments AA′, BB′, CC′, DD′ are vertical.
The *x*-axis is the perpendicular bisector of each line segment.
That is, the *x*-axis divides each line segment into 2 equal parts, and the *x*-axis intersects each line segment at right angles.

In *Practice* question 7, you will investigate a similar reflection in the *y*-axis.

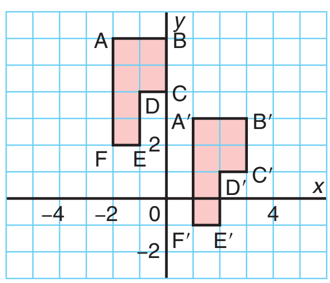
Practice

1. Identify each transformation. Explain your reasoning.

a)



b)

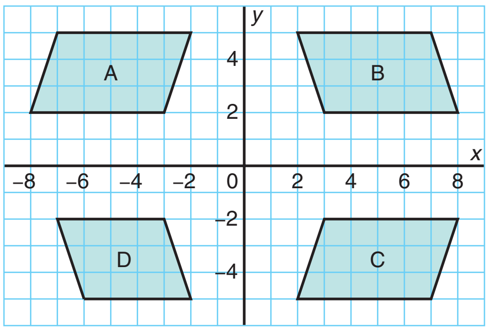


2. Describe the horizontal and vertical distance required to move each point to its image.

a) A(5, −3) to A′(2, 6) **b)** B(−3, 0) to B′(−5, −3) **c)** C(2, −1) to C′(4, 3)

d) D(−1, 2) to D′(−4, 0) **e)** E(3, 3) to E′(−3, 3) **f)** F(4, −2) to F′(4, 2)

3. The diagram shows 4 parallelograms.

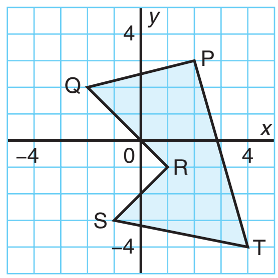


a) Are any 2 parallelograms related by a translation? If so, describe the translation.

b) Are any 2 parallelograms related by a reflection? If so, describe the reflection.

4. Copy this pentagon on grid paper.
Write the coordinates of each vertex.
After each transformation:

- Write the coordinates of the image of each vertex.
- Describe the positional change of the vertices of the pentagon.



a) Draw the image after a translation 3 units left and 2 units up.

b) Draw the image after a reflection in the *x*-axis.

c) Draw the image after a reflection in the *y*-axis.

5. Plot these points on a coordinate grid:
A(1, 3), B(3, −2), C(−2, 5), D(−1, −4), E(0, −3), F(−2, 0)

a) Reflect each point in the *x*-axis.
Write the coordinates of each point and its reflection image.
What patterns do you see in the coordinates?

b) Reflect each point in the *y*-axis.
Write the coordinates of each point and its reflection image.
What patterns do you see in the coordinates?

c) How could you use the patterns in parts a and b to check that you have drawn the reflection image of a shape correctly?

6. a) Plot the points in question 5.
Translate each point 4 units left and 2 units down.
b) Write the coordinates of each point and its translation image.
What patterns do you see in the coordinates?
c) How could you use these patterns to write the coordinates of an image point after a translation, without plotting the points?

7. a) Plot these points on a coordinate grid: P(1, 4), Q(−3, 4), R(−2, −3), S(5, −1)
Join the points to draw quadrilateral PQRS.
Reflect the quadrilateral in the *y*-axis.
b) What do you notice about the line segment joining each point to its image?

8. **Assessment Focus**

When there are 2 transformation images, we use a "double" prime notation for the vertices of the second image.

a) Plot these points on a coordinate grid:
A(2, 4), B(4, 4), C(4, 2), D(6, 2), E(6, 6)
Join the points to draw polygon ABCDE.
b) Translate the polygon 4 units right and 6 units up.
Write the coordinates of each vertex of the image polygon A′B′C′D′E′.
c) Reflect the image polygon A′B′C′D′E′ in the *y*-axis.
Write the coordinates of each vertex of the image polygon A″B″C″D″E″.
d) How does polygon A″B″C″D″E″ compare with polygon ABCDE?

9. a) Plot these points on a coordinate grid: F(−5, 8), G(0, 8), H(−1, 5), J(−5, 5)
Join the points to draw trapezoid FGHJ.
b) Translate the trapezoid 2 units right and 1 unit down.
c) Translate the image trapezoid F′G′H′J′ 2 units right and 1 unit down.
d) Repeat part c four more times for each resulting image trapezoid.
e) Describe the translation that moves trapezoid FGHJ to the final image.

10. **Take It Further** Draw a shape and its image that could represent a translation and a reflection. What attributes does the shape have?

Reflect

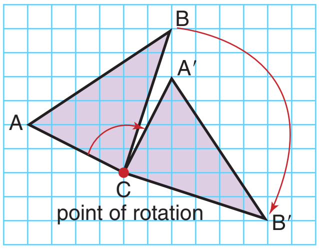
How is a translation different from a reflection?
How are these transformations alike?

8.7 Graphing Rotations

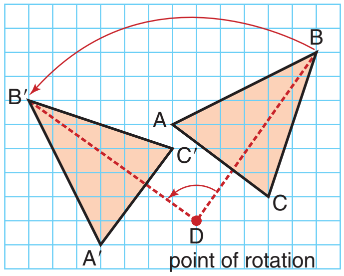
Focus Graph rotation images on a coordinate grid.

Recall that a rotation turns a shape about a point of rotation.
The rotation may be clockwise or counterclockwise.
The point of rotation may be:

On the shape



Off the shape



How would you describe each rotation?

Explore

You will need 0.5-cm grid paper, tracing paper, a protractor, and a ruler.
Draw axes on the grid paper to get 4 quadrants.
Place the origin at the centre of the paper.
Label the axes.
Plot these points: O(0, 0), B(5, 3), and C(5, 4)
Join the points in order, then join C to O.
Use the origin as the point of rotation.

- Rotate the shape 90° counterclockwise.
  Draw its image.
- Rotate the original shape 180° counterclockwise.
  Draw its image.
- Rotate the original shape 270° counterclockwise.
  Draw its image.

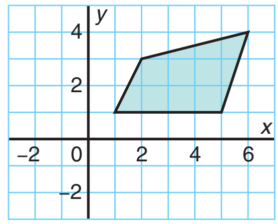
What do you notice about the shape and its 3 images?
What have you drawn?

Reflect & Share

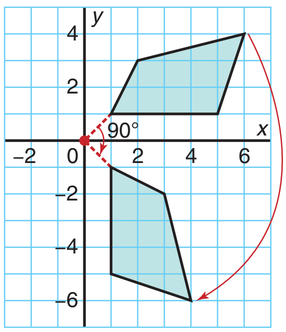
Compare your work with that of another pair of classmates.
What strategies did you use to measure the rotation angle?
Would the images have been different if you had rotated clockwise instead of counterclockwise? Explain your answer.

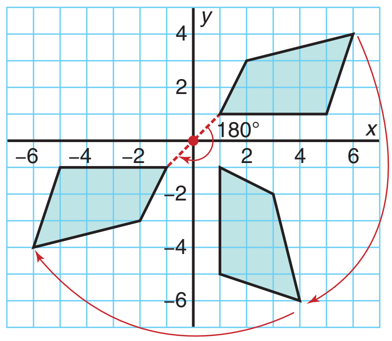
Connect

To rotate the shape at the right clockwise:



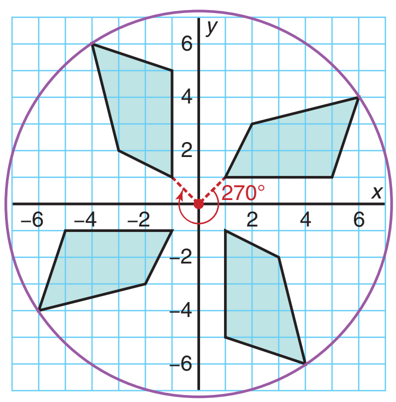
➤ Trace the shape and the axes.
Label the positive y-axis on the tracing paper.
Rotate the tracing paper clockwise about the origin until the positive y-axis coincides with the positive x-axis.
With a sharp pencil, mark the vertices of the image.
Join the vertices to draw the image after a 90° clockwise rotation about the origin, below left.





➤ Place the tracing paper so the shape coincides with its image.
Rotate the tracing paper clockwise about the origin until the positive y-axis coincides with the negative y-axis.
Mark the vertices of the image.
Join the vertices to draw the image of the original shape after a 180° clockwise rotation about the origin, above right.

➤ Place the tracing paper so the shape coincides with its second image.
Rotate the tracing paper clockwise about the origin until the positive y-axis coincides with the negative x-axis.
Mark, then join, the vertices of the image.
This is the image after a 270° clockwise rotation about the origin.



All 4 quadrilaterals are congruent.
A point and all its images lie on a circle, centre the origin.

Example

a) Plot these points: B(−5, 6), C(−3, 4), D(−8, 2)
Join the points to draw △BCD.
Rotate △BCD 90° about the origin, O.
Draw and label the rotation image △B′C′D′.

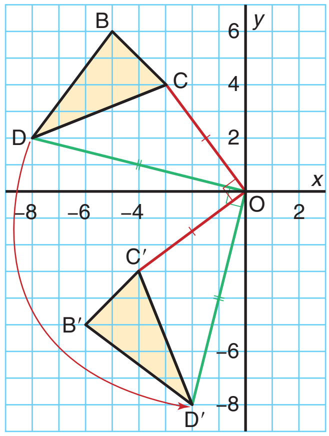
b) Join C, D, C′, D′ to O.
What do you notice about these line segments?

> A counterclockwise rotation is shown by a positive angle such as +90°, or 90°. A clockwise rotation is shown by a negative angle such as −90°.

A Solution

A rotation of 90° is a counterclockwise rotation.

a) Use tracing paper to draw the image △B′C′D′.
Rotate the paper counterclockwise until the positive *y*-axis coincides with the negative *x*-axis.
After a rotation of 90° about the origin:
B(−5, 6) → B′(−6, −5)
C(−3, 4) → C′(−4, −3)
D(−8, 2) → D′(−2, −8)

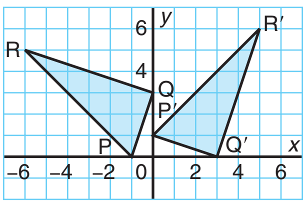


b) From the diagram, OC = OC′ and OD = OD′
∠COC′ = ∠DOD′ = 90°

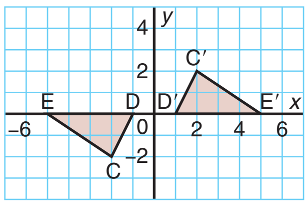
Practice

1. Each grid shows a shape and its rotation image.
Identify the angle and direction of rotation, and the point of rotation.

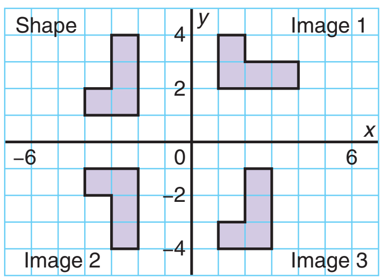
a)



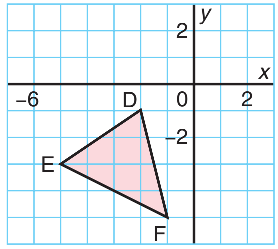
b)



2. Identify the transformation that moves the shape in Quadrant 2 to each image.
Explain how you know.



3. **a)** Copy △DEF on grid paper.
Write the coordinates of each vertex.
After each rotation:
- Write the coordinates of the image of each vertex.
- Describe the positional change of the vertices of the triangle.



b) Rotate △DEF −90° about the origin to its image △D′E′F′.

c) Rotate △DEF +270° about the origin to its image △D″E″F″.

d) What do you notice about the images in parts b and c?
Do you think you would get a similar result with any shape that you rotate −90° and +270°? Explain.

4. Plot each point on a coordinate grid:
A(2, 5), B(−3, 4), C(4, −1)

a) Rotate each point 180° about the origin O to get image points A′, B′, C′.
Write the coordinates of each image point.

b) Draw and measure:
i) OA and OA′ **ii)** OB and OB′ **iii)** OC and OC′
What do you notice?

c) Measure each angle.
i) ∠AOA′ **ii)** ∠BOB′ **iii)** ∠COC′
What do you notice?

d) Describe another rotation of A, B, and C that would result in the image points A′, B′, C′.

5. Repeat question 4 for a rotation of −90° about the origin.

6. **Assessment Focus**

a) Plot these points on a coordinate grid:
A(6, 0), B(6, 2), C(5, 3), D(4, 2), E(2, 2), F(2, 0)
Join the points to draw polygon ABCDEF.

b) Translate the polygon 6 units left and 2 units up.
Write the coordinates of each vertex of the image polygon A′B′C′D′E′F′.

c) Rotate the image polygon A′B′C′D′E′F′ 90° counterclockwise about the origin.
Write the coordinates of each vertex of the image polygon A″B″C″D″E″F″.

d) How does polygon A″B″C″D″E″F″ compare with polygon ABCDEF?

7. Draw a large quadrilateral in the 3rd quadrant.
   a) Rotate the quadrilateral 180° about the origin.
   b) Reflect the quadrilateral in the *x*-axis.
   Then reflect the image in the *y*-axis.
   c) What do you notice about the image in part a and the second image in part b?
   Do you think you would get a similar result if you started:
   i) with a different shape? **ii)** in a different quadrant?
   Investigate to find out. Write about what you discover.

8. **a)** Plot these points on a coordinate grid:
   R(−1, −1), S(−1, 4), T(2, 4), U(2, −1)
   Join the points to draw rectangle RSTU.
   b) Choose a vertex to use as the point of rotation.
   Rotate the rectangle 90° counterclockwise.
   c) Repeat part b two more times for each image rectangle.
   d) Describe the pattern you see in the rectangles.
   e) Is there a transformation that moves rectangle RSTU to the final image directly? Explain.

9. **Take It Further** Plot these points: C(2, 6), D(3, −3), E(5, −7)
   a) Reflect △CDE in the *x*-axis to its image △C′D′E′.
   Rotate △C′D′E′ −90° about the origin to its image △C″D″E″.
   b) Rotate △CDE −90° about the origin to its image △PQR.
   Reflect △PQR in the *x*-axis to its image △P′Q′R′.
   c) Do the final images in parts a and b coincide?
   Explain your answer.

Reflect

When you see a shape and its transformation image on a grid, how do you know what type of transformation it is?
Include examples in your explanation.

Using a Computer to Transform Shapes

Technology

Geometry software can be used to transform shapes.
Use available geometry software.

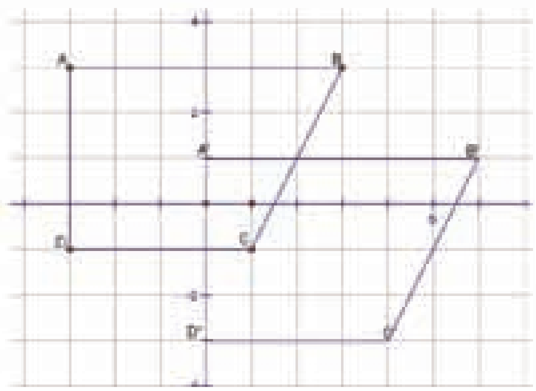
Open a new sketch. Check that the distance units are centimetres. Display a coordinate grid.

Should you need help at any time, use the software's Help Menu.

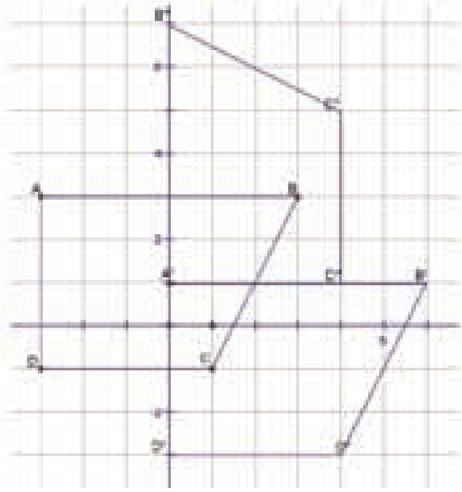
Translating a Shape

Construct a quadrilateral ABCD.
Record the coordinates of each vertex.

Select the quadrilateral. Use the software to translate the quadrilateral 3 units right and 2 units down. Record the coordinates of each vertex of the translation image A′B′C′D′.



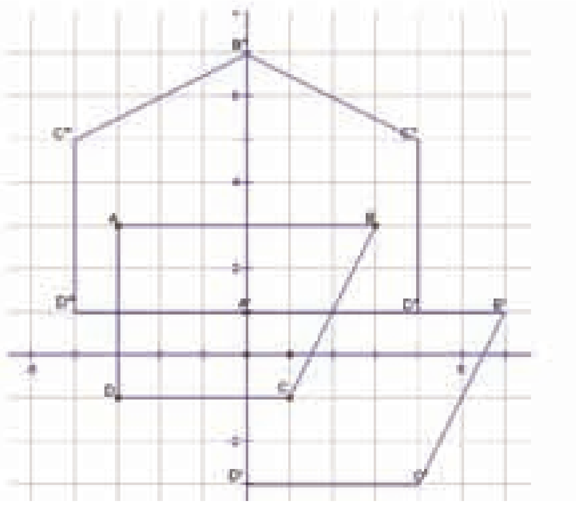
Rotating a Shape



Use the image quadrilateral A′B′C′D′.
Select a vertex of the quadrilateral as the point of rotation. Select quadrilateral A′B′C′D′.
Rotate the quadrilateral 90° counterclockwise.
Record the coordinates of each vertex of the rotation image A″B″C″D″.

Reflecting a Shape

Use the image quadrilateral A″B″C″D″.
Select one side of the quadrilateral as the mirror line.
Select quadrilateral A″B″C″D″.
Reflect the quadrilateral in the mirror line.
Record the coordinates of each vertex of the reflection image A‴B‴C‴D‴.



✓ Check

Create another shape.
Use any or all of the transformations above to make a design that covers the screen.
Colour your design to make it attractive. Print your design.

Making a Study Card

There is a lot of important information in a math unit.

A study card can help you organize and review this information.

How to Start

Read through each lesson in this unit.
As you read, make a study sheet by writing down:

- the main ideas of each lesson
- any new words and what they mean
- any formulas
- one or two examples per lesson
- any notes that might help you remember the concepts

Making a Study Card

- Read over your study sheet.
- Decide which information to include on your study card.
  Only include information you need help remembering.
- Put the information on a recipe card.
  If you have too much information to fit on one card, use two or more cards.

Using Your Study Cards

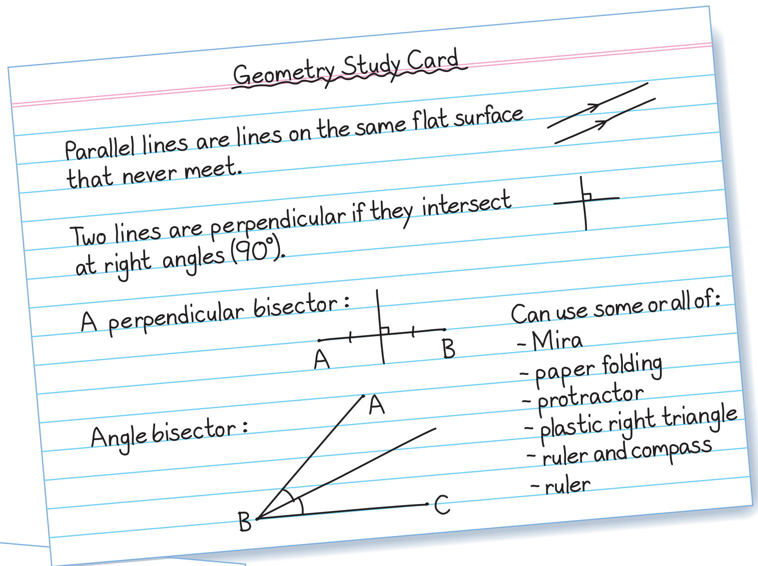
- Use your study cards as you work through the Unit Review.
- When you have finished, you should have some information on your study cards that you do not need help remembering anymore.
- Make a final study card using only one recipe card.

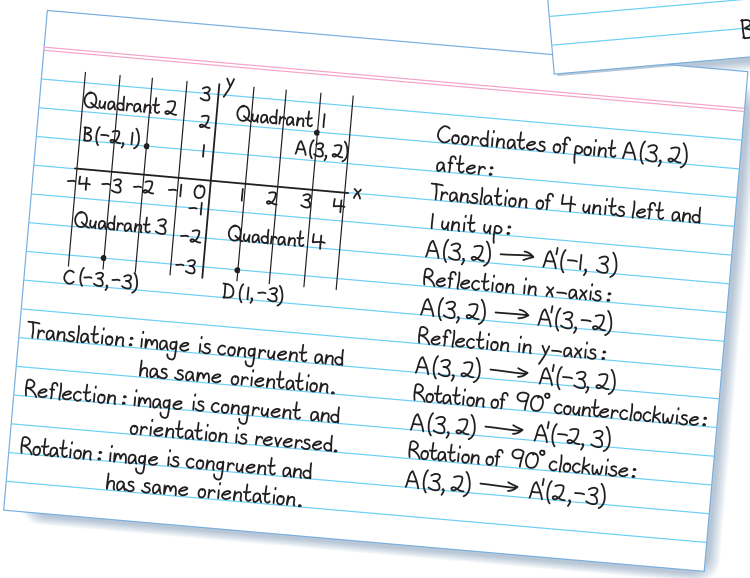
Compare your study card with that of a classmate.
What information does your classmate have that you do not have?
Should everyone's study card be the same? Explain.

The more you use your study card, the less you are going to need it.

Here is an example of a study card for Geometry.

Your study card is your own.
Make it so it helps you.
It may be different from your classmate's study card.





Unit Review

What Do I Need to Know?

☑ Parallel lines

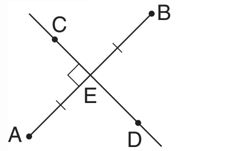
are lines on the same flat surface that never meet.

☑ Perpendicular lines

Two lines are *perpendicular* if they intersect at right angles.

☑ Perpendicular Bisector

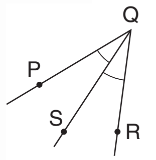
The *perpendicular bisector* of a line segment is drawn at right angles to the segment and divides the segment into two equal parts. Line segment CD is the perpendicular bisector of segment AB.



AE = EB
∠AEC = ∠CEB = 90°

☑ Bisector of an Angle

The *bisector of an angle* divides the angle into two equal angles. Line segment QS is the bisector of ∠PQR.



∠PQS = ∠SQR

☑ Transformations on a Coordinate Grid

A point or shape can be:

- translated (slid)
- reflected in the *x*-axis or the *y*-axis (flipped)
- rotated about the origin (turned)

Math Link

Art
Origami is the Japanese name for the art of paper folding. Use the library or the Internet to get instructions to make a model. Fold a sheet of paper to make your chosen model. Then, unfold the paper and look at the creases. Label as many pairs of parallel line segments as you can.
Do the same with perpendicular line segments, perpendicular bisectors, and angle bisectors.

What Should I Be Able to Do?

LESSON

8.1

1. **a)** Draw line segment FG of length 5 cm.

b) Mark a point H above FG. Draw a line segment parallel to FG that passes through point H.

c) Mark a point J below FG. Draw a line segment parallel to FG that passes through point J.

d) Explain how you can check that the line segments you drew in parts b and c are parallel.

8.2

2. **a)** Draw line segment CD of length 12 cm.

b) Mark a point E above CD. Draw a line segment perpendicular to CD that passes through point E. Label point F where the line segment intersects CD.

c) Join CE and ED. What does EF represent?

8.3

3. **a)** Draw line segment AB. Fold the paper to construct the perpendicular bisector.

b) Draw line segment CD. Use a Mira to construct the perpendicular bisector.

c) Draw line segment EF. Use a ruler and compass to construct the perpendicular bisector.

d) Which of the three methods is most accurate? Justify your answer.

8.4

4. **a)** Draw acute $\angle BAC = 70°$. Fold the paper to construct the angle bisector.

b) Draw right $\angle DEF$. Use a Mira to construct the angle bisector.

c) Draw obtuse $\angle GHJ = 100°$. Use a ruler and compass to construct the angle bisector.

d) Which method is most accurate? Justify your answer.

8.5

5. **a)** On a coordinate grid, plot each point. Join the points in order. Then join D to A. How did you choose the scale?

A(−20, −20) B(30, −20)
C(15, 30) D(−35, 30)

b) Name the quadrant in which each point is located.

c) Identify the shape. Find its area.

6. Do not plot the points. In which quadrant is each point located? How do you know?

a) A(6, −4) **b)** B(−4, −2)
c) C(−3, 2) **d)** D(6, 4)

7. **a)** Find the horizontal distance between each pair of points.

i) A(−5, 1) and B(7, 1)
ii) C(−2, −3) and D(9, −3)

b) Find the vertical distance between each pair of points.

i) E(4, −5) and F(4, 3)
ii) G(−3, −6) and H(−3, 0)

8. Plot each point on a coordinate grid: A(−1, −1), C(3, 1)
A and C are opposite vertices of a rectangle. Find the coordinates of the other 2 vertices.

8.6 **9.** **a)** Plot these points on a coordinate grid.
P(3, 1), Q(7, 1), R(5, 3), S(3, 3)
Join the points to draw trapezoid PQRS.
How do you know it is a trapezoid? Explain.

b) Translate the trapezoid 4 units right. Write the coordinates of each vertex of the image trapezoid P′Q′R′S′.

c) Reflect trapezoid P′Q′R′S′ in the *x*-axis.
Write the coordinates of each vertex of the image trapezoid P″Q″R″S″.

d) How does trapezoid P″Q″R″S″ compare with trapezoid PQRS?

10. Repeat question 9. This time, apply the reflection before the translation. Is the final image the same? Explain.

8.7 **11.** **a)** Plot these points on a coordinate grid:
A (−2, 3) B(−4, 0)
C(−2, −3) D(2, −3)
Join the points to draw quadrilateral ABCD.

b) Draw the image of quadrilateral ABCD after each transformation:
i) a translation 7 units left and 8 units up
ii) a reflection in the *x*-axis
iii) a rotation of +90° about the origin

c) How are the images alike? Different?

12. Use these points:
A(−2, 3), B(0, −1), and C(4, −3)

a) Suppose the order of the coordinates is reversed.
In which quadrant would each point be now?

Draw △ABC on a coordinate grid.

b) Which transformation changes the orientation of the triangle? Justify your answer with a diagram.

c) For which transformation is the image of AC perpendicular to AC? Justify your answer with a diagram.

13. **a)** Plot these points on a coordinate grid.
C(6, −3), D(−4, 3), E(6, 3)
Join the points to draw △CDE.

b) Translate △CDE 5 units left and 4 units up to image △C′D′E′.

c) Rotate △C′D′E′ +90° about the origin to image △C″D″E″.

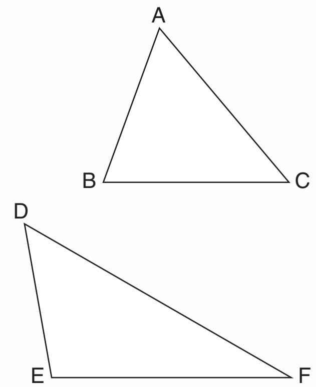
d) How does △C″D″E″ compare with △CDE?

Practice Test

1. Look around the classroom. Where do you see:

a) parallel line segments?
b) perpendicular line segments?
c) perpendicular bisectors?
d) angle bisectors?

2. Your teacher will give you a copy of each of these triangles.



a) Construct the bisector of each angle in △ABC. Use a different method or tool for each bisector.

b) Construct the perpendicular bisector of each side of △DEF. Use a different method or tool for each bisector.

Describe each method or tool used.

3. **a)** On a coordinate grid, draw a triangle with area 12 square units. Place each vertex in a different quadrant.

b) Write the coordinates of each vertex.

c) Explain how you know the area is 12 square units.

d) Translate the triangle 6 units right and 3 units down. Write the coordinates of each vertex of the translation image.

e) Reflect the triangle in the *y*-axis. Write the coordinates of each vertex of the reflection image.

f) Rotate the triangle 90° clockwise about the origin. Write the coordinates of each vertex of the rotation image.

4. **a)** Plot these points on a coordinate grid.
A(−2, 0), B(4, 0), C(3, 4), D(−1, 3)
Join the points to draw quadrilateral ABCD.

b) Translate quadrilateral ABCD 2 units left and 3 units down to the image quadrilateral A′B′C′D′.

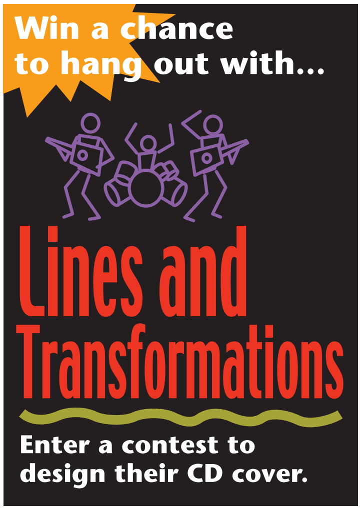
c) Translate quadrilateral A′B′C′D′ 6 units right and 7 units up to the image quadrilateral A″B″C″D″.

d) Describe the translation that moves quadrilateral ABCD to quadrilateral A″B″C″D″.

e) What would happen if the order of the translations was reversed? Explain your answer.

Unit Problem

Design the Cover



You have entered a contest to design the front and back covers of a CD for a new band called *Lines and Transformations*.

Part 1

Your design for the front cover will be created on 4 pieces of paper.
It has to include:

- geometric shapes
- parallel line segments and perpendicular line segments
- geometric constructions

Work in a group of 4.
Brainstorm design ideas for the cover.
Sketch your cover. Show all construction lines.
Each person is responsible for one piece of the cover.
Make sure the pattern or design continues across a seam.
Draw your cover design.
Add colour to make it appealing.

Write about your design.
Explain your choice of design and how it relates to the geometric concepts of this unit.

Check List

Your work should show:

- ✓ a detailed sketch of the front cover, including construction lines
- ✓ a design for the back cover, using transformations
- ✓ your understanding of geometric language and ideas
- ✓ accurate descriptions of construction methods and transformations used

Part 2

Your design for the back cover will be created on a grid.
Each of you chooses a shape from your design for the front cover.
Draw the shape on a coordinate grid.
Use transformations to create a design with your shape.
Colour your design.

Write about your design.
Describe the transformations you used.
Record the coordinates of the original shape and 2 of the images.

Reflect on Your Learning

Write 3 things you now know about parallel and perpendicular line segments that you did not know before.
What have you learned about transformations?

Investigation

Integer Probability

Materials:
- four integer cards labelled −3, −2, +1, +3
- brown paper bag

Work with a partner.

Four integer cards, labelled −3, −2, +1, and +3, are placed in a bag.

James draws three cards from the bag, one card at a time.
He adds the integers.

James predicts that because the sum of all four integers is negative, it is more likely that the sum of any three cards drawn from the bag will be negative.

In this *Investigation*, you will conduct James' experiment to find out if his prediction is correct.

Part 1

➤ Place the integer cards in the bag.
Draw three cards and add the integers.
Is the sum negative or positive?
Record the results in a table.

Integer 1	Integer 2	Integer 3	Sum

➤ Return the cards to the bag. Repeat the experiment until you have 20 sets of results.

➤ Look at the results in your table.
Do the data support James' prediction?
How can you tell?

➤ Combine your results with those of 4 other pairs of classmates.
You now have 100 sets of results.
Do the data support James' prediction?
How can you tell?

➤ Use a diagram or other model to find the theoretical probability of getting a negative sum.
Do the results match your experiment?

➤ Do you think the values of the integers make a difference?
Find 4 integers (2 positive, 2 negative) for which James' prediction is correct.

Part 2

Look at the results of your investigation in *Part 1*.

➤ If the first card James draws is negative, does it affect the probability of getting a negative sum?
Use the results of *Part 1* to support your thinking.

➤ If the first card James draws is positive, does it affect the probability of getting a negative sum?
Use the results of *Part 1* to support your thinking.

Units 1–8 Cumulative Review

UNIT

1 **1.** a) Use algebra. Write a relation for this Input/Output table.

Input n	Output
1	6
2	10
3	14
4	18

b) Graph the relation.

c) Describe the graph.

d) Explain how the graph illustrates the relation.

e) Suggest a real-life situation this graph could represent.

2. The Grade 7 students are organizing an end-of-the-year dance.
The disc jockey charges a flat rate of $85. The cost to attend the dance is $2 per student.

a) How much will the dance cost if 30 students attend?
50 students attend?

b) Write a relation for the cost of the dance when s students attend.

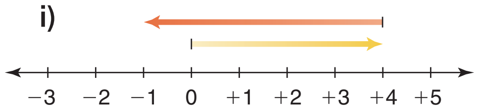
c) Suppose the cost of admission doubles.
Write a relation for the total cost of the dance for s students.

d) Suppose the cost of the disc jockey doubles.
Write a relation for the total cost of the dance for s students.

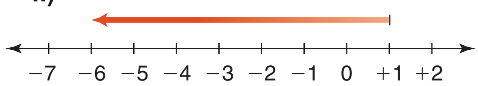
2 **3.** a) Write the addition equation modelled by each number line.

b) Describe a situation that each number line could represent.

i)



ii)



4. On January 11, the predicted high and low temperatures in Flin Flon, Manitoba were −4°C and −13°C.

a) Which is the high temperature and which is the low temperature?

b) What is the difference in temperatures?

3 **5.** Use front-end estimation to estimate each sum or difference.

a) $7.36 + 2.23$ b) $4.255 - 1.386$

c) $58.37 - 22.845$ d) $217.53 + 32.47$

6. A store has a sale.
It will pay the tax if your purchase totals $25 or more.
Justin buys a computer game for $14.95, some batteries for $7.99, and a gaming magazine for $5.95.

a) How much money did Justin spend, before taxes?

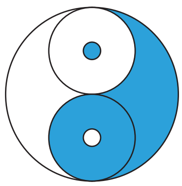
b) Did Justin spend enough money to avoid paying tax?
If your answer is yes, how much more than $25 did Justin spend?
If your answer is no, how much more would he need to spend and not pay the tax?

UNIT

7. Write each fraction as a percent, then as a decimal.

a) $\frac{3}{4}$ b) $\frac{7}{25}$ c) $\frac{9}{10}$ d) $\frac{8}{200}$

4 **8.** This Chinese Yin Yang symbol is made from 5 circles. Suppose the radius of each medium-sized circle is 5 cm. What is the diameter of the largest circle? What assumptions did you make? Explain how you solved the problem.



9. A car tire has radius about 29 cm.

a) What is the diameter of the tire?

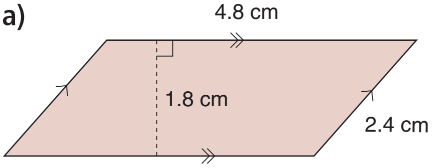
b) Calculate the circumference of the tire.

c) How far has the car tire moved after one complete rotation? Give your answer to the nearest whole number.

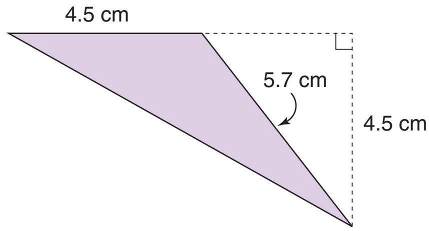
d) About how many rotations will the tire make when the car travels 10 m?

10. Find the area of each shape.

a)



b)



5 **11.** Use a model to show each sum. Sketch the model. Write an addition equation for each picture.

a) $\frac{3}{5} + \frac{2}{10}$ b) $\frac{1}{3} + \frac{1}{12}$

c) $\frac{1}{4} + \frac{7}{8}$ d) $\frac{1}{4} + \frac{5}{6}$

12. A baker's cookie recipe calls for $6\frac{1}{8}$ cups of white sugar and $4\frac{1}{3}$ cups of brown sugar.

a) Estimate how much more white sugar is called for.

b) Calculate how much more white sugar is called for.

c) Draw a diagram to model your calculations in part b.

6 **13.** In a coin toss game, heads score $+1$ and tails score -1.

a) Write an equation you can use to solve each problem.

b) Solve the equation using tiles.

c) Verify each solution. Show your work.

i) Meliq tossed a tail. He then had -2 points. How many points did Meliq have to begin with?

ii) Vera tossed a head. She then had -3 points. How many points did Vera have to begin with?

14. Write an equation you could use to solve each problem. Solve each equation by systematic trial or by inspection.

a) Camille bought 9 teen magazines for $63. She paid the same amount for each magazine. How much did each magazine cost?

b) Nicolas collects fishing lures. He lost 27 of his lures on a fishing trip. Nicolas has 61 lures left. How many lures did he have to begin with?

7 **15.** Mary is a real estate agent in Lethbridge. One month she sold 7 houses at these prices: $171 000, $165 000, $178 000, $161 000, $174 000, $168 000, $240 000

a) Find the median price.

b) Do you think the mean price is greater than or less than the median price? Explain. Calculate to check.

c) What is the range of these prices?

16. Use these data: 28, 30, 30, 31, 32, 33, 34, 35, 37, 38, 39, 41

a) Find the mean, median, and mode.

b) What happens to the mean, median, and mode in each case?

i) Each number is increased by 10.

ii) Each number is doubled.

Explain the results.

17. The masses, in tonnes, of household garbage collected in a municipality each weekday in April are: 285, 395, 270, 305, 320, 300, 290, 310, 315, 295, 310, 295, 305, 325, 315, 310, 305, 300, 325, 305, 305, 300

a) Calculate the mean, median, and mode for the data.

b) What are the outliers? Explain your choice. Calculate the mean without the outliers. What do you notice? Explain.

c) When might you want to include the outliers? Explain.

18. This table shows the hourly wages of the employees at *Tea Break for You*.

Hourly Wage	Number of Employees
$7.50	4
$7.75	6
$8.00	3
$8.50	3
$8.75	2
$10.00	1
$12.50	1

a) Find the mean, median, and mode for these hourly wages.

b) Which measure best represents the wages? Explain.

c) What are the outliers? How is each average affected when the outliers are not included? Explain.

d) Who might earn the wages that are outliers? Explain.

UNIT

19. Is this conclusion true or false? Explain.
The mean test score was 68%. Therefore, one-half the class scored above 68%.

20. Write the probability of each event as many different ways as you can.
a) Roll a 4 on a number cube labelled 1 to 6.
b) December immediately follows November.
c) Pick a red cube from a bag that contains 3 blue cubes, 4 green cubes, and 5 yellow cubes.

21. a) List the possible outcomes for rolling an octahedron labelled 1 to 8 and rolling a die labelled 1 to 6.
b) Why are the events in part a independent?
c) For how many outcomes are both numbers rolled less than 3?

8

22. Draw line segment MN.
Mark a point P not on MN.
Draw a line segment perpendicular to MN that passes through point P.

23. a) Draw line segment FG of length 7 cm. Use a ruler and compass to construct the perpendicular bisector of FG. Explain how you can check that the line you drew is the perpendicular bisector.
b) Draw $\angle PQR = 140°$. Use any method to bisect the angle. Use another method to check that the bisector you have drawn is correct.

24. Suppose you are given the coordinates of a point. You do not plot the point. How can you tell which quadrant the point will be in?

25. a) Plot these points: A(5, 10), B(−5, 10), C(−5, 0), D(−15, 0), E(−15, 10), F(−25, 10), G(−25, −20), H(−15, −20), J(−15, −10), K(−5, −10), L(−5, −20), M(5, −20)
b) Join the points in order. Then join M to A.
c) Explain how you chose the scale.
d) Describe the shape you have drawn.

26. A triangle has vertices C(−1, 5), D(3, 5), and E(3, −1).
a) Plot, then join, the points to draw △CDE.
b) Translate △CDE 2 units left and 4 units up. Write the coordinates of each vertex of the image △C′D′E′.
c) Reflect △C′D′E′ in the *x*-axis. Write the coordinates of each vertex of the image △C″D″E″.
d) Rotate △C″D″E″ 90° counterclockwise about the origin. Write the coordinates of each vertex of the image △C‴D‴E‴.

Answers

Unit 1 Patterns and Relations, page 4

1.1 Patterns in Division, page 8

1. Divisible by 2: parts a, c, and f
Divisible by 5: parts b, d, and f

2. Answers may vary. For example: a number with 0 in ones place is divisible by 2 and by 5. So, it is divisible by 10.

3. Divisible by 4: parts a, b, d, e, and f
Divisible by 8: parts b and f
Divisible by 10: parts c and d

4. Maxine is right. Tony is wrong. A number is divisible by 8 if, when divided by 4, the quotient is even (divisible by 2).

5. Answers may vary. For example: Multiples of 1000 are divisible by 8: 3000, 5000, 8000

6.a) Divisible by 2: 28, 54, 224, 322, 382, 460, 1046, 1088, 1784, 3662
Divisible by 4: 28, 224, 460, 1088, 1784
Divisible by 8: 224, 1088, 1784

c) Answers may vary. For example: 3472, 7000, 9632, all divisible by 8

7. Answers may vary. For example:
a) 0, 4, 8
b) 0, 2, 4, 6, 8
c) 0, 1, 2, 3, 4, 5, 6, 7, 8, 9

8. 1852, 1788, 1992, and 2004 are divisible by 4.
Yes, 1964 is divisible by 4, so it is a leap year.

1.2 More Patterns in Division, page 11

1. Divisible by 3: parts a, b, c, d, e, and f
Divisible by 9: parts a, b, e, and f

2. Answers may vary. For example:
3102, 5100, 2010

3. a, b, c, e, f

4.a) 1, 2, 3, 5, 6, 10, 15, 25, 30, 50, 75, 150
b) 1, 5, 19, 95
c) 1, 3, 9, 13, 39, 117
d) 1, 2, 4, 5, 8, 10, 16, 20, 40, 80

5.

	Divisible by 9	Not divisible by 9
Divisible by 4	144, 252, 468	68, 120, 128, 424
Not divisible by 4	153	235, 361

6. 240

7.a) Answers may vary. For example: 135
b) 1, 3, 5, 9, 15, 27, 45, 135
c) 990; 135

8.a) 2, 5, 8
b) 0, 3, 6, 9
c) 1, 4, 7

9.a) 2 cereal bars
b) 4 cereal bars
c) 24 cereal bars cannot be divided among 0 groups.
d) A whole number cannot be divided among 0 groups.

Unit 1 Reading and Writing in Math: Writing to Explain Your Thinking, page 15

1. 25

2. 22 times

3.a) 41 tiles
b) The 9th term has 37 tiles.

1.3 Algebraic Expressions, page 18

1.a) 3, x, 2 **b)** 5, n, 0
c) 1, w, 3 **d)** 2, p, 4

2. $7p + 9$

3.a) $n + 6$ **b)** $8n$
c) $n - 6$ **d)** $\frac{n}{4}$

4.a) i) \$20.00 **ii)** \$32.00
b) 4t

5.a) $2n + 3$ **b)** $2(n - 5)$ **c)** $\frac{n}{7}+6$
d) $28 - n$ **e)** $n - 28$

6.a) i) $n + 4$
ii) $4 + n$
iii) $n - 4$
iv) $4 - n$

b) In parts i) and ii), the numerical coefficient, the variable, and the constant term are the same. So, the algebraic expressions are the same.
In parts iii) and iv), the numerical coefficients and the constant terms are different. So, the algebraic expressions are different.

7.a) 9 **b)** 12 **c)** 7 **d)** 2 **e)** 13 **f)** 12

8.a) 19 **b)** 3 **c)** 35 **d)** 18 **e)** 21 **f)** 4

9.a) $7 \times 8 + 9 \times 12$
b) $7x + 45$
c) 10 h

10.a) $n = 6$ **b)** $n = 4$ **c)** $n = 2$
d) $n = 3$ **e)** $n = 6$ **f)** $n = 40$

1.4 Relationships in Patterns, page 23

1.a) **i)** The term is twice the term number.
ii) $2n$
b) **i)** The term is 2 more than the term number.
ii) $n + 2$
c) **i)** The term is the term number multiplied by 8.
ii) $8n$
d) **i)** The term is 5 more than the term number.
ii) $n + 5$

2.a) $3n$ **b)** $n + 2$
c) $\frac{n}{2}$ **d)** $4n + 10$

3.a) $10n$
b) $300.00

4.a) $4n$
b) 48 cm
c) Answers may vary. For example:
i) perimeter of an equilateral triangle with side length *s*
ii) perimeter of a regular octagon with side length *t*

5. Answers may vary. For example:
a) Karin's brother is 5 years older than she is.
b) Canoe rental is $15 for the first hour plus $2 per each additional hour.
c) There are 3 candies per person and one left over.

6.a) $65.00; $110.00
b) $9p + 20$
c) $18p + 20$
d) $9p + 40$
e) Answers may vary. For example:
The variable *p* represents any number. So, I can replace *p* to find the value of the algebraic expression for any particular value of the variable.

7.a) $e + 8$
b) $13.00
c) $e + 5$
d) $10.00
e) $3.00

8.a) $4n$
b) $n + 6$
c) $n – 1$

9.a) **i)** The term is double the term number plus one.
ii) $2n + 1$
b) **i)** The term is two less than three times the term number.
ii) $3n – 2$
c) **i)** The term is three less than four times the term number.
ii) $4n – 3$

1.5 Patterns and Relationships in Tables, page 27

1.a)

Input x	Output $2x$
1	2
2	4
3	6
4	8
5	10

The output is double the input.

b)

Input m	Output $10 – m$
1	9
2	8
3	7
4	6
5	5

The output is ten minus the input.

c)

Input p	Output $3x + 5$
1	8
2	11
3	14
4	17
5	20

The output is 5 more than 3 times the input.

2.a) $7n$
b) $3n + 1$
c) $2n – 1$

3. a)

Input n	Output $3n + 4$
1	7
2	10
3	13
4	16

b)

Input n	Output $4n + 3$
1	7
2	11
3	15
4	19

4.a) $3x + 2$
b) $6x – 5$
c) $5x + 3$

5.a) The pattern rule for the input is: Start at 5. Add 10 each time. The pattern rule for the output is: Start at 1. Add 2 each time. When the Input number increases by 10, the Output number increases by 2.
b)

Input x	Output
65	13
75	15
85	17

c) $\frac{x}{5}$ is related to x

Unit 1 Mid-Unit Review, page 29

1. Divisible by 4: parts a, c, d, and e
Divisible by 8: parts c and d
2. Divisible by 3: 54, 123, 3756
Divisible by 5: 85
Divisible by 3 and 5: 735, 1740, 6195
3.a) 1, 5, 17, 85
b) 1, 2, 4, 8, 17, 34, 68, 136
c) 1, 2, 3, 5, 6, 9, 10, 15, 18, 27, 30, 45, 54, 90, 135, 270
4.a) $n + 7$
b) $11n$
c) $\frac{n}{6}$
d) $4n - 3$
e) $2 + 5n$
5.a) **i)** 15
ii) 16
b) **i)** 48
ii) 1
c) **i)** 6
ii) 8
d) **i)** 22
ii) 18
6.a) **i)** The term is the term number multiplied by 6.
ii) $6n$
b) **i)** The term is 4 more than the term number.
ii) $n + 4$
7.a) $12 + 2t$
b) $32.00; $52.00
c) $12 + 4t$
8.a) $4x + 3$
b) $8x - 3$

1.6 Graphing Relations, page 33

1.a) Output: 4, 8, 12, 16, 20
b) Output: 4, 5, 6, 7, 8
c) Output: 10, 14, 18, 22, 26
3.a) Output: 8, 20, 32, 44, 56
b) One square represents 4 units.
c) The graph shows a linear relation: When the Input number increases by 2, the Output number increases by 12.
4.a) 10
b) 5
c) 24

d) Answers may vary. For example: At a bowling alley, shoe rental is $8 and lane rental is $2/h.
5.a) $3n + 5$
c)

Number of Go-Cart Rides	Total Cost ($)
0	5
1	8
2	11
3	14
4	17
5	20

d) **i)** $23.00 **ii)** 8 rides
6.a) ii **b)** iii **c)** i
7.a) $75 - 5s$
b)

Week	Amount Owing
2	65
4	55
6	45
8	35
10	25

c) The graph goes down to the right.
When the number of weeks increases by 2, the amount owing decreases by $10.00.
d) **i)** $10.00
ii) After 15 weeks
8.a) Answers will vary. For example: Maya is paid a flat rate of $6 plus $5 for each item she sells.
b)

Input n	Output $5n + 6$
0	6
1	11
2	16
3	21
4	26
5	31
6	36

c) The graph goes up to the right. When the Input number increases by 1, the Output number increases by 5.
d) Questions may vary. For example: What is the output when the input is 8? (*46*) What is the input when the output is 41? (*7*)

1.7 Reading and Writing Equations, page 36

1.a) $n + 8 = 12$
b) $n - 8 = 12$
2.a) Twelve more than a number is 19.
b) Three times a number is 18.
c) Twelve minus a number is 5.
d) A number divided by 2 is 6.

3.a) $6p = 258$

b) $\frac{s}{2} = 21$

c) $6h = 36$

4. $4s = 156$

5. $p = 6 \times 9$

6.a) C

b) D

c) A

d) B

7. $\frac{n}{4} + 10 = 14$

8.a) **i)** $5s = 295$

ii) $7h = 28$

iii) $2x + 20 = 44$

iv) $n + 7 = 20$

b) Answers may vary. For example:
The equation in part iii is the most difficult because it involves more operations.

c) Answers may vary. For example: One-third the number of books on my shelf is 6.

1.8 Solving Equations Using Algebra Tiles, page 41

1.a) $x = 7$

b) $x = 8$

c) $x = 4$

d) $x = 8$

e) $x = 6$

f) $x = 3$

2.a) $x + 7 = 12$

b) $x = 5$

3. Answers may vary. For example:

a) 6 and 13, 1, x

b) 4 and 12, 1, x

c) 11 and 7, 1, x

d) 16, 2, x

e) 18, 3, x

f) 12, 4, x

4.a) $3x = 12$

b) $x = 4$

5.a) $4x = 20$

b) $x = 5$

6.a) $13 + x = 20$

b) $x = 7$

7.a) $3x + 4 = 16$

b) $x = 4$

8.a) $4x + 2 = 18$

b) $x = 4$

9.a) $3x + 5 = 20$

b) $x = 5$

10. Answers may vary. For example:

a) $3x + 2 = 14$

b) Two more than three times a number is 14.

c) $x = 4$

d) Tina had $14. She bought boxes of cookies at $3 per box. How many boxes did she buy if she was left with $2?

Unit 1 Unit Review, page 44

1. 1, 2, 3, 5, 6, 9, 10, 15, 18, 30, 45, 90

2. Parts a, b, c, d, e, f, h

3. 252 and 432

4.a) Yes. There are numbers divisible by 6 and by 9.

b) Divisible by 6: 330, 858
Divisible by 9: 639, 2295
Divisible by 6 and 9: 5598, 12 006
Divisible by neither 6 nor 9: 10 217, 187

5.a) **i)** $n - 5$

ii) 3

b) **i)** $n + 10$

ii) 18

c) **i)** $3n$

ii) 24

d) **i)** $3n + 6$

ii) 30

6.a) $4n$

b) $n + 3$

c) $\frac{n}{4}$

7.a)

Input n	Output $n + 13$
1	14
2	15
3	16
4	17
5	18

b)

Input n	Output $5n + 1$
1	6
2	11
3	16
4	21
5	26

c)

Input n	Output $6n - 3$
1	3
2	9
3	15
4	21
5	27

8.a) $n + 11$

b) $5n - 3$

9.a) iv

b) i

c) v

10. Answers may vary. For example:

a) **i)** The cost is \$4 plus \$2/h.

ii)

Input m	Output $4 + 2m$
1	6
2	8
3	10
4	12
5	14

iv) The graph goes up to the right. When the Input number increases by 1, the Output number increases by 2.

v) Questions may vary. For example: What is the input when the output is 18? (*7*) What is the output when the input is 6? (*16*)

b) **i)** Anna owes her mother \$15. She pays her \$2/week.

ii)

Input d	Output $15 - 2d$
0	15
1	13
2	11
3	9
4	7

iv) The graph goes down to the right. When the Input number increases by 1, the Output number decreases by 2.

v) What is the input when the output is 3? (*6*) What is the output when the input is 7? (*1*)

11.a) $2c + 6$

b)

c	Amount Paid (\$)
0	6
5	16
10	26
15	36

c) The graph goes up to the right. When the number of children supervised increases by 5, the amount paid increases by \$10.00.

d) **i)** \$56.00
ii) 20 children

12. Answers will vary. For example: May is payed \$24 per day, plus \$2 for each dress she sells.

13.a) $3n = 15$
b) $3n - 4 = 20$

14. $8n = 48$

15.a) **i)** $3x = 36$
ii) $x = 12$
b) **i)** $x + 7 = 18$
ii) $x = 11$
c) **i)** $3x = 24$
ii) $x = 8$
d) **i)** $x + 8 = 21$
ii) $x = 13$

16.a) $4x + 5 = 21$
b) $x = 4$

Unit 1 Practice Test, page 47

1.a) 0, 2, 4, 6, 8
b) 2, 5, 8
c) 2, 6
d) 0, 5
e) 2, 8
f) 6
g) 8
h) 0

2. For $n = 1$, $2 + 3n$ equals $2n + 3$. For $n = 5$, $2n + 3$ equals $3n - 2$.

3.a) $25 + 2v$
b) \$45.00; \$75.00
c) $25 + 3v$; Jamal would pay \$55.00; that is, \$10.00 more.

4.a) **i)** $x + 5 = 22$
ii) $2x = 14$
iii) $3x + 4 = 19$
b) **i)** $x = 17$
ii) $x = 7$
iii) $x = 5$

Unit 2 Integers, page 50

2.1 Representing Integers, page 54

1.a) +1 **b)** +3 **c)** 0 **d)** –1 **e)** –3 **f)** –2

2. Answers may vary. For example:
a) 6 red tiles, or 7 red tiles and 1 yellow tile
b) 7 yellow tiles, or 8 yellow tiles and 1 red tile
c) 6 yellow tiles, or 8 yellow tiles and 2 red tiles
d) 2 red tiles, or 6 red tiles and 4 yellow tiles
e) 9 yellow tiles, or 10 yellow tiles and 1 red tile
f) 4 red tiles, or 5 red tiles and 1 yellow tile
g) 1 yellow tile and 1 red tile, or 3 yellow tiles and 3 red tiles
h) 10 yellow tiles, or 13 yellow tiles and 3 red tiles

3.a)

Number of Yellow Tiles	Number of Red Tiles	Integer Modelled
0	6	–6
1	5	–4
2	4	–2
3	3	0
4	2	+2
5	1	+4
6	0	+6

4.a) I chose +3. I need 3 yellow tiles to model it.
b) I add a zero pair each time. I can model +3 in many ways.
c)

Number of Yellow Tiles	Number of Red Tiles	Integer Modelled
3	0	+3
4	1	+3
5	2	+3
6	3	+3

There are always 3 more yellow tiles than red tiles. As the number of yellow tiles increases, the number of red tiles increases by the same amount.
d) For a negative integer, such as –23, there will always be 23 more red tiles than yellow tiles. For a positive integer, such as +41, there will be 41 more yellow tiles than red tiles.
5.a) 8 **b)** 98
6.a) +9 **b)** –5 **c)** +11 **d)** –9 **e)** –7
7.a) +100; –20
b) +6; –4
c) +12; –8

2.2 Adding Integers with Tiles, page 58

1.a) (+4) + (–2) = +2
b) (+2) + (–3) = –1
c) (–4) + (–2) = –6
d) (+6) + (–3) = +3
e) (+1) + (–4) = –3
f) (+3) + (+2) = +5
2.a) +1 **b)** –1 **c)** 0
3.a) 0 **b)** 0 **c)** 0
The number of red tiles equals the number of yellow tiles each time.
4.a) +5 **b)** +1 **c)** –5
5.a) (+4) + (+3) = +7
b) (–7) + (+5) = –2
c) (–4) + (–5) = –9
d) (+8) + (–1) = +7
e) (–10) + (–6) = –16
f) (+4) + (–13) = –9
6.a) (–3) + (+4) = +1
b) (+5) + (–3) = +2
c) (+15) + (–7) = +8
d) (–3) + (+8) = +5
e) (+12) + (–5) = +7
8.a) (+3) **b)** (–1) **c)** (–2)
d) (+2) **e)** (–1) **f)** (+6)
9.a) –4
b) No, the sum remains the same.
c) Each integer has been replaced by its opposite. The sum is also replaced by its opposite.
10.a) +6 **b)** +4 **c)** –5 **d)** +2
11. a)

+3	–4	+1
–2	0	+2
–1	+4	–3

b)

–1	–6	+1
0	–2	–4
–5	+2	–3

12.a) –8, –12, –16, –20 …
Add –4 each time to get the next term.
b) 0, +3, +6, +9 …
Add +3 each time to get the next term.

2.3 Adding Integers on a Number Line, page 62

1.a) +4 **b)** +2 **c)** –2 **d)** –4
e) –7 **f)** +1 **g)** –1 **h)** +7
2.a) +6 **b)** +2 **c)** –6 **d)** –6
e) –13 **f)** –5 **g)** –3 **h)** +12
3. a), b) The answers are the same.
c) The order in which you add integers does not matter.
4.a) –2 **b)** –3 **c)** +4
5.a) +5; The temperature rose 5°C.
b) +4; Adrian gained $4.
c) +1; The stock was up $1.
6.a) **i)** –2
ii) +5
iii) –6
iv) +8
b) **i)** (+2) + (–2) = 0
ii) (–5) + (+5) = 0
iii) (+6) + (–6) = 0
iv) (–8) + (+8) = 0
c) The sum of two opposite integers is 0.
7. a), b) i) (–5) + (–10) = –15
You take 15 steps backward.
ii) (–5) + (+8) = +3;
You deposit $3.
iii) (–8) + (+6) = –2;
The diver descends 2 m.

iv) (+4) + (–7) = –3;
The snowmobile driver rides 3 km west.

v) (+6) + (–10) = –4;
The person loses 4 kg.

8.a) **i)** (–4) + (+7) = +3
ii) (+8) + (–3) = +5

b) Answers may vary. For example:
i) The temperature dropped 4°C overnight and rose 7°C during the day.
ii) Sarah has $8 and spends $3.

9.a) Always true
b) Never true
c) Always true
d) Sometimes true

10.a) +1 **b)** –5 **c)** –6 **d)** 0

11. +6°C

Unit 2 Mid-Unit Review, page 65

1. Answers may vary. For example:
a) 5 red tiles, or 6 red tiles and 1 yellow tile
b) 1 red tile and 1 yellow tile, or 4 red tiles and 4 yellow tiles
c) 8 yellow tiles, or 9 yellow tiles and 1 red tile
d) 3 red tiles and 2 yellow tiles, or 1 red tile
e) 3 yellow tiles, or 4 yellow tiles and 1 red tile
f) 7 red tiles, or 9 red tiles and 2 yellow tiles

2. 11

3.a) +5 **b)** –2 **c)** 0

4.a) +3 **b)** –5 **c)** –4 **d)** +9 **e)** –12 **f)** +12

5.a) +5 **b)** –6 **c)** –2 **d)** +1 **e)** 0 **f)** +7

6.a) –1
b) Answers may vary. For example:
+2 and –3; +3 and –4; +5 and –6; +6 and –7

7.a) (+50) + (–20) = +30;
Puja had $30.
b) (+5) + (–10) = –5;
The temperature was –5°C.
c) (+124 000) + (–4000) = +120 000;
The population was 120 000.
d) (+12 000) + (–1200) = +10 800;
The plane was cruising at 10 800 m.

8.a) **i)** (–2) + (+6) = +4
ii) (+4) + (–6) = –2
b) Answers may vary. For example:
i) The temperature was –2°C and it rose 6°C.
ii) Karin walked 4 steps forward and 6 steps backward.

9.a) (+1) + (+2) + (+3) + (+4) = +10
b) (–1) + (0) + (+1) = 0 or
(–2) + (–1) + (0) + (+1) + (+2) = 0
c) (–1) + (0) + (+1) + (+2) = +2
d) (+3) + (+4) = +7
e) (–3) + (–2) + (–1) + (0) + (+1) + (+2) + (+3) + (+4) = +4
f) (–7) + (–6) + (–5) + (–4) + (–3) + (–2) + (–1) + (0) + (+1) + (+2) + (+3) + (+4) + (+5) + (+6) + (+7) + (+8) = +8

2.4 Subtracting Integers with Tiles, page 69

1.a) +3 **b)** 0 **c)** –3
d) +2 **e)** –7 **f)** 0

2.a) +3 **b)** –5 **c)** +7
d) –1 **e)** +2 **f)** –9

3.a) –3 **b)** +5 **c)** –7
d) +1 **e)** –2 **f)** +9

4.a) +11 **b)** –10 **c)** –14
d) +14 **e)** –9 **f)** –12

5.a) –1 **b)** –8 **c)** –7
d) +7 **e)** +10 **f)** +11

7.a) **i)** +2 and –2
ii) –1 and +1
iii) +7 and –7
b) When the order in which we subtract two integers is reversed, the answer is the opposite integer.

8. –7

9. I can write as many questions as I want.
For example:
a) (–4) – (–6) = +2
(+7) – (+5) = +2
(+1) – (–1) = +2
b) (–5) – (–2) = –3
(–4) – (+7) = –3
(–1) – (+2) = –3
c) (–3) – (–8) = +5
(+7) – (+2) = +5
(+2) – (–3) = +5
d) (–8) – (–2) = –6
(+3) – (+9) = –6
(–3) – (+3) = –6

10.a) Part i; +4 is greater than –4.
b) Part i; +1 is greater than –1.

11.a) +2 and –3
b) Answers will vary. For example:
Find two integers with a sum of +3 and a difference of +9. Answer: +6 and –3

12.a) (+1) **b)** (+4) **c)** (+5)

13.a) +2 **b)** 0 **c)** 0 **d)** +1 **e)** –3 **f)** 0

14.a) The sum of the numbers in each row, column, and diagonal is –9, so the square is still magic.

–4	+1	–6
–5	–3	–1
0	–7	–2

b) The sum of the numbers in each row, column, and diagonal is +6, so the square is still magic.

+1	+6	–1
0	+2	+4
+5	–2	+3

2.5 Subtracting Integers on a Number Line, page 73

1.a) +1 **b)** +7 **c)** –3 **d)** –7 **e)** +4 **f)** +4

2.a) –1, –7, +3, +7, –4, –4

b) The answers in part a are the opposites of those in question 1. When the order of the integers is reversed, the difference changes to its opposite.

3.a) +5 **b)** +10 **c)** –14
d) –15 **e)** –8 **f)** 0

4.a) (+6) + (–4) = +2
b) (–5) + (–4) = –9
c) (–2) + (+3) = +1
d) (+4) + (+2) = +6
e) (+1) + (–1) = 0
f) (+1) + (+1) = +2

5.a) +12°C or –12°C
b) +7°C or –7°C
c) +13°C or –13°C

6.a) +8 or –8
b) +5 or –5
c) +9 or –9

7.a) **i)** (+13) – (–4) = +17; +17°C
ii) (–10) – (–22) = +12; +12°C
iii) (+12) – (–3) = +15; +15°C
iv) (+13) – (+7) = +6; +6°C
b) Calgary

8.a) –17
b) +17; the answers in parts a and b are opposite integers.
c) Each integer was replaced with its opposite. The differences are opposite integers: +17 and –17

9. Answers may vary. For example:
(–6) – (–10) = +4
(+6) – (+2) = +4
(–1) – (–5) = +4

10.a) (+6) – (+5) = +1
(+5) – (+5) = 0
(+4) – (+5) = –1
(+3) – (+5) = –2
(+2) – (+5) = –3
b) (+7) – (+4) = +3
(+7) – (+3) = +4
(+7) – (+2) = +5
(+7) – (+1) = +6
(+7) – (0) = +7
(+7) – (–1) = +8
(+7) – (–2) = +9
(+7) – (–3) = +10
c) (+8) – (+7) = +1
(+7) – (+7) = 0
(+6) – (+7) = –1
(+5) – (+7) = –2
(+4) – (+7) = –3
(+3) – (+7) = –4
(+2) – (+7) = –5
(+1) – (+7) = –6
0 – (+7) = –7
(–1) – (+7) = –8
(–2) – (+7) = –9
(–3) – (+7) = –10

11.a) –6, –10, –14, –18;
Start at +6. Subtract +4 each time.
b) +3, +5, +7, +9;
Start at –3. Subtract –2 each time.
c) +26, +33, +40, +47;
Start at +5. Subtract –7 each time.
d) –2, –3, –4, –5;
Start at +1. Subtract +1 each time.

12.a) +1 **b)** +1 **c)** –4 **d)** +2 **e)** +12 **f)** –11

Unit 2 Unit Review, page 79

1.a) 5 **b)** 17 **c)** 37 **d)** 0

2.a) +8 **b)** –5 **c)** +12 **d)** –7 **e)** –9

3.a) –3 **b)** +1 **c)** –1 **d)** 0

4.a) (–6) + (+4) = –2
b) (–25) + (+13) = –12
c) (+15) + (–23) = –8
d) (–250) + (+80) = –170

5. Answers may vary. For example:
a) (–5) + (0) = –5;
(–3) + (–2) = –5;
(–1) + (–4) = –5;
(+1) + (–6) = –5

b) (+4) + (0) = +4;
(+2) + (–2) = +4;
(–2) + (+6) = +4;
(–4) + (+8) = +4

6. (–10) + (+17) = +7;
The new temperature is +7°C.

7.a) **i)** (–4) + (+5) = +1
ii) (+2) + (–4) = –2
b) Answers may vary. For example:
i) Sasha takes 4 steps backward and 5 steps forward.
ii) The temperature is +2°C and then drops 4°C.

8.a) +2 **b)** –1 **c)** –5 **d)** +2

9.a) +2 **b)** +2 **c)** –10 **d)** –2

10. The difference of two positive integers is positive if the first integer is greater than the second integer. The difference of two positive integers is negative if the first integer is less than the second integer.

11.a) +9°C **b)** 0°C **c)** –6°C **d)** –7°C

12.a) +3 **b)** +6 **c)** +4 **d)** –5
e) –4 **f)** –5 **g)** –2 **h)** +5

13.a) +5 **b)** –10 **c)** +1 **d)** 0 **e)** +6 **f)** –1

14.a) +12°C or –12°C
b) –150 m or +150 m

15.a) –9 m or +9 m
b) +14 m or –14 m

16.a) +12 kg or –12 kg
b) –1 kg or +1 kg

17.a) +1 **b)** –2 **c)** +3 h or –3 h

18. Answers may vary. For example:
a) (+10) – (+4) = +6
(+8) – (+2) = +6
(+6) – (0) = +6
(+4) – (–2) = +6
(+2) – (–4) = +6
b) (–5) – (–2) = –3
(–1) – (+2) = –3
(+3) – (+6) = –3
(0) – (+3) = –3
(–3) – (0) = –3

Unit 2 Practice Test, page 81

1.a) –3 **b)** –10 **c)** –10
d) +6 **e)** –4 **f)** +23

2.a) +8 **b)** –15 **c)** –11
d) +7 **e)** +2 **f)** +4

3.a) The sum of two integers is zero when the integers are opposites.
b) The sum of two integers is negative when both integers are negative; or when one integer is positive and the other is negative, and the negative integer has a longer arrow on the number line.
c) The sum of two integers is positive when both integers are positive; or when one integer is positive and the other is negative, and the positive integer has a longer arrow on the number line.

4.a) 6 different scores
b) (+10) + (+10) = +20
(+10) + (+5) = +15
(+10) + (–2) = +8
(+5) + (+5) = +10
(+5) + (–2) = +3
(–2) + (–2) = –4

5. +373°C or –373°C

6. There are 4 possible answers: +7, +13, –1, and +5.
For 4 integers in a row, the addition and/or subtraction signs can be arranged as shown:
+ + +; + + –; + – +; + – –; – + +; – + –; – – +; – – –

Unit 2 Unit Problem: What Time Is It?, page 82

1.a) 0:00 a.m.
b) 5:00 a.m.
c) 9:00 a.m.
d) 6:00 a.m.

2. 10:00 a.m. the next day

3. Atsuko needs to fly out at 3:00 p.m. Tokyo time.
Paula needs to fly out at 7:00 a.m. Sydney time.

Unit 3 Fractions, Decimals, and Percents, page 84

3.1 Fractions to Decimals, page 88

1.a) **i)** $0.\overline{6}$
ii) 0.75
iii) 0.8
iv) $0.8\overline{3}$
v) $0.\overline{857\,142}$
b) **i)** repeating
ii) terminating
iii) terminating
iv) repeating
v) repeating

2.a) $\frac{9}{10}$

b) $\frac{26}{100} = \frac{13}{50}$

c) $\frac{45}{100} = \frac{9}{20}$

d) $\frac{1}{100}$

e) $\frac{125}{1000} = \frac{1}{8}$

3. a) **i)** $0.\overline{037}$

ii) $0.\overline{074}$

iii) $0.\overline{1}$

b) As the numerator of the fraction increases by 1, the corresponding decimal increases by $0.\overline{037}$ each time.

c) **i)** $0.\overline{148}$

ii) $0.\overline{185}$

iii) $0.\overline{296}$

4.a) $\frac{4}{10}$, 0.4

b) $\frac{25}{100}$, 0.25

c) $\frac{52}{100}$, 0.52

d) $\frac{38}{100}$, 0.38

e) $\frac{74}{1000}$, 0.074

5.a) $\frac{2}{3}$ **b)** $\frac{5}{9}$ **c)** $\frac{41}{99}$ **d)** $\frac{16}{99}$

6. a) $0.\overline{571\,428}$ **b)** $0.\overline{4}$

c) $0.\overline{54}$ **d)** $0.\overline{538\,461}$

7. 0.294 117 647; Use long division.

8. 0.2

a) 0.8 **b)** 1.4 **c)** 1.8 **d)** 2.2

9.a) **i)** $0.\overline{001}$

ii) $0.\overline{002}$

iii) $0.\overline{054}$

iv) $0.\overline{113}$

b) The numerator of the fraction becomes the repeating digits in the decimal. If the numerator is a two-digit number, the first repeating digit is 0.

c) **i)** $\frac{4}{999}$

ii) $\frac{89}{999}$

iii) $\frac{201}{999}$

iv) $\frac{326}{999}$

10.a) iii **b)** i **c)** iv **d)** ii

11.a) 1.0, 2.0, 1.5, $1.\overline{6}$, 1.6, 1.625; The decimals are greater than or equal to 1 and less than or equal to 2.

b) $1.\overline{615\,384}$, $1.\overline{619\,047}$, 1.617 647…, $1.\overline{618}$

12.a) $1.\overline{142\,857}$; Six digits repeat.

b) $0.\overline{285\,714}$, $0.\overline{428\,571}$, $0.\overline{571\,428}$, $0.\overline{714\,285}$, $0.\overline{857\,142}$; The tenth digit increases from least to greatest; the other digits follow in a clockwise direction around the circle.

13.a) **i)** 0.875; terminating

ii) $0.2\overline{7}$; repeating

iii) 0.3; terminating

iv) $0.\overline{296}$; repeating

v) 0.16; terminating

b) **i)** $2 \times 2 \times 2$

ii) $2 \times 3 \times 3$

iii) 2×5

iv) $3 \times 3 \times 3$

v) 5×5

c) When the prime factors of the denominator are 2 and 5 only, the corresponding decimal is terminating. When the denominator has any other prime factors, the fraction can be written as a repeating decimal.

d) **i)** No

ii) Yes

iii) No

iv) Yes

3.2 Comparing and Ordering Fractions and Decimals, page 94

1. Answers may vary.

For example: $\frac{1}{7}$, $\frac{4}{7}$, $\frac{8}{7}$, $\frac{18}{7}$, $\frac{24}{7}$

2. From greatest to least: $\frac{11}{3}$, $2\frac{5}{6}$, $2\frac{1}{2}$

3.a) 1, $\frac{7}{6}$, $1\frac{2}{9}$, $\frac{15}{12}$

b) $\frac{7}{6}$, $1\frac{3}{4}$, 2, $\frac{7}{3}$

c) $\frac{15}{10}$, $\frac{7}{4}$, 2, $\frac{11}{5}$

d) $2\frac{1}{3}$, $\frac{10}{4}$, 3, $\frac{9}{2}$

4.a) $3\frac{1}{2}$, $\frac{13}{4}$, $3\frac{1}{8}$; 3.5, 3.25, 3.125

b) $1\frac{1}{12}, \frac{5}{6}, \frac{9}{12}, \frac{2}{3}$; $1.08\overline{3}, 0.8\overline{3}, 0.75, 0.\overline{6}$

c) $\frac{3}{2}, 1\frac{2}{5}, \frac{4}{3}$; $1.5, 1.4, 1.\overline{3}$

5.a) $1, 1.25, 1.6, \frac{7}{4}, 1\frac{4}{5}$

b) $1.875, 2, \frac{5}{2}, 2\frac{5}{8}, 2\frac{3}{4}$

6.a) $\frac{17}{5}, 3\frac{1}{4}, 3.2, \frac{21}{7}, 2.8, 2$

7. Answers may vary. For example:

a) $\frac{27}{16}$ **b)** 2.25

8. Answers may vary. For example:

a) $\frac{11}{14}$ **b)** $1\frac{1}{2}$ **c)** 1.35 **d)** 0.55

9.a) $\frac{11}{4}$; $2\frac{1}{2} = \frac{10}{4}$ which is less than $\frac{11}{4}$.

b) $3\frac{2}{5}$; $\frac{2}{5}$ is close to $\frac{1}{2}$, so $3\frac{2}{5}$ is closer to $3\frac{1}{2}$.

10.a) $6\frac{2}{20}$ should be the second number in the set:

$\frac{29}{5}, 6\frac{2}{20}, 6\frac{2}{10}, 6.25$

b) $\frac{3}{2}$ should be the first number in the set:

$\frac{3}{2}, 1\frac{7}{16}, 1\frac{3}{8}, 1.2, \frac{3}{4}$

11.a) From least to greatest: $\frac{11}{6}, 1.875, \frac{9}{4}$

b) Corey sold the most pizzas; Amrita sold the fewest pizzas.

c) Use equivalent fractions.

d) $\frac{11}{6}, 1.875, 2\frac{1}{5}, \frac{9}{4}$

3.3 Adding and Subtracting Decimals, page 98

1.a) 2 – 0 = 2
b) 71 + 6 = 77
c) 125 + 37 = 162
d) 9 – 1 = 8

2. 0.067 km

3.a) $819.24
b) $248.26

4. a) 12.7 kg
b) No; 12.7 is greater than 10.5.
c) 2.2 kg

5. Use front-end estimation: 49; 51.485

6.a) Robb family: $428.79; Chan family: $336.18
b) $92.61

7. Answers may vary.
For example: 216.478 and 65.181

8. Answers may vary.
For example: 0.312 and 5.476

9.a) The student did not line up the digits of like value.
b) 4.437

10. Answers may vary.
For example: 1.256 and 2.044

11.a) Start at 2.09. Add 0.04 each time.
b) Start at 5.635. Subtract 0.25 each time.

3.4 Multiplying Decimals, page 102

1.a) 1.7 × 1.5 = 2.55
b) 2.3 × 1.3 = 2.99

2.a) 3.9
b) 0.92
c) 0.56

3. Answers may vary. For example: I chose part a from question 2. I used 2 flats: 2 × 1 = 2;
16 rods: 16 × 0.1 = 1.6;
30 small cubes: 30 × 0.01 = 0.3.
The area of the plot is: 2 + 1.6 + 0.3 = 3.9

4.a) 15.54 **b)** 2.67 **c)** 0.54

5. 161.65; I estimated 150, so the answer is reasonable.

6.a) 83.6; 836; 8360; 83 600; Multiply by multiples of 10. The digits in the product move one place to the left each time. Or, the decimal point moves one place to the right.
b) 0.836; 0.0836; 0.008 36; 0.000 836; Multiply by multiples of 0.1. The digits in the product move one place to the right each time. Or, the decimal point moves one place to the left.

7. 9.18 m^2

8.a) 12.922 2
b) 174.315 96
c) 1.333 072

9.a) 936.66 km
b) 852.24 km

10.a) $2.43 **b)** $12.50 **c)** $0.62

11. Answers may vary.
For example: 1.2 and 0.3 or 0.2 and 1.8

12.a) 216
b) **i)** 21.6
ii) 2.16
iii) 2.16
iv) 0.0216

13.a) **i)** 11.34
ii) 0.0962
iii) 8.448
iv) 1.1106

b) The number of decimal places in the product is the sum of the number of decimal places in the question.

c) 9.1; Yes, the rule applies, but the product must be written as 9.10. The calculator does not show the product this way.

3.5 Dividing Decimals, page 106

1.a) 8 **b)** 4 **c)** 4.5 **d)** 5.5

2.a) 12.45; 1.245; 0.1245; 0.012 45; Divide by multiples of 10. The digits in the quotient move one place to the right each time. Or, the decimal point moves one place to the left.

b) 1245; 12 450; 124 500; 1 245 000; Divide by multiples of 0.1. The digits in the quotient move one place to the left each time. Or, the decimal point moves one place to the right.

3. All division statements are equivalent.

4.a) 11.9 **b)** 976.5 **c)** 39.15

5.a) 2.5 **b)** 3.2 **c)** 1.6 **d)** 2.4

6.a) 3.5 **b)** 1.5 **c)** 7.1 **d)** 24.1

7. 87

8. 27.9 m

9.a) About $3

b) $3.35

c) About 3 kg

10.a) About 12 pieces; Assumptions may vary.

b) No, he needs 14 pieces and he has material for 12.

c) If Alex cannot use the 0.28-m piece left after he cut twelve 0.8-m pieces, he needs 1.6 m of fabric. If he can use it, he only needs 1.32 m of fabric.

d) Yes; Alex would only need 0.7 m × 14 = 9.8 m of fabric.

11. Answers may vary. For example: 0.312 and 2.6

12. $9.25; The result should be written to the nearest hundredth.

13. 237 is greater than 10 times 7 and less than 100 times 7, so the quotient should be between 10 and 100: 237 ÷ 7 = 33.857

a) 338.57 **b)** 33.857 **c)** 3.3857 **d)** 33.857

3.6 Order of Operations with Decimals, page 109

1.a) 6.5 **b)** 6.2 **c)** 14 **d)** 1498

2.a) 58 **b)** 211 **c)** 12

3.a) 4.4

b) 2.2

4.a) 345.68 **b)** 18.038

c) 163 **d)** 116.54

5.a) Aida

b) Ioana: 12 × (4.8 ÷ 0.3 – 3.64 × 3.5) = 39.12
Norman: (12 × 4.8 ÷ 0.3 – 3.64) × 3.5 = 659.26

6. 41.21

7. Answers may vary. For example:
0.1 + 0.2 + 0.3 + 0.4 = 1,
(0.6 × 0.5 + 0.7) × 0.2 ÷ 0.1 = 2,
(0.8 + 0.7) × 0.6 ÷ 0.3 = 3,
0.6 ÷ 0.2 + 0.1 + 0.9 = 4,
0.9 ÷ 0.3 + 0.4 ÷ 0.2 = 5

Unit 3 Mid-Unit Review, page 110

1.a) **i)** $0.0\overline{3}$

ii) $0.0\overline{6}$

iii) $0.0\overline{9}$

b) Start at $0.0\overline{3}$. Add $0.0\overline{3}$ each time.

c) **i)** $\frac{5}{33}$

ii) $\frac{12}{33}$

2.a) 0.125; terminating

b) 0.6; terminating

c) $0.\overline{6}$; repeating

d) $0.\overline{538\ 461}$; repeating

3.a) $\frac{1}{5}$ **b)** $\frac{8}{9}$ **c)** $\frac{1}{200}$ **d)** $\frac{23}{99}$

4. From least to greatest:

a) $\frac{11}{6}$, 2, $2\frac{1}{4}$, $\frac{8}{3}$ **b)** $1\frac{3}{4}$, $\frac{23}{8}$, 3.5

c) 1, $\frac{13}{10}$, $1\frac{3}{5}$, 1.75, $\frac{9}{5}$

5. Answers may vary. For example:

a) 1.5 **b)** 2.4 **c)** 1.5

6.a) 25.72 **b)** 137.521 **c)** 17.1

7.a) 3.585 kg **b)** 9.25 kg

8.a) 7.44 **b)** 4.706 **c)** 58.95

9. 9.94 km^2

10. The division statements are equivalent.

11.a) 16.26 **b)** 50.5 **c)** 18.431

3.7 Relating Fractions, Decimals, and Percents, page 112

1.a) $\frac{3}{20}$, 15%, 0.15

b) $\frac{2}{5}$, 40%, 0.4

c) $\frac{4}{5}$, 80%, 0.8

2.a) $\frac{1}{50}$, 0.02

b) $\frac{9}{100}$, 0.09

c) $\frac{7}{25}$, 0.28

d) $\frac{19}{20}$, 0.95

3.a) 0.2, 20%
b) 0.06, 6%
c) 0.16, 16%
d) 0.65, 65%
e) 0.8, 80%

4. Janet; 82% is greater than 80%.

5. 15%

6.a) 25% **b)** 50% **c)** 6% **d)** 10%

3.8 Solving Percent Problems, page 115

1.a) 3 **b)** 10 **c)** 6.48 **d)** 75.04

2.a) \$45.00 **b)** \$42.00 **c)** \$36.00

3. a) \$40.50 **b)** \$22.00 **c)** \$35.00

4. a) \$3.63 **b)** \$11.30 **c)** \$3.27

5.a) **i)** \$7.74
ii) \$136.74
b) **i)** \$1.50
ii) \$26.49
c) **i)** \$2.58
ii) \$45.55

6. About 192 bands

7.a) Answers may vary. For example: Some items will be 60% off, others will be reduced by less. Or, the sale prices will be at least 40% the original price.
b) Scarves and hats
c) Sweaters: About \$20.00 (\$14.99 off sale price), ski jackets: \$60.00 (\$52.49 off sale price), leather gloves: \$28.00 (\$10.49 off sale price)

8.a) \$199.99 – \$199.99 × 0.25 = \$149.99
b) \$199.99 × 0.75 = \$149.99
c) Yes

Unit 3 Unit Review, page 121

1.a) 0.6; terminating
b) $0.8\overline{3}$; repeating
c) 0.375; terminating
d) 0.15; terminating

2.a) $\frac{11}{20}$ **b)** $1\frac{1}{3}$ **c)** $\frac{4}{5}$ **d)** $\frac{7}{99}$

3.a) From least to greatest:
$\frac{3}{6}, \frac{5}{8}, 1\frac{1}{16}, 1.1, \frac{5}{4}$

4. For example:
a) 2.25; From least to greatest:
$2.25,\ 2\frac{1}{3},\ \frac{17}{6},\ 2\frac{11}{12}$

b) $1\frac{3}{15}$; From least to greatest:
$\frac{3}{5}, \frac{9}{10}, \frac{21}{20}, 1.1, 1\frac{3}{15}$

5. Answers will vary.
For example: 1.78 and 1.63

6. 0.72 s

7.a) \$118.58
b) \$59.29

8. \$1.56

9. i) a, b, c
ii) d, e, f; part d: 4.1875; part e: 5.2; part f : 24.2

10. 6.25 m

11.a) 43.79
b) 5.855

12.a) **i)** 10.68
ii) 10.92
iii) 9.48
iv) 11.56
b) When the position of the brackets changes, the order of operations changes.

13.a) $\frac{4}{5}$, 0.8

b) $\frac{3}{25}$, 0.12

c) $\frac{1}{50}$, 0.02

d) $\frac{63}{100}$, 0.63

14.a) 0.56, 56%
b) 0.95, 95%
c) 0.14, 14%
d) 0.2, 20%

15. 28 students

16.a) \$33.15 **b)** \$21.75 **c)** \$31.50

17.a) \$34.19 **b)** \$31.79 **c)** \$2.40

18. \$6.55

Unit 3 Practice Test, page 123

1.a) $\frac{1}{250}$ **b)** $\frac{16}{25}$ **c)** $\frac{1}{3}$
d) 0.255 **e)** 0.75

2.a) \$90.00
b) No. The equipment costs \$107.80.
c) \$17.80

3. Yes

4.a) 34.74 **b)** 15.67

5. 26 cats
6.a) $58.50 **b)** $19.50 **c)** $3.51 **d)** $62.01

Cumulative Review Units 1–3, page 126

1. Divisible by 4: 320, 488, 2660
Divisible by 6: 762, 4926
Divisible by 4 and by 6: 264, 504
Not divisible by 4 or by 6: 1293
2.a) 5 strawberries **b)** 8 strawberries
c) I cannot divide 40 strawberries among 0 people.
3.a) $\frac{n}{12}$
b) $n + 11$
c) $n - 8$
4.a) When the Input number increases by 1, the Output number increases by 2.
b)

Input x	Output
1	4
2	6
3	8
4	10
5	12
6	14

c) $2x + 2$; The table shows how $2x + 2$ relates to x.
5.a) 3, s, 2
b) 7, p
c) 1, c, 8
d) 11, w, 9
6.a) $5 + 3c$
b)

Additional Half Hours	Cost ($)
0	5
1	8
2	11
3	14
4	17

c) The graph goes up to the right. When the number of additional half hours increases by 1, the cost increases by $3.
d) **i)** $23.00
ii) 8 additional half hours
7.a) $x = 5$
b) $x = 2$
8.a) 11 red tiles
b) 3 ways: 3 red tiles, or 4 red tiles and 1 yellow tile, or 5 red tiles and 2 yellow tiles
9.a) 0
b) –2
c) –12
d) +2
10.a) **i)** +10, –5
ii) +25, –10
iii) –9, +12
b) **i)** (+10) + (–5) = +5; I deposit $5.
ii) (+25) + (–10) = +15; The balloon rises 15 m.
iii) (–9) + (+12) = +3; I ride the elevator up 3 floors.
11.a) 115 m or –115 m
b) –75 m or 75 m
12.a) –4
b) –6
c) +10
d) –6
13.a) **i)** $0.\overline{03}$
ii) $0.\overline{06}$
iii) $0.\overline{09}$
b) As the numerator of the fraction increases by 1, the corresponding decimal increases by $0.\overline{03}$ each time.
c) **i)** $\frac{5}{33}$
ii) $\frac{8}{33}$
iii) $\frac{10}{33}$
14.a) From greatest to least:
$5\frac{1}{3}, 5.3, \frac{21}{4}, 4.9, \frac{24}{5}$
15. 1.873 m
16.a) 7.82
b) 3.96
c) 15.17
d) 4.93
17.a) 21 bottles
b) 0.375 L
18.a) **i)** $7.80
ii) $137.79
b) **i)** $1.08
ii) $19.06

Unit 4 Circles and Area, page 128

4.1 Investigating Circles, page 131

1.a) 12 cm
b) 16 cm
2.a) 14 cm
b) 8 cm
3.a) 1.9 cm
b) 15 cm

4. 0.6 m
5.c) 360°
d) The sum of the angles at the centre is 360°.
6. 15 glasses; Assumptions may vary. For example: All glasses are cylindrical and they can touch.
7. Answers may vary. For example:
15 cm, 7.5 cm; 2.5 cm, 1.25 cm; 9.6 cm, 4.8 cm; 8.8 cm, 4.4 cm; 1.5 cm, 0.75 cm; 1.8 cm, 0.9 cm; 2.6 cm, 1.3 cm
8. Answers may vary. For example:
Fix one end of a measuring tape on the circumference. Walk around the circle with the measuring tape at ground level, until you reach the maximum distance across the circle, which is the diameter. The centre of the circle is the midpoint of the diameter.

4.2 Circumference of a Circle, page 136

1.a) About 31.42 cm **b)** About 43.98 cm
c) About 47.12 cm
2.a) About 7.64 cm; about 3.82 cm
b) About 0.76 m; about 0.38 m
c) About 12.73 cm; about 6.37 cm
3. Less than; π is greater than 3.
4.a) About 7.5 m
b) About $33.98, assuming the edging does not have to be bought in whole metres
5.a) The circumference doubles.
b) The circumference triples.
6. About 71.6 cm
7. No, because π never terminates or repeats. So, the circumference will never be a whole number.
8.a) A dotted line with the marks equally spaced apart
b) About 289 cm, or 2.89 m
c) About 346 times
9.a) About 40 075 cm
b) There would be a gap of about 160 m under the ring. You would be able to crawl, walk, and drive a school bus under the ring.

Unit 4 Mid-Unit Review, page 138

2. Answers may vary, but diameters should be less than 20 cm and greater than 10 cm.
3.a) 3.9 cm **b)** 4.1 cm **c)** 5 cm **d)** 12.5 cm
4. No, two circles with the same radius are the same (congruent).
5.a) About 37.70 cm **b)** About 50.27 cm
6.a) **i)** About 207.35 cm
ii) About 232.48 cm
iii) About 188.50 cm
b) The tire has the greatest circumference; it has the greatest diameter, too.
7. About 24.38 m
8.a) About 40.7 cm
b) About 18.0 cm
c) About 7.2 cm
9. About 78.54 cm

4.3 Area of a Parallelogram, page 139

1.iii) **a)** 20 cm^2
b) 9 cm^2
c) 30 cm^2
2.a) 312 cm^2
b) 195 mm^2
c) 384 cm^2
3.b) The 3 parallelograms have equal areas: 21 cm^2
4. Yes; Parallelograms with the same base and height have equal areas.
5.b) 10 cm^2
6.a) 5 m **b)** 3 mm **c)** 6 cm
7. Answers may vary. For example:
a) $b = 5$ cm, $h = 2$ cm
b) $b = 6$ cm, $h = 3$ cm
c) $b = 7$ cm, $h = 4$ cm
8. The area of the parallelogram is 16 cm^2. The student may have used the side length, 5 cm, as the height of the parallelogram.
9. No, the areas of Shape A and Shape B are equal.
10.a) 95.04 m^2
b) 132 m^2
c) 36.96 m^2; 18.45 m^2 each

4.4 Area of a Triangle, page 145

2.a) 21 cm^2 **b)** 12.5 cm^2 **c)** 12 cm^2
d) 12 cm^2 **e)** 10 cm^2 **f)** 8 cm^2
3.b) In a right triangle, two heights coincide with the sides.
4.a) 21 cm^2
c) Each parallelogram has area 42 cm^2.
5.a) 4 cm **b)** 16 m **c)** 32 mm
6.b) All triangles in part a have the same area: 6 cm^2
7.a) $b = 4$ cm, $h = 7$ cm or $b = 2$ cm, $h = 14$ cm
b) $b = 10$ cm, $h = 2$ cm or $b = 4$ cm, $h = 5$ cm
c) $b = 4$ cm, $h = 4$ cm or $b = 2$ cm, $h = 8$ cm
8.a) **i)** The area doubles.
ii) The area is 4 times as great.
iii) The area is 9 times as great.
b) I can triple the base or the height of the triangle.

9.a) 11.7 m^2
b) About 3 cans of paint
10.a) 17 triangles: 12 small, 4 medium, 1 large
b) 1 small triangle is $\frac{1}{4}$ of a medium triangle and $\frac{1}{16}$ of the large triangle.
1 medium triangle is $\frac{1}{4}$ of the large triangle and 4 times as great as a small triangle.
The large triangle is 4 times as great as a medium triangle and 16 times as great as a small triangle.
c) 12 parallelograms: 9 small, 3 medium
d) 27.6 cm^2 **e)** 6.9 cm^2
f) 1.725 cm^2
g) Small: 3.45 cm^2; medium: 13.8 cm^2
11.a) 92.98 m^2
b) At least 33 sheets of plywood

4.5 Area of a Circle, page 151

1.a) About 12.57 cm^2
b) About 153.94 cm^2
c) About 153.94 cm^2
d) About 706.86 cm^2
2.a) About 28.27 cm^2
b) About 113.10 cm^2
c) About 254.47 cm^2
d) About 452.39 cm^2
3.a) The area is 4 times as great.
b) The area is 9 times as great.
c) The area is 16 times as great.
4.a) The area of the circle is approximately halfway between the area of the smaller square and the area of the larger square: About 75 cm^2
b) About 78.54 cm^2
c) Answers may vary.
5.a) About 104 cm^2
b) About 16 cm^2
6.a) About 0.0707 m^2
b) About 1.0603 m^2; about 3.3929 m^2; about 5.6549 m^2
7.a) About 113.10 cm^2
b) About 19.63 cm^2
c) About 34.58 cm^2
8. Two large pizzas are the better deal.

4.6 Interpreting Circle Graphs, page 158

1.a) Traditional dance lessons
b) Powwow drum classes; traditional dance lessons
c) Stick games: 175 students; Powwow drum classes: 200 students; traditional dance lessons: 125 students
2.a) 0 to 12 years and 13 to 19 years
b) **i)** 112 500 viewers
ii) 62 500 viewers
iii) 25 000 viewers
3.a) 161 t
b) 805 t
4.a) French: $550; History: $1050; Science: $750; Biography: $550; Geography: $450; Fiction: $900; Reference: $750
b) The total amount of money spent on each type of book should be $5000.
5.a) 10%
b) Saskatchewan, Manitoba, Alberta, British Columbia
c) Saskatchewan: 968 300 people; about 968 000 people
Manitoba: 1 161 960 people; about 1 162 000 people
Alberta: 3 292 220 people; about 3 292 000 people
British Columbia: 4 260 520 people; about 4 261 000 people
6.a) 25 students
b) Autumn: $\frac{7}{2}$; 28%; winter: $\frac{3}{25}$; 12%; spring: $\frac{5}{25}$; 20%; summer: $\frac{10}{25}$; 40%
c) All percents in part b should add up to 100.
7.a) Morning Snack Mix: sunflower seeds 30 g, almonds 54 g, raisins 25.5 g, peanuts 40.5 g
Super Snack Mix: raisins 19.5 g, banana chips 34.5 g, cranberries 25.5 g, papaya chunks 40.5 g, pineapple chunks 30 g
b) Morning Snack Mix: 51 g of raisins
Super Snack Mix: 39 g of raisins
I assumed the percents of the ingredients in both snack mixes remain the same.

4.7 Drawing Circle Graphs, page 163

1.a) 50 students
b) Blue: $\frac{12}{50}=\frac{6}{25}$; brown: $\frac{24}{50}=\frac{12}{25}$; green: $\frac{8}{50}=\frac{4}{25}$; grey: $\frac{6}{50}=\frac{3}{25}$
c) Blue: 24%; brown: 48%; green: 16%; grey: 12%
2.a) 92 people

b) MAJIC99: $\frac{88}{400}=\frac{11}{50}$, 22%;

EASY2: $\frac{92}{400}=\frac{23}{100}$, 23%;

ROCK1: $\frac{120}{400}=\frac{3}{10}$, 30%;

HITS2: $\frac{100}{400}=\frac{1}{4}$, 25%

3.a) 40 000 000 U.S. residents

b) $\frac{1\,200\,000}{40\,000\,000}=\frac{12}{400}=\frac{3}{100}$

c) 10%

4.a) Yes, each number of students can be written as a fraction of the whole.

b) No, data cannot be written as a fraction of the whole.

5. Asia: about 367 million km^2
Africa: about 244 million km^2
South America: about 147 million km^2
Antarctica: about 98 million km^2
Europe: about 86 million km^2
Australia: about 61 million km^2

Unit 4 Unit Review, page 168

1. Answers may vary. For example: Use a pencil, a string, and a pin.

2.a) 6 cm **b)** 10 cm **c)** 3.5 cm

3.a) 30 cm **b)** 44 cm **c)** 8.4 cm

4. About 34.85 m

5.a) About 75.40 m **b)** 14 m **c)** About 87.96 m

6.a) About 94.25 mm **b)** About 131.95 mm

c) Mel's dial; it has the greater radius.

7. Answers may vary. For example: 6 cm and 4 cm; 4 cm and 6 cm; 8 cm and 3 cm; 3 cm and 8 cm; 2 cm and 12 cm; 12 cm and 2 cm; 1 cm and 24 cm; 24 cm and 1 cm

8.a) 3.84 m^2

b) **i)** 0.96 m^2 **ii)** 13.44 m^2

9.a) Answers may vary. For example: b = 1 cm, h = 24 cm; b = 2 cm, h = 12 cm; b = 3 cm, h = 8 cm; b = 4 cm, h = 6 cm; b = 6 cm, h = 4 cm; b = 8 cm, h = 3 cm; b = 12 cm, h = 2 cm; b = 24 cm, h = 1 cm

b) The area of the parallelograms in question 7 is double the area of the triangles in part a.

10. $1265.63

11.a) About 201.06 m^2 **b)** About 50.27 m

12.a) The circumference is halved.

b) The area is one-quarter of what it was.

13. About 637.94 cm^2

14. I calculated the area of each shape: about 55.42 cm^2, 54 cm^2, 56 cm^2
The shape in part c will require the most paint.

15.a) Laura received the most votes.

b) Jarrod: 140 votes; Laura: 280 votes; Jeff: 80 votes

16.a) Lake Huron

b) Lake Superior has the greatest surface area.

c) 26 840 km^2

17.a) Water: 62%, protein: 17%, fat: 15%, nitrogen: 3%, calcium: 2%, other: 1%

b) 37.2 kg

18.a) Manitoba: 10%, Saskatchewan: 10%, Quebec: 30%, Ontario: 50%

Unit 4 Practice Test, page 171

2.a) About 31.42 cm **b)** About 78.54 cm^2

3. 360°

4.a) 63 cm^2 **b)** 9 cm^2

5.a) Too many to count

b) No, because π never terminates or repeats. So, the area will never be a whole number.

6.b) No. The circle represents the whole and each percent can be written as a fraction of the whole.

Unit 5 Operations with Fractions, page 176

5.1 Using Models to Add Fractions, page 179

1.a) $\frac{2}{4}+\frac{1}{2}=1$ **b)** $\frac{2}{3}+\frac{4}{6}=1\frac{1}{3}$ **c)** $\frac{7}{10}+\frac{4}{5}=1\frac{1}{2}$

2.a) $\frac{7}{8}+\frac{1}{2}=1\frac{3}{8}$ **b)** $\frac{3}{10}+\frac{2}{5}=\frac{7}{10}$ **c)** $\frac{2}{3}+\frac{1}{2}=1\frac{1}{6}$

d) $\frac{2}{3}+\frac{5}{6}=1\frac{1}{2}$ **e)** $\frac{3}{6}+\frac{1}{12}=\frac{7}{12}$ **f)** $\frac{1}{4}+\frac{2}{8}=\frac{1}{2}$

g) $\frac{1}{3}+\frac{1}{2}=\frac{5}{6}$ **h)** $\frac{1}{2}+\frac{4}{10}=\frac{9}{10}$

3. $\frac{1}{2}$ h

4.a) **i)** $\frac{2}{5}$

ii) 1

iii) $\frac{7}{10}$

iv) $\frac{2}{3}$

b) Answers may vary. For example: Use fraction circles. Or, add numerators.

5.a) $\frac{3}{4}$; less **b)** $\frac{9}{5}=1\frac{4}{5}$; greater

c) 1; equal **d)** $\frac{4}{10}=\frac{2}{5}$; less

6. Answers may vary. For example: $\frac{1}{6}$ and $\frac{2}{3}$

7.a) $\frac{1}{8}$; $\frac{1}{4}$; $\frac{3}{8}$

b) $\frac{3}{4}$; $\frac{1}{4}$

5.2 Using Other Models to Add Fractions, page 183

1.a) $\frac{2}{4},\frac{3}{6},\frac{4}{8}$ **b)** $\frac{2}{8}$ **c)** $\frac{4}{6},\frac{6}{9}$

2.a) $\frac{3}{4}+\frac{7}{8}=\frac{13}{8}$ **b)** $\frac{5}{6}+\frac{2}{3}=\frac{9}{6}$ **c)** $\frac{3}{2}+\frac{3}{4}=\frac{9}{4}$

3. Answers may vary. For example:

a) The greater denominator is a multiple of the lesser denominator. The greater denominator shows which number line to use to get the answer.

b) One denominator is a multiple of the other.

4.a) $\frac{7}{6}$ **b)** $\frac{11}{12}$ **c)** $\frac{7}{10}$ **d)** $\frac{1}{4}$

5.a) $\frac{5}{6}$ **b)** $\frac{19}{12}$ **c)** $\frac{11}{10}$ **d)** $\frac{13}{15}$

6. Answers may vary. For example:

a) The least common multiple of the denominators shows which number line to use to get the answer.

b) The denominators are not multiples, nor factors of each other.

c) Use a number line divided in fractions whose denominator is given by the least common multiple of the unrelated denominators.

7.a) $\frac{13}{21}$ **b)** $\frac{35}{36}$ **c)** $\frac{57}{40}$ **d)** $\frac{29}{35}$

8. $\frac{19}{12}$

9.a) There are 36 possible fractions:

$\frac{1}{1},\frac{1}{2},\frac{1}{3},\frac{1}{4},\frac{1}{5},\frac{1}{6},\frac{2}{1},\frac{2}{2},\frac{2}{3},\frac{2}{4},\frac{2}{5},\frac{2}{6},\frac{3}{1},\frac{3}{2},\frac{3}{3},\frac{3}{4},\frac{3}{5},\frac{3}{6},$

$\frac{4}{1},\frac{4}{2},\frac{4}{3},\frac{4}{4},\frac{4}{5},\frac{4}{6},\frac{5}{1},\frac{5}{2},\frac{5}{3},\frac{5}{4},\frac{5}{5},\frac{5}{6},\frac{6}{1},\frac{6}{2},\frac{6}{3},\frac{6}{4},\frac{6}{5},\frac{6}{6}$

Answers may vary.

For example: $\frac{3}{4}+\frac{5}{6}=\frac{19}{12}=1\frac{7}{12}$; $\frac{3}{4}+\frac{1}{2}=\frac{5}{4}=1\frac{1}{4}$

b) $\frac{4}{6}+\frac{2}{5}=\frac{16}{15}$

10. Answers may vary. For example:

$\frac{7}{10}+\frac{4}{5}=\frac{3}{2}$; $\frac{3}{4}+\frac{3}{4}=\frac{3}{2}$

11. Yes, $\frac{7}{4}<2$

12. 2 cups

5.3 Using Symbols to Add Fractions, page 188

1.a) Eighths **b)** Twenty-fourths

c) Ninths **d)** Fifteenths

2.a) 1 **b)** 8 **c)** 2 **d)** 20

3.a) $\frac{7}{9}$ **b)** $\frac{5}{6}$ **c)** $\frac{15}{8}=1\frac{7}{8}$ **d)** $\frac{11}{12}$

4.a) About 1; $\frac{11}{10}=1\frac{1}{10}$ **b)** About $\frac{1}{2}$; $\frac{19}{24}$

c) About 2; $\frac{29}{18}=1\frac{11}{18}$ **d)** About $1\frac{1}{2}$; $\frac{37}{28}=1\frac{9}{28}$

e) About $\frac{1}{2}$; $\frac{11}{15}$ **f)** About 1; $\frac{31}{30}=1\frac{1}{30}$

5. $\frac{3}{16}$

6. $\frac{3}{4}+\frac{4}{5}$ is greater.

7. Statement b is true: $\frac{3}{10}+\frac{1}{5}+\frac{1}{2}=1$

Statement a is false: $\frac{1}{10}+\frac{3}{5}+\frac{1}{2}=\frac{12}{10}=\frac{6}{5}>1$

8. About $\frac{29}{30}$

9. Sums in parts a, e, and f are correct.

10.a) $\frac{13}{8}=1\frac{5}{8}$ **b)** $\frac{43}{20}=2\frac{3}{20}$ **c)** $\frac{35}{18}=1\frac{17}{18}$

Unit 5 Mid-Unit Review, page 190

1. $\frac{3}{5}+\frac{3}{10}=\frac{9}{10}$

2. $\frac{11}{12}$ h

3.a) $\frac{1}{2}+\frac{5}{12}=\frac{11}{12}$ **b)** $\frac{2}{3}+\frac{3}{4}=\frac{17}{12}=1\frac{5}{12}$

4.a) $\frac{5}{8}$ **b)** $\frac{5}{6}$ **c)** $\frac{13}{12}=1\frac{1}{12}$ **d)** $\frac{9}{10}$

5. $\frac{3}{2}=1\frac{1}{2}$; Methods may vary. For example: Use Pattern Blocks. Or, use fraction circles. Or, use equivalent fractions.

6.a) $\frac{9}{8}=1\frac{1}{8}$ **b)** $\frac{14}{15}$ **c)** $\frac{3}{8}$ **d)** $\frac{17}{12}=1\frac{5}{12}$

7. No; $\frac{59}{60}<1$

8. a) i) $\frac{3}{4}$

ii) $\frac{1}{2}$

iii) $\frac{1}{4}$

iv) $\frac{1}{2}$

b) Puzzles and games

5.4 Using Models to Subtract Fractions, page 193

1. Answers may vary. For example:

a) $\frac{4}{8}$ and $\frac{5}{8}$ **b)** $\frac{3}{12}$ and $\frac{4}{12}$

c) $\frac{4}{6}$ and $\frac{1}{6}$ **d)** $\frac{6}{10}$ and $\frac{5}{10}$

2.a) $\frac{1}{3}$; Less than $\frac{1}{2}$ **b)** $\frac{3}{4}$; Greater than $\frac{1}{2}$

c) $\frac{1}{3}$; Less than $\frac{1}{2}$ **d)** $\frac{1}{6}$; Less than $\frac{1}{2}$

3.a) $\frac{1}{4}$ **b)** $\frac{3}{5}$ **c)** $\frac{1}{3}$ **d)** $\frac{1}{4}$

4.a) Subtract the numerators only. The denominator remains the same.

b) Examples may vary.

5.a) $\frac{7}{9}-\frac{1}{3}=\frac{4}{9}$ **b)** $\frac{7}{8}-\frac{3}{4}=\frac{1}{8}$

c) $\frac{8}{10}-\frac{2}{5}=\frac{4}{10}=\frac{2}{5}$ **d)** $\frac{11}{12}-\frac{2}{3}=\frac{3}{12}$

6.a) $\frac{1}{8}$ **b)** $\frac{1}{5}$ **c)** $\frac{3}{8}$ **d)** $\frac{7}{12}$

7. $\frac{1}{6}$

8. $\frac{1}{4}$

9. No. Spencer needs $\frac{1}{12}$ cup more.

10. Answers may vary. For example:

a) $\frac{2}{3}-\frac{1}{3}=\frac{1}{3}$ **b)** $\frac{4}{5}-\frac{1}{5}=\frac{3}{5}$ **c)** $\frac{2}{3}-\frac{2}{4}=\frac{1}{6}$

11.a) More: $\frac{3}{4}-\frac{1}{8}=\frac{5}{8}>\frac{1}{2}$ **b)** $\frac{1}{8}$

12.a) iii **b)** Use estimation.

5.5 Using Symbols to Subtract Fractions, page 197

1.a) $\frac{2}{5}$ **b)** $\frac{1}{3}$ **c)** $\frac{1}{3}$ **d)** $\frac{2}{7}$

2.a) $\frac{1}{2}$ **b)** $\frac{1}{8}$ **c)** $\frac{4}{5}$ **d)** $\frac{1}{12}$

3.a) $\frac{1}{12}$ **b)** $\frac{2}{15}$ **c)** $\frac{19}{20}$ **d)** $\frac{1}{10}$

4.a) $\frac{1}{6}$ **b)** $\frac{11}{12}$ **c)** $\frac{17}{30}$ **d)** $\frac{1}{12}$

5. Walnuts; $\frac{1}{12}$ cup more

6.a) Terri; $1\frac{5}{12}>1\frac{1}{4}$

b) $\frac{1}{6}$ h

7. Answers may vary. For example: $\frac{9}{4}-\frac{3}{2}=\frac{3}{4}$

8. The other fraction is between $\frac{1}{2}$ and $\frac{3}{4}$.

9. 18 min

5.6 Adding with Mixed Numbers, page 202

1.a) $\frac{3}{2}$ **b)** $\frac{17}{4}$ **c)** $\frac{7}{4}$ **d)** $\frac{18}{5}$

2.a) $3\frac{2}{5}$ **b)** $2\frac{1}{4}$ **c)** $4\frac{1}{2}$ **d)** $4\frac{2}{3}$

3.a) $1\frac{1}{2}$ **b)** $2\frac{1}{3}$ **c)** $4\frac{1}{6}$ **d)** $6\frac{1}{6}$

4.a) 6 **b)** $4\frac{3}{4}$ **c)** $7\frac{7}{9}$ **d)** $8\frac{2}{5}$

5.a) $3\frac{3}{8}$ **b)** $3\frac{1}{12}$ **c)** $5\frac{1}{8}$ **d)** $4\frac{1}{10}$

6.a) $3\frac{7}{10}$ **b)** $2\frac{7}{10}$ **c)** $5\frac{7}{10}$ **d)** $7\frac{7}{10}$

7.a) $3\frac{7}{12}$ **b)** $2\frac{2}{5}$ **c)** $3\frac{7}{20}$ **d)** $2\frac{13}{14}$

e) $6\frac{13}{24}$ **f)** $5\frac{4}{15}$ **g)** $7\frac{11}{40}$ **h)** $6\frac{1}{12}$

8. $6\frac{7}{15}$ h

9.a) Estimates may vary. For example: About $3\frac{1}{2}$

b) $3\frac{5}{8}$

10. $9\frac{5}{12}$ cups

11.a) $3\frac{7}{10}$

b) $\frac{8}{5}$ and $\frac{21}{10}$

c) $\frac{37}{10}$

12. $4\frac{5}{12}$ h

13. $1\frac{2}{5}$ or $\frac{7}{5}$; equivalent fractions may vary.

5.7 Subtracting with Mixed Numbers, page 207

1.a) $1\frac{1}{5}$ **b)** $2\frac{1}{4}$ **c)** 3 **d)** $\frac{5}{3}=1\frac{2}{3}$

2.a) $1\frac{1}{3}$ **b)** 2 **c)** $\frac{1}{2}$ **d)** $1\frac{3}{4}$

3.a) $2\frac{1}{6}$ **b)** $1\frac{1}{6}$ **c)** $2\frac{1}{6}$ **d)** $4\frac{1}{6}$

4.a) About $2\frac{1}{2}$; $\frac{9}{4} = 2\frac{1}{4}$

b) About $1\frac{1}{2}$; $\frac{3}{2} = 1\frac{1}{2}$

c) About $\frac{1}{2}$; $\frac{13}{20}$

d) About $1\frac{1}{2}$; $\frac{13}{20} = 1\frac{3}{10}$

5.a) **i)** $\frac{11}{5} = 2\frac{1}{5}$ **ii)** $\frac{25}{7} = 3\frac{4}{7}$

iii) $\frac{25}{6} = 4\frac{1}{6}$ **iv)** $\frac{50}{9} = 5\frac{5}{9}$

6.a) $2\frac{11}{20}$ **b)** $1\frac{2}{5}$ **c)** $2\frac{5}{12}$ **d)** $2\frac{1}{21}$

7. i) **a)** $2\frac{3}{10}$ **b)** $\frac{23}{10}$

c) Answers may vary. For example: The first method is easier because $\frac{3}{5}$ is greater than $\frac{3}{10}$.

ii) **a)** $1\frac{7}{10}$ **b)** $\frac{17}{10}$

c) Answers may vary. For example: The second method is easier because $\frac{3}{5}$ is less than $\frac{3}{10}$.

8. $1\frac{17}{40}$ cups

9. $\frac{11}{12}$ h

10.a) $\frac{19}{24}$ **b)** $\frac{31}{18}$ or $1\frac{13}{18}$

c) $\frac{44}{15}$ or $2\frac{14}{15}$ **d)** $\frac{101}{40}$ or $2\frac{21}{40}$

11.a) Estimates may vary.

For example: About $1\frac{1}{2}$

b) $\frac{35}{24}$ or $1\frac{11}{24}$ **d)** $\frac{29}{24}$ or $1\frac{5}{24}$

12. Answers may vary. For example: $\frac{21}{8}$ or $2\frac{5}{8}$

Unit 5 Unit Review, page 213

1.a) $\frac{13}{12}$ **b)** 1 **c)** $\frac{11}{12}$ **d)** $\frac{7}{10}$

2.a) $\frac{11}{9}$ **b)** $\frac{3}{2}$ **c)** $\frac{3}{4}$ **d)** $\frac{9}{8}$

3. Answers may vary. For example: $\frac{1}{4}+\frac{3}{8}=\frac{5}{8}$

4. Answers may vary. For example:

a) $\frac{12}{20}$ and $\frac{15}{20}$ **b)** $\frac{2}{5}$ and $\frac{1}{5}$

c) $\frac{8}{18}$ and $\frac{9}{18}$ **d)** $\frac{15}{24}$ and $\frac{4}{24}$

5.a) $\frac{4}{5}$ **b)** $\frac{13}{14}$ **c)** $\frac{29}{30}$ **d)** $\frac{17}{20}$

6.a) $1-\frac{1}{3}=\frac{4}{6}$ **b)** $\frac{7}{10}-\frac{2}{5}=\frac{3}{10}$

c) $\frac{10}{12}-\frac{3}{4}=\frac{1}{12}$ **d)** $\frac{5}{8}-\frac{1}{4}=\frac{3}{8}$

7.a) $\frac{3}{5}$ **b)** $\frac{1}{2}$ **c)** $\frac{5}{12}$

8.a) Javon; $\frac{5}{6} > \frac{7}{9}$ **b)** $\frac{1}{18}$

9.a) $\frac{1}{2}$ **b)** $\frac{3}{2}=1\frac{1}{2}$

c) $\frac{27}{20}=1\frac{7}{20}$ **d)** $\frac{19}{12}=1\frac{7}{12}$

10. Answers will vary. For example:

a) $\frac{4}{3}-\frac{5}{6}=\frac{1}{2}$ **b)** $\frac{31}{36}-\frac{1}{9}=\frac{3}{4}$ **c)** $\frac{17}{20}-\frac{3}{4}=\frac{1}{10}$

d) $\frac{5}{2}-\frac{7}{3}=\frac{1}{6}$ **e)** $\frac{5}{6}-\frac{7}{12}=\frac{1}{4}$

11.a) Brad **b)** $\frac{1}{8}$ bottle

12. $\frac{3}{8}$

13.a) $6\frac{2}{3}$ **b)** $1\frac{7}{12}$ **c)** $5\frac{1}{2}$ **d)** $6\frac{13}{20}$

14.a) $4\frac{1}{2}$ **b)** $4\frac{5}{8}$ **c)** $10\frac{1}{10}$ **d)** $8\frac{2}{9}$

15. $3\frac{5}{8}$ h

16.a) $\frac{33}{8}$, or $4\frac{1}{8}$ **b)** $\frac{25}{9}$, or $2\frac{7}{9}$

c) $\frac{19}{12}$, or $1\frac{7}{12}$ **d)** $\frac{47}{24}$, or $1\frac{23}{24}$

17.a) The second recipe; $1\frac{7}{9} > 1\frac{3}{4}$

b) $\frac{1}{8}$ cup

18.a) $\frac{25}{6}$, or $4\frac{1}{6}$ **b)** $\frac{49}{30}$, or $1\frac{19}{30}$

c) $\frac{169}{24}$, or $7\frac{1}{24}$ **d)** $\frac{3}{4}$

19. $\frac{5}{6}$ h

Unit 5 Practice Test, page 215

1.a) 2 **b)** $\frac{19}{30}$

c) $\frac{1}{4}$ **d)** $\frac{29}{18} = 1\frac{11}{18}$

2. Answers may vary. For example:

a) $\frac{1}{5} + \frac{2}{5} = \frac{3}{5}$ **b)** $\frac{1}{35} + \frac{4}{7} = \frac{3}{5}$

3. Answers may vary. For example:

a) $\frac{3}{8} - \frac{1}{8} = \frac{1}{4}$ **b)** $\frac{3}{4} - \frac{1}{2} = \frac{1}{4}$

4.a) $\frac{343}{40}$, or $8\frac{23}{40}$ **b)** $\frac{13}{10}$, or $1\frac{3}{10}$

5. $7\frac{3}{4}$ h; Answers may vary. For example: No, Lana cannot do all the jobs. If she allows at least 3 h to travel from one place to another and $\frac{1}{2}$ h for her lunch break, her total time is $11\frac{1}{4}$ h.

6.a) $\frac{1}{2} + \frac{1}{4} = \frac{3}{4}$ **b)** $\frac{1}{2} + \frac{1}{8} = \frac{5}{8}$

7. Answers may vary. For example:

Counter 1: $\frac{1}{6}$ and $\frac{7}{12}$, Counter 2: $\frac{5}{12}$ and $\frac{2}{3}$

Unit 6 Equations, page 218

6.1 Solving Equations, page 223

1.a) equation **b)** expression **c)** expression
d) equation **e)** expression **f)** equation

2.a) $w = 12$ **d)** $x = 96$ **f)** $z = 11$

3.a) $x - 10 = 35$ **b)** $x = 45$

4.a) $7 + n = 18$; $n = 11$ **b)** $n - 6 = 24$; $n = 30$
c) $5n = 45$; $n = 9$ **d)** $\frac{n}{6} = 7$; $n = 42$
e) $4n + 3 = 19$; $n = 4$

5.a) $14x = 182$; $x = 13$ **b)** $b - 14 = 53$; $b = 67$
c) $100 = 56 + 11p$; $p = 4$

6. For example: **a)** $4s = 48$ **b)** $s = 12$

7. For example: **a)** $\frac{p}{6} = 11$ **b)** $p = 66$

8. Answers may vary. For example:
a) The perimeter of a triangle is 27 cm. Write an equation you can solve to find the side length of the triangle.
b) $27 = 3t$ **c)** $t = 9$

9.a) $130 = 10 + 24f$ **b)** $f = 5$

10.a) $n = 9$ **b)** $n = 12$ **c)** $n = 15$ **d)** $n = 81$

11.a) $x = 3$ **b)** $y = 6$ **c)** $z = 2166$ **d)** $x = 5$

6.2 Using a Model to Solve Equations, page 229

1.a) A = 30 g **b)** B = 65 g
c) C = 50 g **d)** D = 21 g

2.b) **i)** $x = 7$
ii) $x = 14$
iii) $y = 3$
iv) $m = 7$
v) $k = 8$
vi) $p = 21$

3.i) **a)** $5 + n = 24$ **b)** $n = 19$
ii) **a)** $n + 8 = 32$ **b)** $n = 24$
iii) **a)** $3n = 42$ **b)** $n = 14$
iv) **a)** $2n + 5 = 37$ **b)** $n = 16$

4.a) $60 = 12h$; $h = 5$ m **b)** $112 = 8h$; $h = 14$ cm
c) $169 = 13h$; $h = 13$ m

5.a) Left pan: x and 35 g; right pan: 35 g and 25 g
b) $x = 25$

6. Problems may vary. For example:
a) Helen is 16 years old. Kian is 4 years younger than Helen. How old is Kian?
b) Helen is 4 years older than Kian. Kian is 16 years old. How old is Helen?
c) Part a: $x = 12$; part b: $x = 20$

7. Answers may vary. The sum of the digits should be a multiple of nine. For example:
$5 + x + 7 = 18$, $x = 6$;
567 is divisible by 9.

6.3 Solving Equations Involving Integers, page 234

1.a) $x = 4$ **b)** $x = 7$ **c)** $x = 10$
d) $x = 12$ **e)** $x = 13$ **f)** $x = 14$

2.a) $n = 13$ **b)** $x = 2$ **c)** $p = 7$
d) $x = -5$ **e)** $s = -14$ **f)** $x = 3$

3. $x = 17$

4. $f - 6 = 5$; $f = 11$

5.a) $t - 8 = -3$ **b)** $t = 5$

6.a) $x = 7$ **b)** $n = 19$

7.a) $n + 2 = 4$; +2
b) $n - 2 = 1$; +3
c) $n - 4 = -2$; +2

Unit 6 Mid-Unit Review, page 236

1.a) **i)** $5 + d = 12$; $d = 7$
ii) $2d = 12$; $d = 6$
b) **i)** $67 + s = 92$; $s = 25$
ii) $3w + 8 = 29$; $w = 7$

2.i) **a)** $n + 9 = 17$ **c)** $n = 8$
ii) **a)** $3n = 21$ **c)** $n = 7$
iii) **a)** $7 + 2n = 19$ **c)** $n = 6$

3. $40 = 14 + 2B$; Bill is 13 years old.

4.i) **a)** $n - 8 = 7$ **c)** $n = 15$
ii) **a)** $t - 6 = -4$ **c)** $t = 2$
iii) **a)** $m - 7 = 5$ **c)** $m = 12$

6.4 Solving Equations Using Algebra, page 238

1.a) $x = 62$ **b)** $x = 12$ **c)** $x = 17$
2.a) $19 + n = 42$; $n = 23$
b) $3n + 10 = 25$; $n = 5$
c) $15 + 4n = 63$; $n = 12$
3.a) $27 = 5 + 2J$ **b)** $J = 11$
4.a) $33 = 3 + 6h$ **b)** $h = 5$
5.a) $25 = 4 + 7x$ **b)** $x = 3$
6.a) $56 = 24 + 4s$ **b)** $s = 8$
7.a) $72 + 24w = 288$ **b)** $w = 9$; After 9 weeks
8. Problems may vary. For example:
a) Sarah spent $9 at the bowling alley. How many games did she bowl?
b) $9 = 3 + 2g$; $g = 3$
9.a) 17 **b)** 13 **c)** 27

6.5 Using Different Methods to Solve Equations, page 243

1.a) $x = 8$ **b)** $x = 21$ **c)** $x = 64$ **d)** $x = 50$
2. Methods may vary.
a) $x = 7$ **b)** $x = 17$
c) $x = 54$ **d)** $x = -13$
e) $x = 9$ **f)** $x = 7$
g) $x = 7$ **h)** $x = 11$
3.a) $x + 7 = 21$; $x = 14$
4. $\frac{c}{8} = 4$; $c = 32$
6.a) For example: $20 + 8m = 92$; $m = 9$
b) Methods may vary. For example: I used algebra.
7.a) $37 = 5 + 4g$; $g = 8$ **b)** $37 = 10 + 9g$; $g = 3$
8.a) $85 = 40 + 15n$; $n = 3$
b) $140 = 90 + 10n$; $n = 5$
9.b) Answers may vary. For example: $15 + 8 + 12 = 35$ or $25 + 8 + 2 = 35$

Unit 6 Reading and Writing in Math: Decoding Word Problems, page 247

1. One group of 6 rows by 6 columns; 4 groups of 3 rows by 3 columns; 9 groups of 2 rows by 2 columns
2. 144 fence posts
3. 12:21, 1:01, 1:11, 1:21, 1:31, 1:41, 1:51, 2:02, 2:12, 2:22, 2:32, 2:42, 2:52, 3:03, 3:13, 3:23, 3:33, 3:43, 3:53, 4:04, 4:14, 4:24, 4:34, 4:44, 4:54, 5:05, 5:15, 5:25, 5:35, 5:45, 5:55, 6:06, 6:16, 6:26, 6:36, 6:46, 6:56, 7:07, 7:17, 7:27, 7:37, 7:47, 7:57, 8:08, 8:18, 8:28, 8:38, 8:48, 8:58, 9:09, 9:19, 9:29, 9:39, 9:49, 9:59, 10:01, 11:11

Unit 6 Unit Review, page 248

1. $x = 13$; Jan started with 13 stamps.
2.a) $5 + n = 22$; $n = 17$ **b)** $n - 7 = 31$; $n = 38$
c) $6n = 54$; $n = 9$ **d)** $\frac{n}{8} = 9$; $n = 72$
e) $9 + 3n = 24$; $n = 5$
3.a) $m - 36 = 45$; $m = 81$ **b)** $13b = 208$; $b = 16$
c) $\frac{d}{15} = 17$; $d = 255$
4.a) $27 = 15 + x$; $x = 12$
b) $25 = 2x + 11$; $x = 7$
5.a) $x = 6$ cm **b)** $x = 16$ cm
6.a) $81 = 25 + 8c$; $c = 7$
7.a) $x = 3$ **b)** $n = -3$ **c)** $w = 15$ **d)** $x = 15$
8.a) $5 + x = -7$, $y - 5 = 7$
b) $x = -12$, $y = 12$
9. i) a) $-8 + x = 3$ **b)** $x = 11$
ii) a) $3 + y = -1$ **b)** $y = -4$
10.a) $56 = 7n$ **b)** $n = 8$
11.a) $400 = 140 + x$ **b)** $x = 260$
12.a) $228 = 4p$ **b)** $p = 57$
13.a) $x = 19$ **b)** $x = 7$ **c)** $x = 45$ **d)** $x = 8$
14.a) $x = 12$ **b)** $x = -10$ **c)** $x = 3$
d) $x = 7$ **e)** $x = 99$ **f)** $x = 13$
15. $25 = 1 + 3b$; $b = 8$
16.a) $545 = 125 + 12m$ **b)** $m = 35$

Unit 6 Practice Test, page 251

1.a) $x = 2$ **b)** $p = 14$
c) $c = 63$ **d)** $q = 13$
2.a) $44 = 4h$; $h = 11$
b) $50 = 2b + 32$; $b = 9$
3.a) 10 km
b) 48 km
c) 58 km
4.a) $47 = 12 + 5d$; $d = 7$ **b)** $107 = 12 + 5d$; $d = 19$
5.a) $75 + 3 \times 25$ **b)** $204 = 75 + 3s$; $s = 43$

Cumulative Review Units 1–6, page 254

1.a) 1, 2, 3, 4, 5, 6, 8, 10, 12, 15, 20, 24, 30, 40, 60, 120
b) 1, 2, 3, 4, 6, 7, 12, 14, 21, 28, 42, 84
c) 1, 2, 3, 4, 6, 8, 9, 12, 18, 24, 27, 36, 54, 72, 108, 216
2.a) $x + 7 = 19$
b) $x = 12$
3.a) –8 **b)** –10 **c)** +9
4.a) –6 **b)** +12
c) +6 **d)** –12
5. Answers may vary. For example:
a) $1.\overline{6}$ **b)** 0.6

c) 2.2 **d)** 2.75
6. 56.16 m^2
7.a) $71.99
b) $82.07
8.a) Too many to count
b) Too many to count
9.a) About 37.7 cm
10. Greatest area: part b; least area: part c
11.a) 50 m
b) About 7.96 m
c) About 199.06 m^2
12.a) 120 students
b) Black: $\frac{60}{120}=\frac{1}{2}$; brown: $\frac{20}{120}=\frac{1}{6}$;
blonde: $\frac{30}{120}=\frac{1}{4}$; red: $\frac{10}{120}=\frac{1}{12}$
c) Black: 50%; brown: about 17%;
blonde: 25%; red: about 8%
13. $\frac{17}{24}$ cup of sugar
14.a) $\frac{23}{30}$ **b)** $\frac{5}{12}$
c) $\frac{13}{24}$ **d)** $\frac{17}{36}$
15. $\frac{5}{8}$
16.a) $8\frac{11}{12}$ **b)** $\frac{19}{30}$
c) $5\frac{4}{15}$ **d)** $1\frac{5}{24}$
17.a) **i)** $s = 5$
ii) $s = 9$
iii) $s = 9$
iv) $s = 6$
18.a) $x = 6$ **b)** $x = 17$
19.a) $7x + 5 = 250$
b) $x = 35$; Juan worked 35 h.
20.a) $x + 3 = 10$; $x = 7$
Shin's score after Round One was +7.
b) $x – 1 = –4$; $x = –3$
Lucia's score after Round One was –3.

Unit 7 Data Analysis, page 256

7.1 Mean and Mode, page 260

1.a) 4
b) 3
c) 3
2.a) 6
b) 34
3.a) 4
b) no mode
4.a) $13
b) $15
c) The mean is $14.50. The mode remains the same: $15
5.a) Mean: 29.5; mode: 18
b) Answers will vary. For example: 10, 13, 15, 15, 21, 28, 36, 36, 45, 54, 60
6.

	Mean	Mode
a) Games Played	55	no mode
b) Goals	23.25	no mode
c) Assists	29	39
d) Points	52.25	no mode

7.a) Volleyball and soccer
b) I could count the number of bars of equal length.
The length which occurs most often is the mode. Mode: 750 people
c) About 1003
8.a) Any pair of numbers whose sum is 11: 0 and 11, 1 and 10, 2 and 9, 3 and 8, 4 and 7, 5 and 6
b) 3 and 8

7.2 Median and Range, page 264

1.a) Median: 90; range: 20
b) Median: 25.5 kg; range: 73 kg
2.a) Class A: 12.5; Class B: 12
b) Class A: 7; Class B: 4
c) Class A; Class A's median mark is greater.
3.a) **i)** Mean: 7; median: 7; no mode
ii) Mean: 60; median: 60; modes: 50, 70
iii) Mean: 56; median: 68; mode: 71
iv) Mean: 13; median: 13; mode: 13
b) i, ii, and iv; iv; iii
4. Answers may vary. For example:
a) 85, 90, 100, 100, 110, 115
b) 80, 85, 100, 100, 105, 110
5. Answers may vary. For example (in cm):
a) 135, 143, 146, 155, 158, 158, 160, 163, 164, 166
b) 150, 154, 158, 163, 163, 163, 165, 170, 174, 178
6.a) Median: 120 s; mode: 118 s
b) 122 s
c) The mean would be most affected.
The mean increases to 135.7 s.
The mode remains 118 s.
The median increases to 122 s.

7. a)

	Mean	Median	Mode
Games	12.4	12	11
Goals	9.7	6	3
Assists	10.9	10.5	4
Points	20.7	17	10
Penalty Minutes	18.3	8	2 and 8

8. Edward's answer is correct.

7.3 The Effects of Outliers on Average, page 269

1.a) Mean: 4.96 min; median: 5 min; mode: 5 min
b) The outliers are 0, 1, 2.
c) Mean: about 5.8 min; median: 5 min; mode: 5 min
The mean increases. The median and the mode remain the same.
2.a) Mean: 21.35 min; median: 18 min; mode: 15 min
b) The outlier is 95 min.
Explanations may vary.
c) Mean: about 17.47 min; median: 18 min; mode: 15 min
The mean decreases. The median and the mode remain the same.
d) About 18 min; Bryan should use the median time to answer.
3.a) Mean: 34.4; median: 36; mode: 36
b) The outlier is 4.
c) Mean: about 36.6; median: 36; mode: 36
The mean increases. The median and the mode remain the same.
d) No. The outlier is a recording error.
4.a) Mean: about 67.6; median: 68; modes: 65 and 68
b) The outlier is 0.
c) Mean: about 73.7; median: 68; modes: 65 and 68
The mean increases. The median and the modes remain the same.
d) No. The outlier does not represent the data.
5.a) Examples will vary. For example: The outliers should be ignored when reporting pulse rates.
b) Examples will vary. For example: The outliers cannot be ignored when reporting average daily temperatures.
6.a) 460 raisins
b) **i)** Mean: About 454.5; median: 465; no mode
ii) 400 and 499
iii) Mean: About 455.2; median: 465; no mode; The mean increases. The median and the mode remain the same.
iv) Yes. These outliers provide important information.
v) No. The mean is significantly less than 460.
7.a) Mean: 5; median: 5; mode: 5 **b)** 19

7.4 Applications of Averages, page 273

1.a) Mean: About 26.4°C; median: 27°C; modes: 23°C and 28°C
b) The mean best describes the daily high temperature.
c) No. Explanations may vary. For example: The weather channel reported one of the mode temperatures. The mean and the median are significantly higher than 23°C.
2.a) Math: Mean: About 74.6; median: 75; no mode
Music: Mean: About 77.3; median: 81; mode: 81
French: Mean: About 74.4; median: 74; mode: 74
b) The mean is not one of Caitlin's marks. The median is the middle value in each ordered set of marks. The mode represents the mark that occurs most often.
c) Caitlin is best at music because the mean, median, and mode are highest for this subject. Caitlin is worst at French because the mean, median, and mode are lowest for this subject.
3.a) Week 1: Mean: \$825; median: \$800; no mode
Week 2: Mean: \$825; median: \$775; no mode
b) Mean: \$825; median: \$787.50; mode: \$600
c) The means are the same. The medians and modes are different.
d) The median best represents the tips earned.
4.a) Mean: About \$62 667; median: \$50 000; modes: \$50 000 and \$28 000
b) \$102 000
c) **i)** Mean **ii)** The lesser mode
5.a) Yes **b)** No
6.a) Mode **b)** Mean **c)** Median
7.a) Mean: About 395.3 g; median: 395 g; mode: 405 g
b) 25 g **c)** Mode
8. a) **i)** 85% **ii)** 90% **iii)** 95%

b) No, Andrew cannot get a mean mark of 84% or higher because he would need a math mark greater than 100%.

9. No, Celia's reasoning is not correct. Her mean mark is 83.5%.

Technology: Using Spreadsheets to Investigate Averages, page 277

1.a) Mean: About \$15.68; median: \$15; mode: \$9

2.a) Mean: About \$51.23; median: \$47.19; mode: \$34.45

3. Mean: 110.9; median: 113; no mode

Unit 7 Mid-Unit Review, page 278

1.a) Mean: 165 cm; median: 166 cm; mode: 170 cm

b) 20 cm

2. Answers may vary. For example: 13, 15, 23, 24, 25; 5, 17, 23, 25, 30

3.a) Mean: About \$82.13; median: \$75; mode: \$75

b) The outlier, \$20, may be a recording error. The outlier, \$229, may be the rate charged for a luxury suite.

c) Mean: About \$76.07; median: \$75; mode: \$75
The mean decreases. The median and the mode remain the same.

d) The outlier, \$20, is a recording error and should not be used. The outlier, \$229, is an actual rate and should be used.

4.a) Mean: About 99.8 g; median: About 99.8 g; mode: 100.3 g

b) Mode

5.b) False

7.5 Different Ways to Express Probability, page 282

1.a) $\frac{1}{3}$, or about 33.3%, or 1:3

b) 0, or 0%

c) $\frac{2}{16}$, or $\frac{1}{8}$, or 12.5%, or 1:8

b) 1, or $\frac{100}{100}$, or 100%, or 1:1

2.a) $\frac{14}{54}$, or $\frac{7}{27}$, or about 26%, or 7:27

b) $\frac{12}{54}$, or $\frac{2}{9}$, or about 22%, or 2:9

3.a) $\frac{1}{250}$, or 0.4%, or 1:250

b) $\frac{10}{250}$, or $\frac{1}{25}$, or 4%, or 1:25

c) $\frac{225}{250}$, or $\frac{9}{10}$, or 90%, or 9:10

4.a) $\frac{5}{20}$, or $\frac{1}{4}$, or 25%, or 1:4

b) $\frac{11}{20}$, or 55%, or 11:20

c) 1, or 100%, or 1:1

d) 0, or 0%, or 0:20

e) $\frac{1}{20}$, or 5%, or 1:20

5.a) $\frac{1}{8}$, or 12.5%, or 1:8

b) $\frac{7}{8}$, or 87.5%, or 7:8

c) $\frac{4}{8}$, or $\frac{1}{2}$, or 50%, or 1:2

d) $\frac{4}{8}$, or $\frac{1}{2}$, or 50%, or 1:2

e) 0, or 0%, or 0:8 **f)** 1, 100%, 1:1

6. Answers may vary. For example:
You roll a die.

a) The probability of getting a number less than 10

b) The probability of getting an even number

c) The probability of getting a 4

d) The probability of getting a 7

7. I divided the spinner into 10 equal sectors: 2 red, 5 yellow, 1 blue, and 2 green

8.a) The third candy is most likely white.

b) $\frac{3}{7}$, or about 43%, or 3:7

c) $\frac{4}{7}$, or about 57%, or 4:7

7.6 Tree Diagrams, page 287

1.a) 3H, 3T, 4H, 4T, 5H, 5T, 6H, 6T, 7H, 7T, 8H, 8T
The outcome of rolling a die does not depend on the outcome of tossing a coin.

b) 1B, 1Y, 1P, 2B, 2Y, 2P, 3B, 3Y, 3P, 4B, 4Y, 4P
The outcome of rolling a tetrahedron does not depend on the outcome of spinning the pointer on a spinner.

c) 1, 1; 1, 2; 1, 3; 1, 4; 1, 5; 1, 6; 2, 1; 2, 2; 2, 3; 2, 4; 2, 5; 2, 6; 3, 1; 3, 2; 3, 3; 3, 4; 3, 5; 3, 6; 4, 1; 4, 2; 4, 3; 4, 4; 4, 5; 4, 6; 5, 1; 5, 2; 5, 3; 5, 4; 5, 5; 5, 6; 6, 1; 6, 2; 6, 3; 6, 4; 6, 5; 6, 6
The outcome of rolling one die does not depend on the outcome of rolling the other die.

2. Aseea; $\frac{3}{4}$ is greater than $\frac{1}{3}$.

3. Answers may vary. For example:
The probability of rolling an even number
4. The probability of rolling both numbers greater than 4 is: $\frac{4}{36}$, or $\frac{1}{9}$
5. a)

Seat Colour / Paint Colour

	Black	Blue	Red	Silver	Gold
Grey	Gr, Bla	Gr, Blu	Gr, R	Gr, S	Gr, G
Black	Bla, Bla	Bla, Blu	Bla, R	Bla, S	Bla, Go

b) $\frac{2}{10}$, or $\frac{1}{5}$, or 20%
6. The player should choose to roll the tetrahedron twice to have the greatest probability of winning.

Unit 7 Unit Review, page 292

1.a) Under par: 10; at par: 2; over par: 7
b) 26
c) Mean: About 34.3; median: 35; mode: 33
2. Answers will vary.
For example: 4, 5, $6\frac{1}{2}$, $6\frac{1}{2}$, 7, $7\frac{1}{2}$, 8,
or 4, 5, 5, 6, 7, 8, 8, 9
3.a) Mean: 12.6 h; median: 13.5 h; mode: 15 h
b) 3 h
c) Mean: About 13.7 h; median: 15 h; mode: 15 h
The mean and median decrease. The mode remains the same.
d) No. The outlier is not typical of the number of hours Josephine works in a week.
4.a) Mean: About 46.3 min; median: 40.5 min; mode: 47 min
b) 8 min, 74 min, 125 min
Mean: About 40.1 min; the mean decreases. So, it is greatly affected by the outliers.
c) Median
d) Yes, the outliers are actual times spent by students on math homework.
5.a) Mean: 122 s; median: 119.5 s; mode: 118 s
b) Median c) 19 s
d) Annette must get a time greater than or equal to 120 s in her next run.
e) 113 s; unlikely
6.a) Mode b) Median c) Mean d) Median
7.a) $\frac{10}{20}$, or $\frac{1}{2}$, or 50%, or 1:2
b) $\frac{5}{20}$, or $\frac{1}{4}$, or 25%, or 1:4
c) $\frac{8}{20}$, or $\frac{2}{5}$, or 40%, or 2:5
d) 0, or 0%, or 0:20 e) 1, or 100%, or 1:1
8.a) 2, 3, 4, 6, 8, 9, 12
b) The probability of getting a product of 2, 3, 8, 9, and 12: $\frac{1}{9}$
The probability of getting a product of 4 and 6: $\frac{2}{9}$
c) 2, 3, 8, 9, and 12; 4 and 6
d) $\frac{8}{9}$
9.b) i) $\frac{1}{3}$ ii) $\frac{1}{3}$
iii) $\frac{1}{9}$ iv) $\frac{1}{9}$
11. No, each player has a 50% probability of winning and each prize has a greater value than the cost.

Unit 7 Practice Test, page 295

1.a) 243.25 s b) 208 s
c) 158 s d) 237.5 s
2.a) Mean: about 7.8; median: 7.25; mode: 7
b) 18
c) Mean: about 7.3; median: 7; mode: 7
The mean and the median decrease.
The mode remains the same.
d) No. The outlier is a recording error.
3.a) ii) b) i c) iv d) iii

Unit 8 Geometry, page 298

8.1 Parallel Lines, page 302

1. Parts a and c
4. Answers may vary. For example:
Use tracing paper.
5. Answers may vary. For example:
Shelves on a bookshelf
6. JE and AB, CL and BK, BE and AF, BF and GK, AF and GK

8.2 Perpendicular Lines, page 305

1. Parts a and b
4. Answers may vary. For example: Book covers, desks, floor, ceiling
5. AE and FR, BR and KL, AE and AC, AC and BL, FH and GJ, ED and DL, FR and RB

8.3 Constructing Perpendicular Bisectors, page 308

1.b) The distance from C and the distance from D to any point on the perpendicular bisector are the same.

2.b) Any point on the perpendicular bisector is the same distance from E as from F.

4.b) The distances from A and from B to the point on the perpendicular bisector are equal.

5.a) Circles intersect only once, at the midpoint of the line segment.

b) Circles do not intersect.

7. Answers may vary. For example: Ceiling or floor tiles

9.a) Connect the points to form a triangle; draw the perpendicular bisector of each side. The point where the bisectors meet is the centre of the circle through the points.

b) Repeat the construction in part a.

8.4 Constructing Angle Bisectors, page 312

1. Yes

2. Yes

3.a) The two angles formed by the bisector will measure 25°.

b) The two angles formed by the bisector will measure 65°.

4. Methods may vary. For example: Use a Mira; use a plastic right triangle; use paper folding.

5. Answers may vary. For example: A ruler and a compass allow for a more accurate construction.

6.c) Two; Opposite angles have the same bisector.

7. c) i) Yes **ii)** Yes **iii)** No

8. Answers may vary. For example: Frame of a kite

9.a) The two angles are equal.

b) The centre of the circle is at the intersection of the folded creases.

c) The folding constructed angle bisectors.

Unit 8 Mid-Unit Review, page 314

2.a) AH and CE and FL and GN, AC and HE, FH and EN

b) EH and FL, AC and CE, CE and EH, AH and HE, AH and AC, GN and EH

3.c) Angle measures should be equal.

4.c) Isosceles triangle; AD = BD; CD bisects $\angle$ADB.

5.c) Angle measures should be equal.

8.5 Graphing on a Coordinate Grid, page 318

1. Each grid square represents 5 units. A(10, 15); B(0, 25); C(5, –10); D(–30, 0); E(0, –25); F(0, 0); G(–5, –5); H(–25, 15); J(20, 0); K(–25, –30).

2.a) B, E, and F

b) D, F, and J

c) B, E, and F; H and K

d) D, F, and J; A and H

e) F and G

f) none

3. Answers may vary. For example: Each grid square represents 5 units. O is the origin.

5. Quadrant 3; Quadrant 1; Quadrants 2 and 4

6.c) 16-sided shape with 4 lines of symmetry that intersect at (0, 2). The vertical line of symmetry coincides with the *y*-axis.

8.a) 8 cm

b) 11 cm

10. Too many to count. For example: A(0,0), B(4, 0), C(5, 3), D(1, 3)

11.b) N(–15, –10)

12.a) Answers may vary. For example: Each grid square represents 2 units.

b) 442 units^2

13. Answers may vary. For example: C(2, 10) and D(–4, 10); C(2, –2) and D(–4, –2); C(–1, 7) and D(–1, 1)

8.6 Graphing Translations and Reflections, page 322

1.a) Reflection

b) Translation

2.a) 3 units left and 9 units up

b) 2 units left and 3 units down

c) 2 units right and 4 units up

d) 3 units left and 2 units down

e) 6 units left

f) 4 units up

3.a) A and C; C is the image of A after a translation 10 units right and 7 units down.

b) B and C; C is the image of B after a reflection in the *x*-axis.

4. P(2, 3), Q(–2, 2), R(1, –1), S(–1, –3), T(4, –4)

a) P'(–1, 5), Q'(–5, 4), R'(–2, 1), S'(–4, –1), T'(1, –2); the pentagons have the same orientation.

b) P'(2, –3), Q'(–2, –2), R'(1, 1), S'(–1, 3), T'(4, 4); the pentagons have different orientations.

c) P'(–2, 3), Q'(2, 2), R'(–1, –1), S'(1, –3), T'(–4, –4); the pentagons have different orientations.

5.a) A'(1, –3), B'(3, 2), C'(–2, –5), D'(–1, 4), E'(0, 3), F(–2, 0); the sign of each *y*-coordinate changes.

b) A'(–1, 3), B'(–3, –2), C'(2, 5), D'(1, –4), E'(0, –3), F(2, 0); the sign of each *x*-coordinate changes.

c) The coordinates of the image should match the patterns in parts a and b.

6.b) A(1, 3), B(3, –2), C(–2, 5), D(–1, –4), E(0, –3), F(–2, 0); A'(–3, 1), B'(–1, –4), C'(–6, 3), D'(–5, –6), E'(–4, –5), F'(–6, –2); Each *x*-coordinate decreases by 4. Each *y*-coordinate decreases by 2.

c) Use the pattern in part b: add the number of units moved to the right or subtract the number of units moved to the left from the *x*-coordinate. Add the number of units moved up or subtract the number of units moved down from the *y*-coordinate.

7.b) The line segments are horizontal. The *y*-axis is the perpendicular bisector of each line segment.

8.b) A'(6, 10), B'(8, 10), C'(8, 8), D'(10, 8), E'(10, 12)

c) A"(–6, 10), B"(–8, 10), C"(–8, 8), D"(–10, 8), E"(–10, 12)

d) Answers may vary. For example: ABCDE and A"B"C"D"E" are congruent, but have different orientations.

9.e) Translation 12 units right and 6 units down

10. Answers may vary. For example: The shape has a line of symmetry that is parallel to the mirror line.

8.7 Graphing Rotations, page 327

1.a) 90° clockwise about the origin or 270° counterclockwise about the origin

b) 180° about the origin

2. The shape was rotated 90° clockwise about the origin (Image 1), reflected in the *x*-axis (Image 2), translated 5 units right and 5 units down (Image 3).

3.a) D(–2, –1), E(–5, –3), F(–1, –5)

b) D'(–1, 2), E'(–3, 5), F'(–5, 1)

c) D"(–1, 2), E"(–3, 5), F"(–5, 1)

d) Yes. The images in parts b and c are the same.

4.a) A'(–2, –5), B'(3, –4), C'(–4, 1)

b) **i)** OA = OA'
ii) OB = OB'
iii) OC = OC'

c) **i)** 180° **ii)** 180° **iii)** 180°
All angles measure 180°.

d) A rotation of –180° about the origin

5.a) A'(5, –2), B'(4, 3), C'(–1, –4)

b) **i)** OA = OA' **ii)** OB = OB'
iii) OC = OC'

c) **i)** 90° **ii)** 90° **iii)** 90°
All angles measure 90°.

d) A rotation of 270° about the origin

6.a) A(6, 0), B(6, 2), C(5, 3), D(4, 2), E(2, 2), F(2, 0)

b) A'(0, 2), B'(0, 4), C'(–1, 5), D'(–2, 4), E'(–4, 4), F'(–4, 2)

c) A"(–2, 0), B"(–4, 0), C"(–5, –1), D"(–4, –2), E"(–4, –4), F"(–2, –4)

d) Answers may vary. For example: ABCDEF and A"B"C"D"E"F" are congruent and have the same orientation.

7.c) The images coincide. A rotation of 180° is equivalent to a reflection in one axis followed by a reflection in the other axis.
i) Yes
ii) Yes

8. Answers may vary. For example:

b) Rotation about U: R'(2, –4), S'(–3, –4), T'(–3, –1), U(2, –1)

c) Second rotation about U: R"(5, –1), S"(5, –6), T"(2, –6), U(2, –1)
Third rotation about U: R'"(2, 2), S'"(7, 2), T'"(7, –1), U(2, –1)

d) After each 90° rotation counterclockwise about a vertex, the horizontal sides of rectangle RSTU become vertical and the vertical sides become horizontal.

e) Yes. A 90° rotation clockwise about U

9.a) C'(2, –6), D'(3, 3), E'(5, 7); C'(–6, –2), D'(3, –3), E'(7, –5)

b) P'(6, –2), Q'(–3, –3), R'(–7, –5); P'(6, 2), Q'(–3, 3), R'(–7, 5)

c) No

Unit 8 Unit Review, page 335

2.c) The height of ΔCDE

5.a) Scales may vary. For example: Each grid square represents 5 units.

b) A: Quadrant 3, B: Quadrant 4, C: Quadrant 1, D: Quadrant 2

c) Parallelogram; Area = 2500 units^2

6.a) Quadrant 4

b) Quadrant 3

c) Quadrant 2

d) Quadrant 1

7.a) **i)** 12 units **ii)** 11 units
b) **i)** 8 units **ii)** 6 units
8. (–1, 1) and (3, –1)
9.a) PQRS has only one pair of parallel sides.
b) P'(7, 1), Q'(11, 1), R'(9, 3), S'(7, 3)
c) P"(7, –1), Q"(11, –1), R"(9, –3), S"(7, –3)
d) PQRS and P"Q"R"S" are congruent, but have different orientations.
10.b) P'(3, –1), Q'(7, –1), R'(5, –3), S'(3, –3)
c) P"(7, –1), Q"(11, –1), R"(9, –3), S"(7, –3)
Yes, the image remains the same when the translation and rotation are reversed.
11.c) All the images are congruent.
Under the translation and rotation, the images have the same orientation as quadrilateral ABCD. Under the reflection, the orientation of the image is changed.
12.a) A would be in Quadrant 4, B would be on the negative *x*-axis, between Quadrants 2 and 3, C would be in Quadrant 2.
b) Reflection
c) A 90° or 270° (–90°) rotation
13.b) C'(1, 1), D'(–9, 7), E'(1, 7)
c) C"(–1, 1), D"(–7, –9), E"(–7, 1)
d) ABC and A"B"C" are congruent; they have the same orientation.

Unit 8 Practice Test, page 337

4.b) A'(–4, –3), B'(2, –3), C'(1, 1), D'(–3, 0)
c) A'(2, 4), B'(8, 4), C'(7, 8), D'(3, 7)
d) A translation 4 units right and 4 units up
e) The image remains the same.

Cumulative Review Units 1–8, page 342

1.a) $4n + 2$
c) The graph goes up to the right.
When the Input number increases by 1, the Output number increases by 4.
2.a) $145; $185
b) $85 + 2s$
c) $85 + 4s$
d) $170 + 2s$
3.a) **i)** $(+4) + (-5) = -1$
ii) $(+1) + (-7) = -6$
4.a) High: –4°C; low: –13°C
b) +9°C or –9°C
5.a) About 9
b) About 3
c) About 35
d) About 249
6.a) $28.89 **b)** Yes; Justin spent $3.89 more.
7.a) 75%, 0.75
b) 28%, 0.28
c) 90%, 0.9
d) 4%, 0.04
8. 20 cm; I assume the medium-sized circles touch the large circle and each other.
9.a) About 58 cm
b) About 182.21 cm
c) About 182 cm
d) About 5 rotations
10.a) 8.64 cm^2
b) 10.125 cm^2
11.a) $\frac{8}{10} = \frac{4}{5}$
b) $\frac{5}{12}$
c) $\frac{9}{8} = 1\frac{1}{8}$
d) $\frac{13}{12} = 1\frac{1}{12}$
12.a) About 2 cups more
b) $\frac{43}{24} = 1\frac{19}{24}$ cups
13.a) **i)** $x - 1 = -2$
ii) $x + 1 = -3$
b) **i)** $x = -1$
ii) $x = -4$
14.a) $9x = 63$; $x = 7$; $7
b) $x - 27 = 61$; $x = 88$; 88 lures
15.a) $171 000
b) The mean prize is greater than the median: About 179 571
c) 79 000
16.a) Mean = 34; median = 33.5; mode = 30
b) **i)** Mean = 44; median = 43.5; mode = 40
The mean, median, and mode increase by 10.
ii) Mean = 68; median = 67; mode = 60
The mean, median, and mode double.
17.a) Mean $\doteq 308.4$; median = 305; mode = 305
b) Outlier: 395
Mean $\doteq 304.3$; median = 305; mode = 305
The mean decreases. The median and the mode remain the same.
18.a) Mean = $8.30, median $\doteq$ $7.88; mode = $7.75
b) Mean
c) Outliers: $10.00 and $12.50
Mean $\doteq$ $7.97; median = $7.75; mode = $7.75
The mean and the median decrease.
The mode remains the same.
19. False

20.a) $\frac{1}{6}, 0.1\overline{6}$, about 16%

b) $\frac{100}{100}$, 1, 100%

c) 0, 0%

21.a) There are 48 possible outcomes: 1, 1; 1, 2; 1, 3; 1, 4; 1, 5; 1, 6; 2, 1; 2, 2; 2, 3; 2, 4; 2, 5; 2, 6; 3, 1; 3, 2; 3, 3; 3, 4; 3, 5; 3, 6; 4, 1; 4, 2; 4, 3; 4, 4; 4, 5; 4, 6; 5, 1; 5, 2; 5, 3; 5, 4; 5, 5; 5, 6; 6, 1; 6, 2; 6, 3; 6, 4; 6, 5; 6, 6; 7, 1; 7, 2; 7, 3; 7, 4; 7, 5; 7, 6; 8, 1; 8, 2; 8, 3; 8, 4; 8, 5; 8, 6

b) The outcome of rolling an octahedron does not depend on the outcome of rolling a die.

c) $\frac{4}{48} = \frac{1}{12}$, or $0.08\overline{3}$, or about 8.3%

24. Answers may vary. For example: If both coordinates are positive, the point is in Quadrant 1. If the *x*-coordinate is negative and the *y*-coordinate is positive, the point is in Quadrant 2. If both coordinates are negative, the point is in Quadrant 3. If the *x*-coordinate is positive and the *y*-coordinate is negative, the point is in Quadrant 4.
If the *x*-coordinate is 0, the point is on the *y*-axis. If the *y*-coordinate is 0, the point is on the *x*-axis.

25.a) Each grid square represents 5 units.

d) H

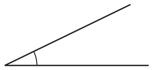
26.b) C′(–3, 9), D′ (1, 9), E′ (1, 3)

c) C″ (–3, –9), D″ (1, –9), E″ (1, –3)

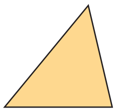
d) C‴(9, –3), D‴ (9, 1), E‴ (3, 1)

Illustrated Glossary

acute angle: an angle measuring less than 90°



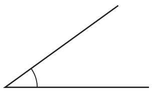
acute triangle: a triangle with three acute angles



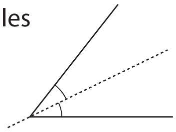
algebra tiles: a collective term for unit tiles and variable tiles

algebraic expression: a mathematical expression containing a variable: for example, $6x - 4$ is an algebraic expression

angle: formed by two rays from the same endpoint



angle bisector: the line that divides an angle into two equal angles



approximate: a number close to the exact value of an expression; the symbol $\doteq$ means "is approximately equal to"

area: the number of square units needed to cover a region

array: an arrangement in rows and columns

average: a single number that represents a set of numbers (see *mean, median,* and *mode*)

bar graph: a graph that displays data by using horizontal or vertical bars

bar notation: the use of a horizontal bar over a decimal digit to indicate that it repeats; for example, $1.\bar{3}$ means 1.333 333 …

base: the side of a polygon or the face of an object from which the height is measured

bisector: a line that divides a line segment or an angle into two equal parts

capacity: the amount a container can hold

Cartesian Plane: another name for a coordinate grid (see *coordinate grid*)

central angle: the angle between the two radii that form a sector of a circle

certain event: an event with probability 1, or 100%

chance: a description of a probability expressed as a percent

circle graph: a diagram that uses parts of a circle to display data

circumcentre: the point where the perpendicular bisectors of the sides of a triangle intersect (see *circumcircle*)

circumcircle: a circle drawn through all vertices of a triangle and with its centre at the circumcentre of the triangle

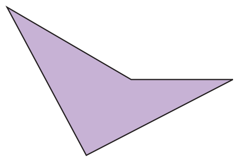
circumference: the distance around a circle, also known as the perimeter of the circle

common denominator: a number that is a multiple of each of the given denominators; for example, 12 is a common denominator for the fractions $\frac{1}{3}, \frac{5}{4}, \frac{7}{12}$

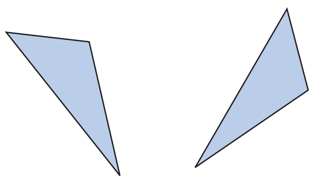
common factor: a number that is a factor of each of the given numbers; for example, 3 is a common factor of 15, 9, and 21

composite number: a number with three or more factors; for example, 8 is a composite number because its factors are 1, 2, 4, and 8

concave polygon: has at least one angle greater than 180°



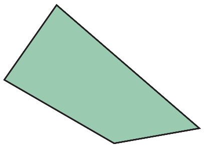
congruent: shapes that match exactly, but do not necessarily have the same orientation



consecutive numbers: integers that come one after the other without any integers missing; for example, 34, 35, 36 are consecutive numbers, so are −2, −1, 0, and 1

constant term: the number in an expression or equation that does not change; for example, in the expression $4x + 3$, 3 is the constant term

convex polygon: has all angles less than 180°

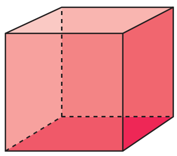


coordinate axes: the horizontal and vertical axes on a grid

coordinate grid: a two-dimensional surface on which a coordinate system has been set up

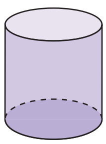
coordinates: the numbers in an ordered pair that locate a point on the grid (see *ordered pair*)

cube: an object with six congruent square faces



cubic units: units that measure volume

cylinder: an object with two parallel, congruent, circular bases

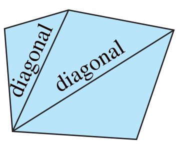


data: facts or information

database: an organized collection of facts or information, often stored on a computer

denominator: the term below the line in a fraction

diagonal: a line segment that joins two vertices of a shape, but is not a side



diameter: the distance across a circle, measured through its centre

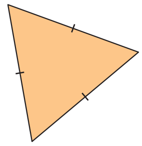
digit: any of the symbols used to write numerals; for example, in the base-ten system the digits are 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9

dimensions: measurements, such as length, width, and height

discount: the amount by which a price is reduced

equation: a mathematical statement that two expressions are equal

equilateral triangle: a triangle with three equal sides



equivalent: having the same value; for example, $\frac{2}{3}$ and $\frac{6}{9}$ are equivalent fractions; 2:3 and 6:9 are equivalent ratios

estimate: a reasoned guess that is close to the actual value, without calculating it exactly

evaluate: to substitute a value for each variable in an expression

even number: a number that has 2 as a factor; for example, 2, 4, 6

event: any set of outcomes of an experiment

experimental probability: the probability of an event calculated from experimental results

expression: a mathematical phrase made up of numbers and/or variables connected by operations

factor: to factor means to write as a product; for example, $20 = 2 \times 2 \times 5$

formula: a rule that is expressed as an equation

fraction: an indicated quotient of two quantities

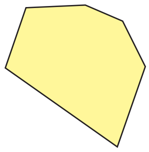
fraction strips: strips of paper used to model fractions

frequency: the number of times a particular number occurs in a set of data

greatest common factor (GCF): the greatest number that divides into each number in a set; for example, 5 is the greatest common factor of 10 and 15

height: the perpendicular distance from the base of a shape to the opposite side or vertex; the perpendicular distance from the base of an object to the opposite face or vertex

hexagon: a six-sided polygon



horizontal axis: the horizontal number line on a coordinate grid

image: the shape that results from a transformation

impossible event: an event that will never occur; an event with probability 0, or 0%

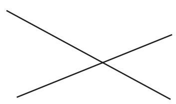
improper fraction: a fraction with the numerator greater than the denominator; for example, both $\frac{6}{5}$ and $\frac{5}{3}$ are improper fractions

independent events: two events in which the result of one event does not depend on the result of the other event

inspection: solving an equation by finding the value of the variable by using addition, subtraction, multiplication, and division facts

integers: the set of numbers
$\ldots -3, -2, -1, 0, +1, +2, +3, \ldots$

intersecting lines: lines that meet or cross; lines that have one point in common



inverse operation: an operation that reverses the result of another operation; for example, subtraction is the inverse of addition, and division is the inverse of multiplication

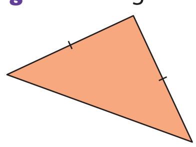
irrational number: a number that cannot be represented as a terminating or repeating decimal; for example, π

isosceles acute triangle: a triangle with two equal sides and all angles less than 90°

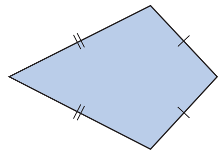
isosceles obtuse triangle: a triangle with two equal sides and one angle greater than 90°

isosceles right triangle: a triangle with two equal sides and a 90° angle

isosceles triangle: a triangle with two equal sides



kite: a quadrilateral with two pairs of equal adjacent sides



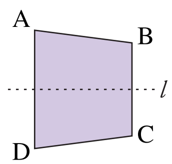
legend: part of a circle graph that shows what category each sector represents

linear relation: a relation whose points lie on a straight line

line graph: a graph that displays data by using points joined by line segments

line segment: the part of a line between two points on the line

line symmetry: a shape has line symmetry when it can be divided into 2 congruent parts, so that one part concides with the other part when the shape is folded at the line of symmetry; for example, line l is the line of symmetry for shape ABCD



lowest common multiple (LCM): the lowest multiple that is the same for two numbers; for example, the lowest common multiple of 12 and 21 is 84

magic square: an array of numbers in which the sum of the numbers in any row, column, or diagonal is always the same

magic sum: the sum of the numbers in a row, column, or diagonal of a magic square

mass: the amount of matter in an object

mean: the sum of a set of numbers divided by the number of numbers in the set

measure of central tendency: a single number that represents a set of numbers (see *mean, median,* and *mode*)

median: the middle number when data are arranged in numerical order; if there is an even number of data, the median is the mean of the two middle numbers

midpoint: the point that divides a line segment into two equal parts

mixed number: a number consisting of a whole number and a fraction; for example, $1\frac{1}{18}$ is a mixed number

mode: the number that occurs most often in a set of numbers

multiple: the product of a given number and a natural number; for example, some multiples of 8 are 8, 16, 24, …

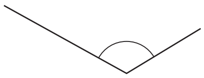
natural numbers: the set of numbers 1, 2, 3, 4, 5, …

negative number: a number less than 0

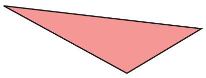
numerator: the term above the line in a fraction

numerical coefficient: the number by which a variable is multiplied; for example, in the expression $4x + 3$, 4 is the numerical coefficient

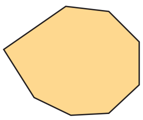
obtuse angle: an angle greater than 90° and less than 180°



obtuse triangle: a triangle with one angle greater than 90°



octagon: an eight-sided polygon



odd number: a number that does not have 2 as a factor; for example, 1, 3, 7

operation: a mathematical process or action such as addition, subtraction, multiplication, or division

opposite integers: two integers with a sum of 0; for example, $+3$ and -3 are opposite integers

ordered pair: two numbers in order, for example, (2, 4); on a coordinate grid, the first number is the horizontal coordinate of a point, and the second number is the vertical coordinate of the point

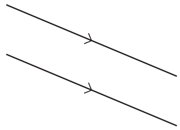
order of operations: the rules that are followed when simplifying or evaluating an expression

origin: the point where the *x*-axis and the *y*-axis intersect

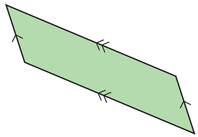
outcome: a possible result of an experiment or a possible answer to a survey question

outlier: a number in a set that is significantly different from the other numbers

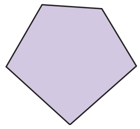
parallel lines: lines on the same flat surface that do not intersect



parallelogram: a quadrilateral with both pairs of opposite sides parallel



pentagon: a five-sided polygon



percent: the number of parts per 100; the numerator of a fraction with denominator 100

percent circle: a circle divided into 10 congruent sectors, with each sector further divided into 10 parts; each part is 1% of the circle

perimeter: the distance around a closed shape

perpendicular bisector: the line that is perpendicular to a line segment and divides the line segment into two equal parts

perpendicular lines: intersect at 90°

polygon: a closed shape that consists of line segments; for example, triangles and quadrilaterals are polygons

polyhedron (*plural*, **polyhedra**)**:** an object with faces that are polygons

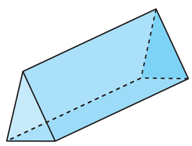
population: the set of all things or people being considered

positive number: a number greater than 0

prediction: a statement of what you think will happen

prime number: a whole number with exactly two factors, itself and 1; for example, 2, 3, 5, 7, 11, 29, 31, and 43

prism: an object that has two congruent and parallel faces (the *bases*), and other faces that are parallelograms

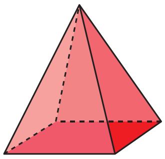


probability: the likelihood of a particular outcome; the number of times a particular outcome occurs, written as a fraction of the total number of outcomes

product: the result when two or more numbers are multiplied

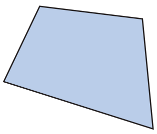
proper fraction: a fraction with the numerator less than the denominator; for example, $\frac{5}{6}$

pyramid: an object that has one face that is a polygon (the *base*), and other faces that are triangles with a common vertex



quadrant: one of four regions into which coordinate axes divide a plane

quadrilateral: a four-sided polygon



quotient: the result when one number is divided by another

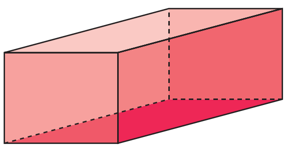
radius (*plural,* **radii**)**:** the distance from the centre of a circle to any point on the circle

range: the difference between the greatest and least numbers in a set of data

ratio: a comparison of two or more quantities with the same unit

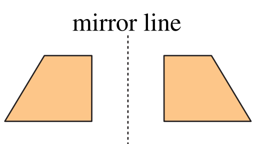
rectangle: a quadrilateral that has four right angles

rectangular prism: a prism that has rectangular faces

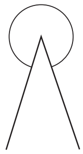


rectangular pyramid: a pyramid with a rectangular base

reflection: a transformation that is illustrated by a shape and its image in a mirror line



reflex angle: an angle between 180° and 360°



regular hexagon: a polygon that has six equal sides and six equal angles

regular octagon: a polygon that has eight equal sides and eight equal angles

regular polygon: a polygon that has all sides equal and all angles equal

related denominators: two fractions where the denominator of one fraction is a factor of the other; their lowest common denominator is the greater of the two denominators

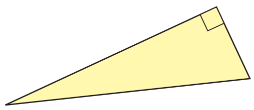
relation: a variable compared to an expression that contains the variable

repeating decimal: a decimal with a repeating pattern in the digits that follow the decimal point; it is written with a bar above the repeating digits; for example, $\frac{1}{11} = 0.\overline{09}$

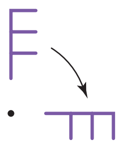
rhombus: a parallelogram with four equal sides

right angle: a 90° angle

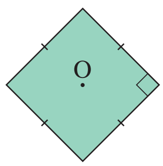
right triangle: a triangle that has one right angle



rotation: a transformation in which a shape is turned about a fixed point



rotational symmetry: a shape that coincides with itself in less than one full turn about its centre is said to have rotational symmetry; for example, a square has rotational symmetry



sample/sampling: a representative portion of a population

sample space: a list of all possible outcomes for an experiment that has independent events

scale: the numbers on the axes of a graph

scalene triangle: a triangle with all sides different

sector: part of a circle between two radii and the included arc

sector angle: see *central angle*

simplest form: a ratio with terms that have no common factors, other than 1; a fraction with numerator and denominator that have no common factors, other than 1

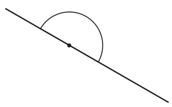
spreadsheet: a computer-generated arrangement of data in rows and columns, where a change in one value results in appropriate calculated changes in the other values

square: a rectangle with four equal sides

square number: the product of a number multiplied by itself; for example, 25 is the square of 5

statistics: the branch of mathematics that deals with the collection, organization, and interpretation of data

straight angle: an angle measuring 180°



surface area: the total area of the surface of an object

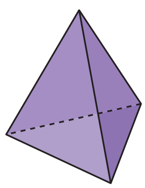
symmetrical: possessing symmetry (see *line symmetry* and *rotational symmetry*)

systematic trial: solving an equation by choosing a value for the variable, then checking by substituting

term: (of a fraction) the numerator or the denominator of the fraction

terminating decimal: a decimal with a certain number of digits after the decimal point; for example, $\frac{1}{8} = 0.125$

tetrahedron: an object with four triangular faces; a triangular pyramid

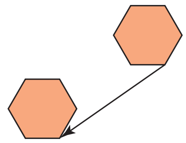


theoretical probability: the number of favourable outcomes written as a fraction of the total number of possible outcomes

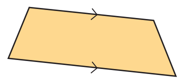
three-dimensional: having length, width, and depth or height

transformation: a translation, rotation, or reflection

translation: a transformation that moves a point or a shape in a straight line to another position on the same flat surface



trapezoid: a quadrilateral that has at least one pair of parallel sides



tree diagram: a diagram that resembles the roots or branches of a tree, used to count outcomes

triangle: a three-sided polygon

two-dimensional: having length and width, but no thickness, height, or depth

unit fraction: a fraction that has a numerator of 1

unit price: the price of one item, or the price of a particular mass or volume of an item

unit tile: a tile that represents $+1$ or -1

unrelated denominators: two fractions where the denominators have no common factors; their lowest common denominator is the product of the two denominators

variable: a letter or symbol representing a quantity that can vary

variable tile: a tile that represents a variable

vertex (*plural,* vertices): the corner of a shape or object

vertical axis: the vertical number line on a coordinate grid

volume: the amount of space occupied by an object

whole numbers: the set of numbers 0, 1, 2, 3, …

***x*-axis:** the horizontal number line on a coordinate grid

***y*-axis:** the vertical number line on a coordinate grid

zero pair: two opposite numbers whose sum is equal to zero

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